How to Track Qubits Through Space and Time

Stronger guarantees for position-based cryptography, and how to achieve them

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Columbia University

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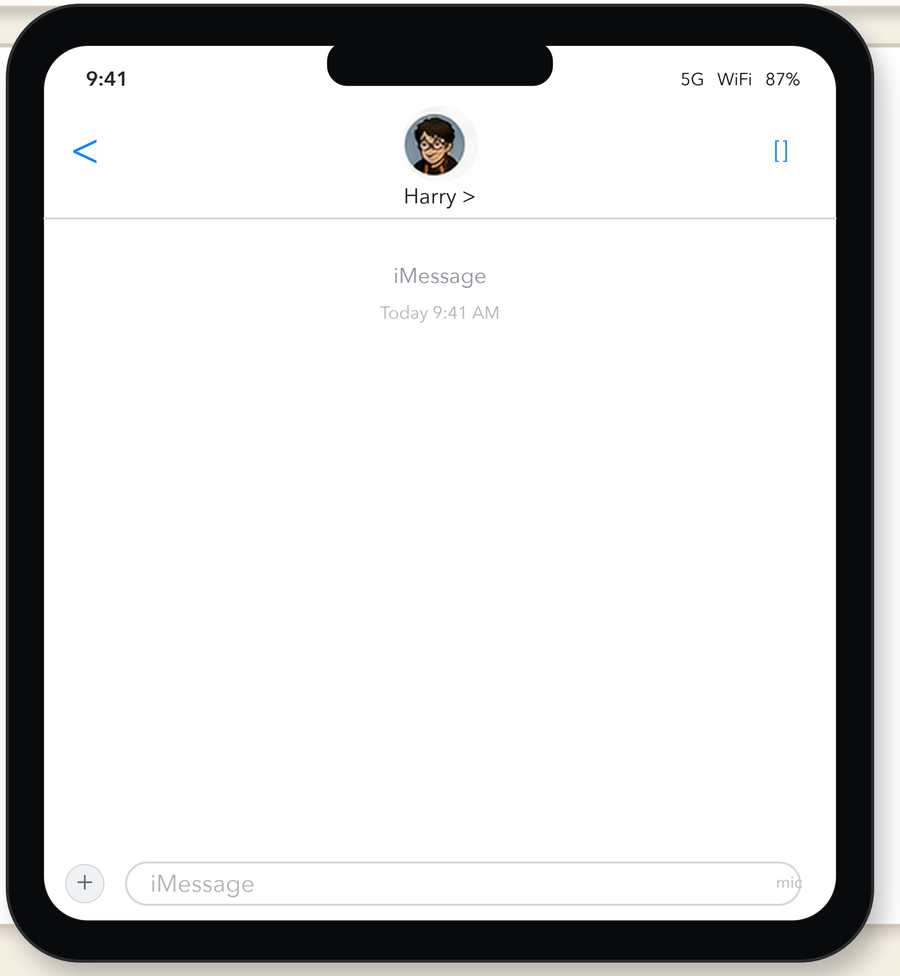
Scenario: A Friend Needs Help

> When position-based cryptography comes in handy.



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Can we authenticate a message by its positional origin?

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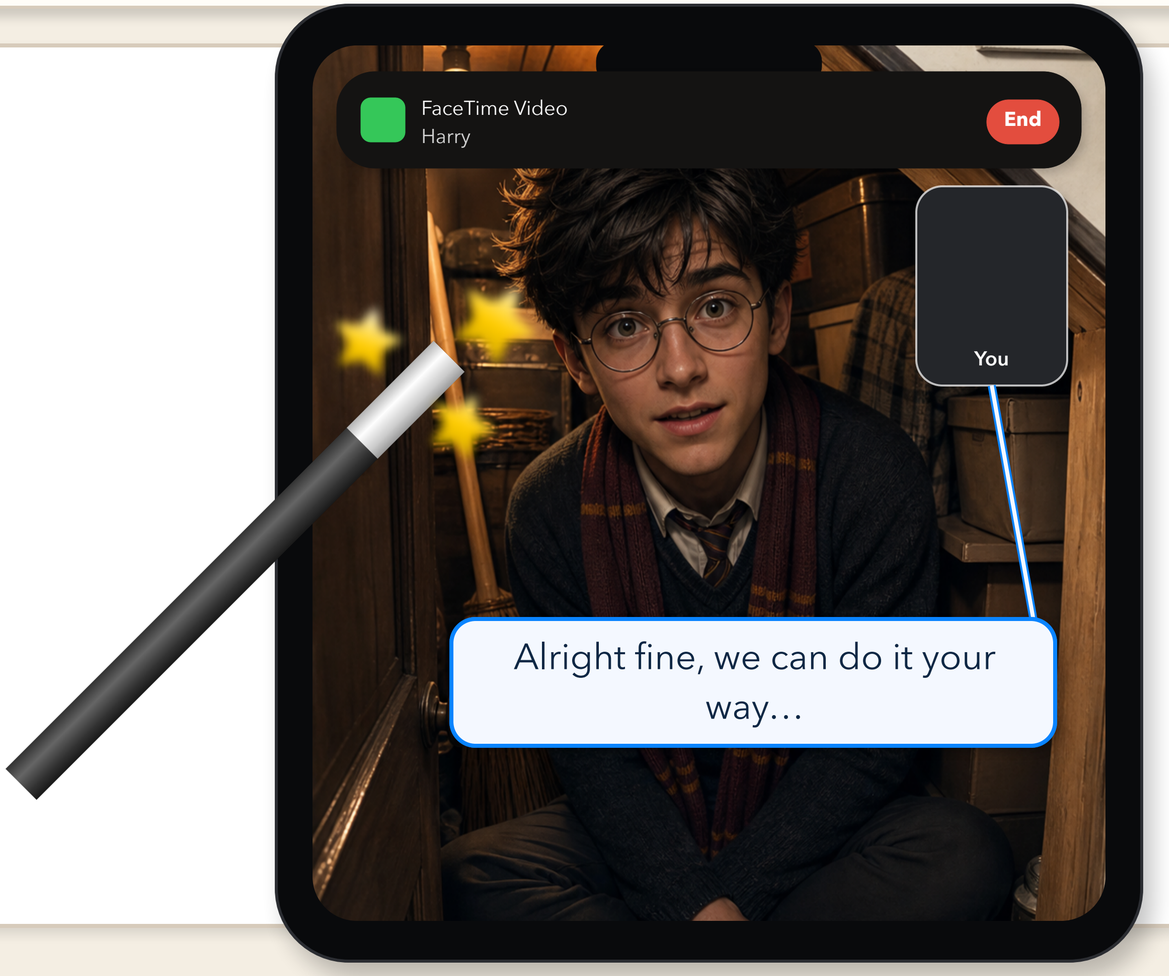

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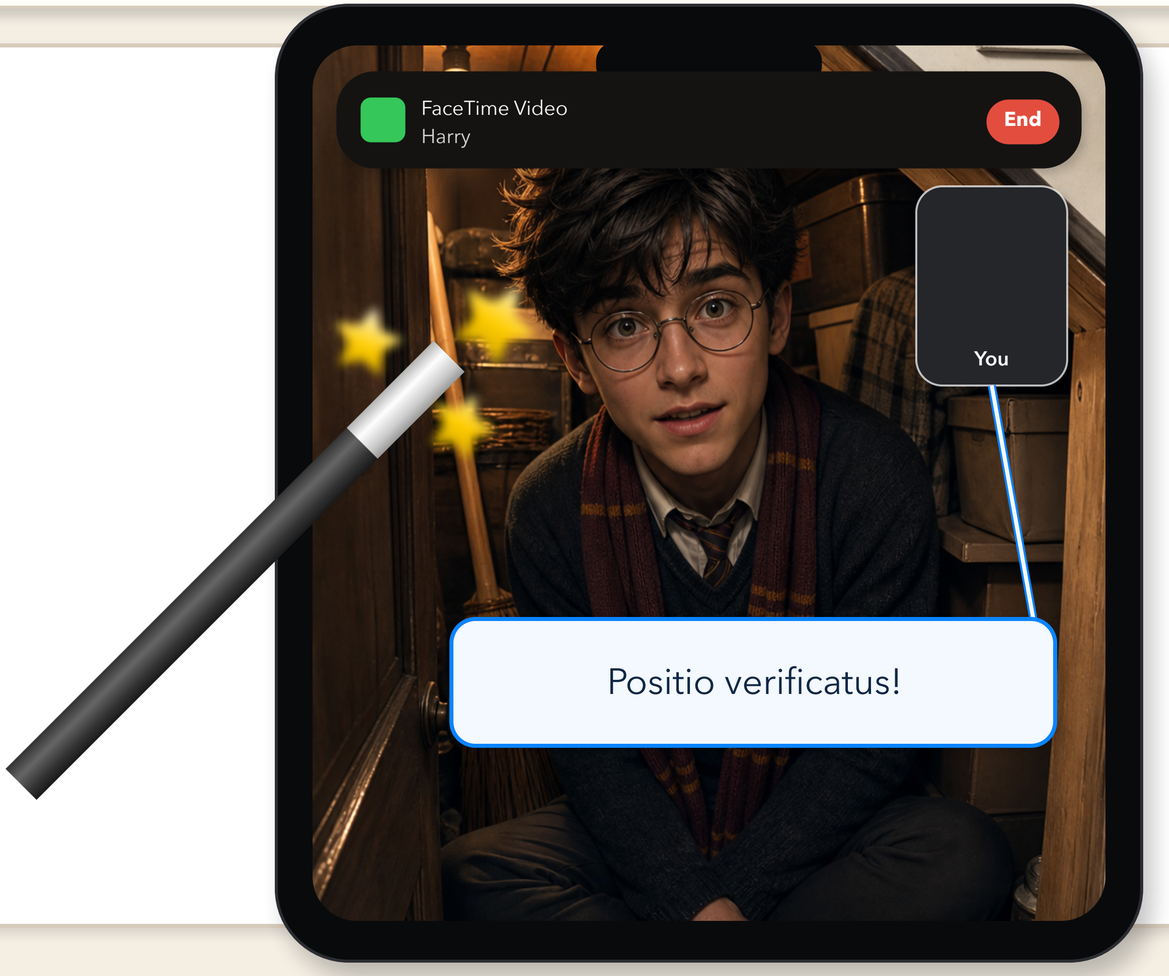

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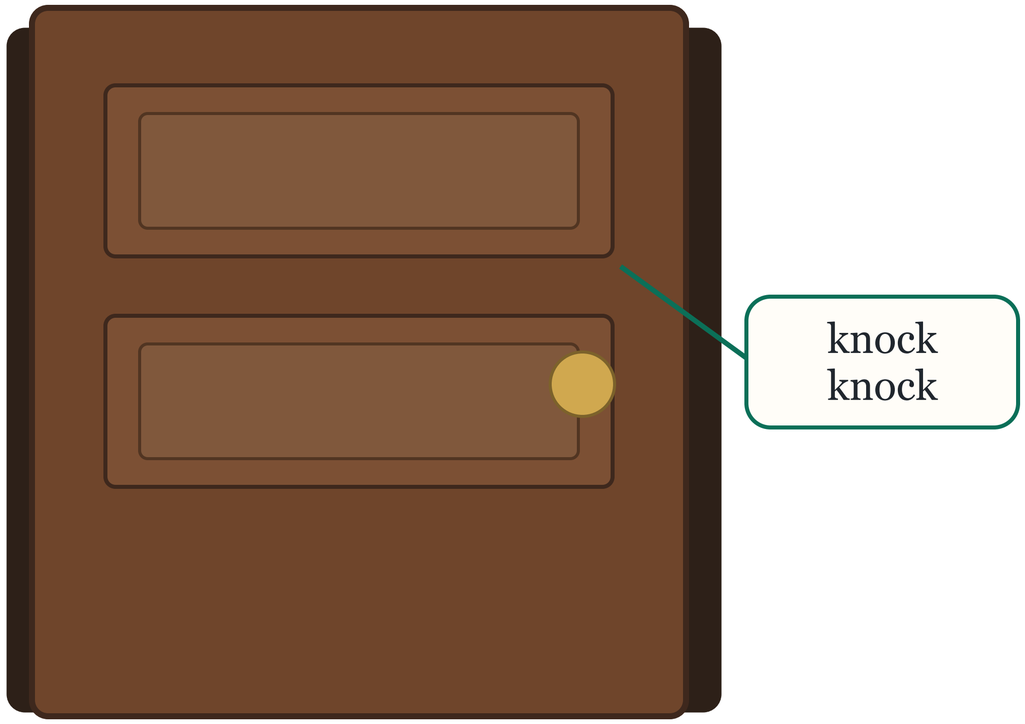
Scenario 2: Realistic Clones

What cryptographic credential can distinguish us from *any* other entity?



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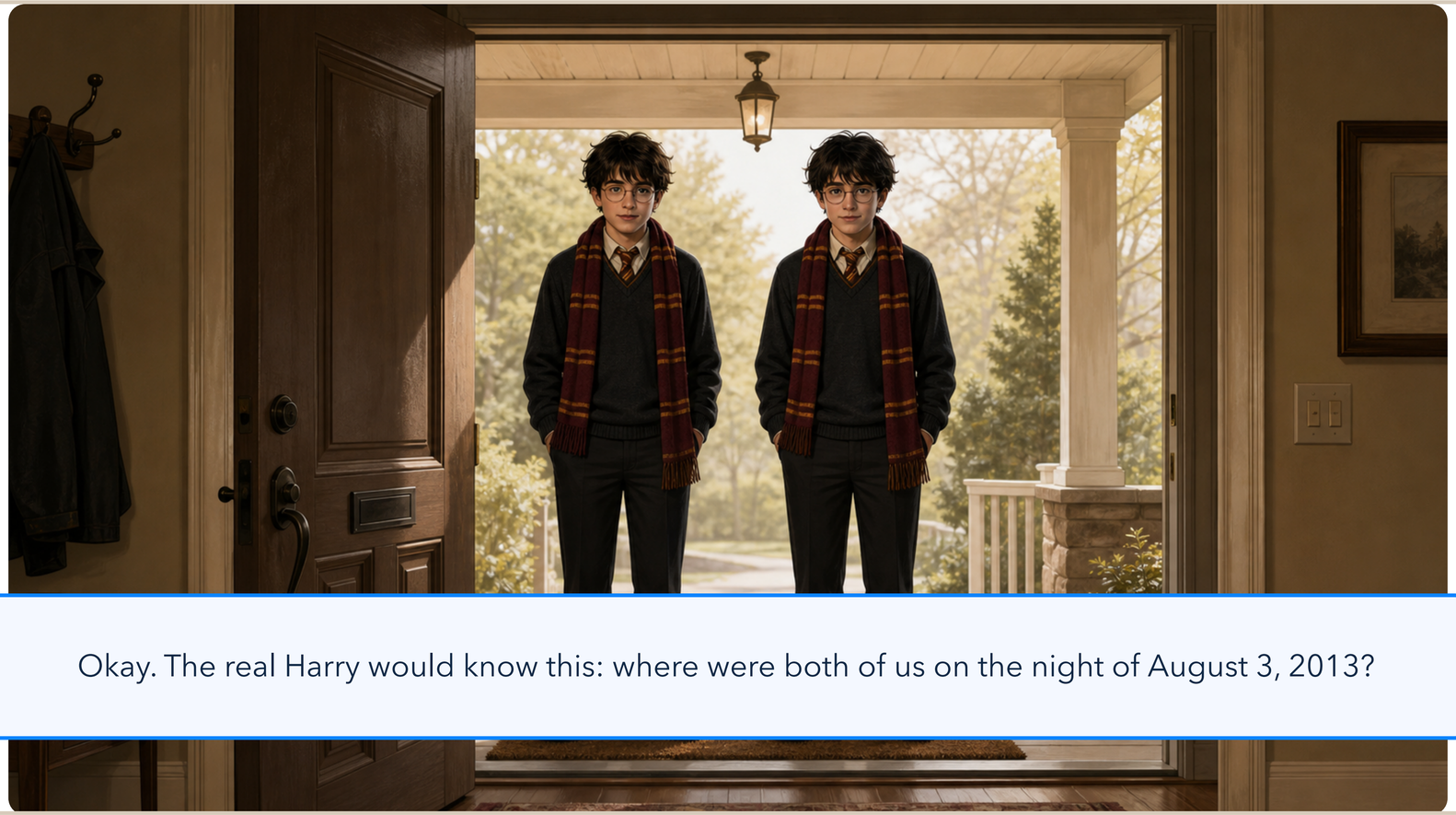

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This Work: **Strengthening Position-Based Guarantees**

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Talk Outline:

1. **Background — position-based cryptography**
2. **Introduce quantum localization schemes**
3. **Construct entanglement localization and trajectory verification**
4. **Bonus: functionality localization, with applications to PBA**

Position as a Credential

> Shifting identity from *what you know* to *where you are*

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- Have access to a secure facility (e.g. bank vault, military base)
- Located within a particular jurisdiction

Could be more robust!

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Enables a new kind of cryptography: position-based cryptography

Position-Based Cryptography

Position-Based Authentication

Position-Based Key Exchange

Position-Based Cryptography

Position-Based Authentication

Prover's goal: Send a message m to the verifiers, authenticated with his location.

E.g. m = "Cashapp me $500"

Verifier's goal: Accept m *only* if the sender was at some location L.

E.g. L = "cupboard under the stairs"

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Position-Based Key Exchange

Prover's goal: establish a shared key with the verifiers.

Verifier's goal: *only* someone at location L can learn the key.

Application: sending a message which can only be read at a specific location.

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Position Verification

Prover's goal: convince verifier he is at location L

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Position-Based Cryptography

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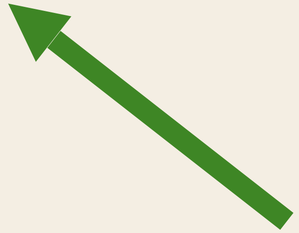
[BCFGGOS'10, U'14]: PV -> PBA



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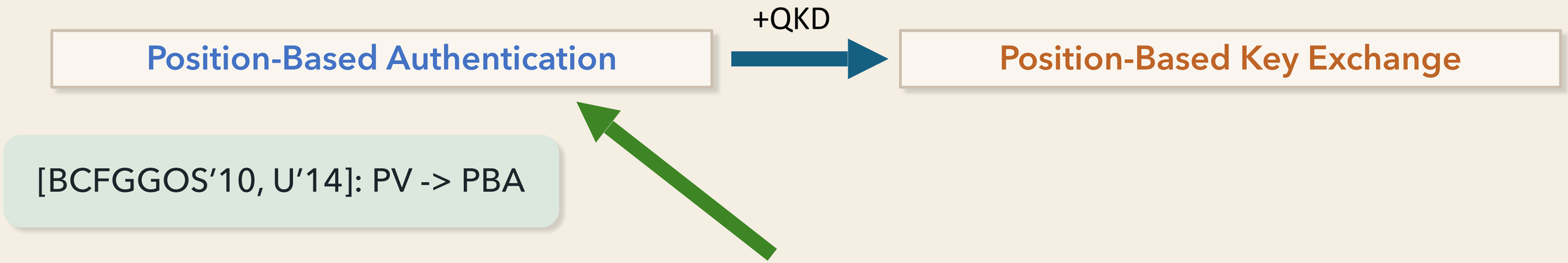

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Position Verification

The core building block of position-based cryptography.

Location L

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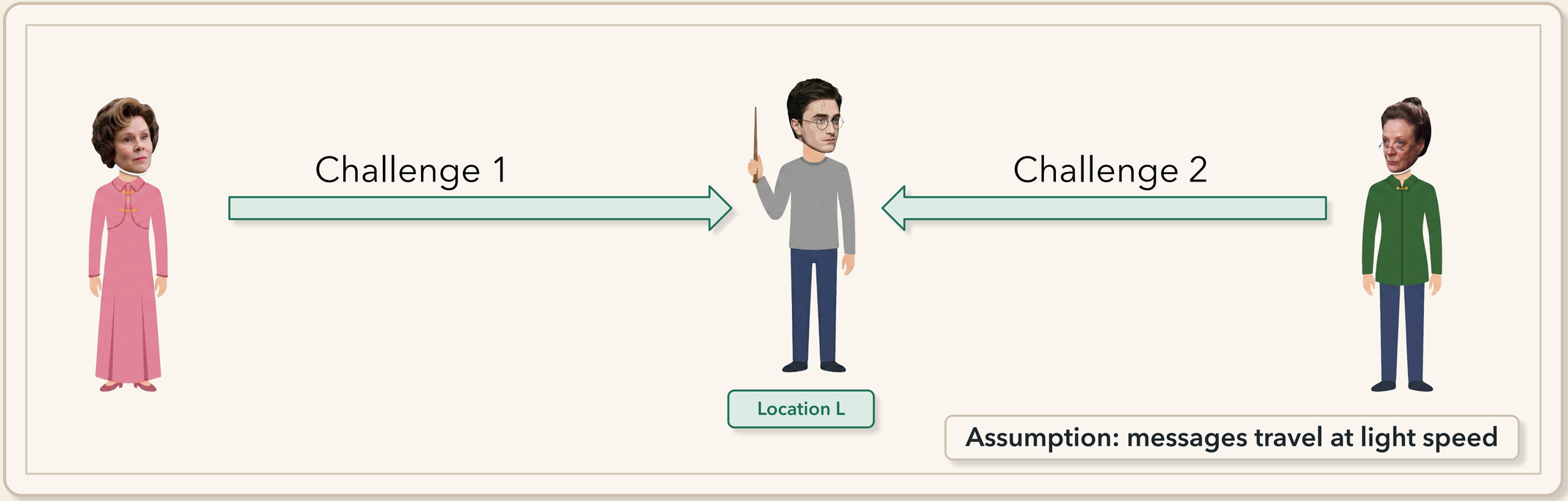

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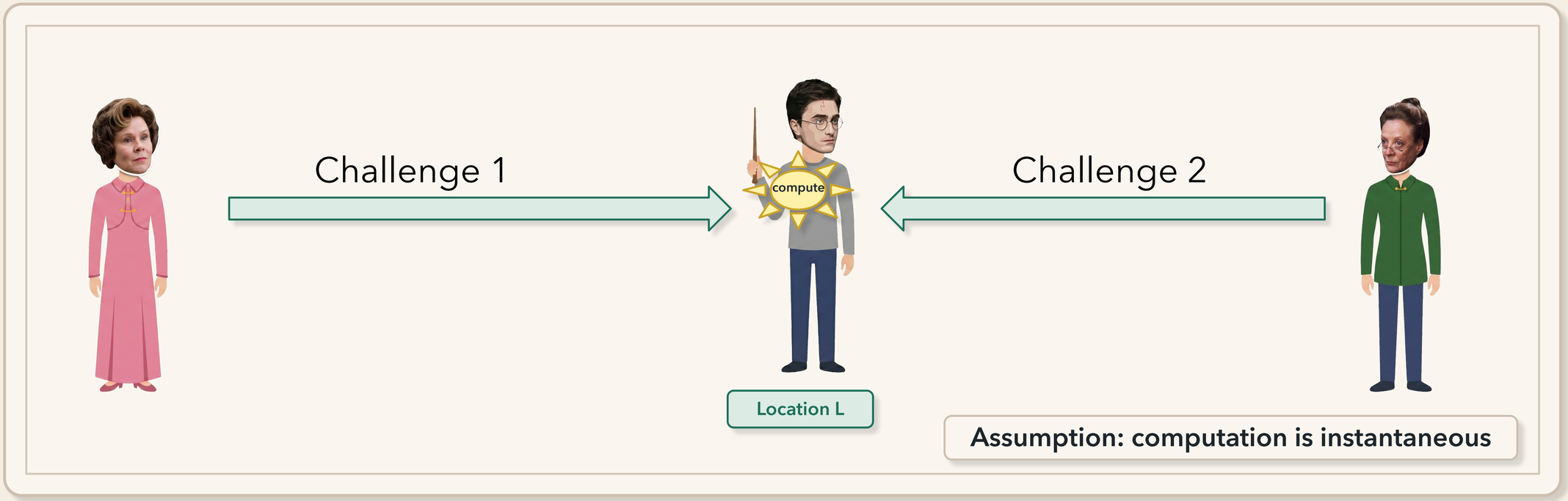

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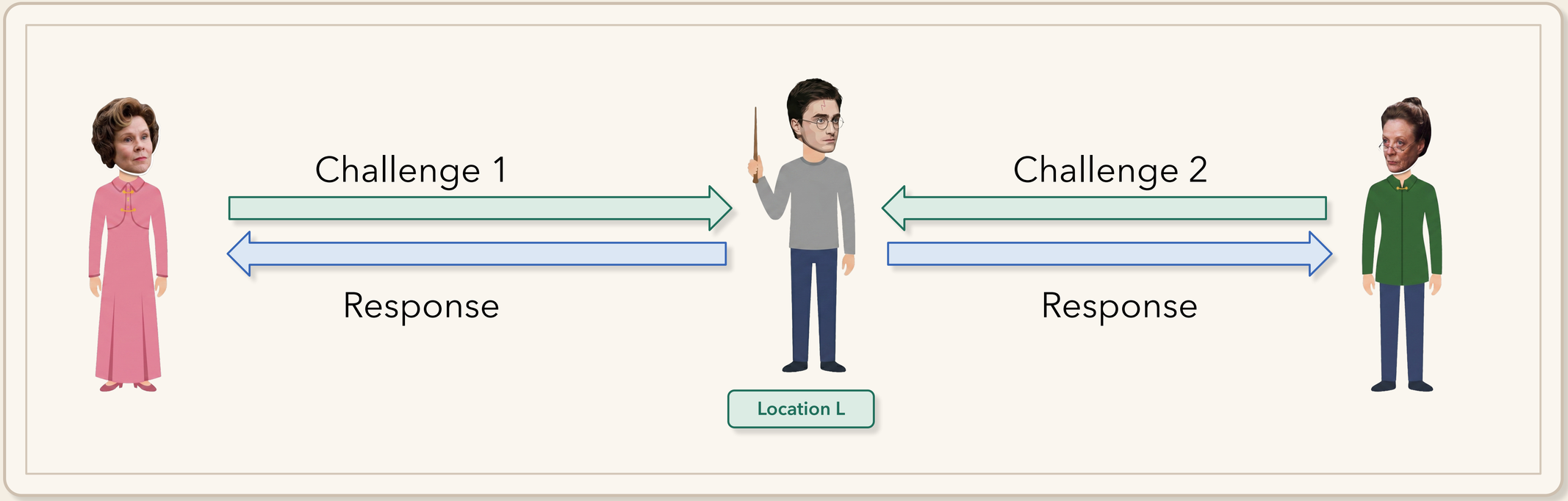

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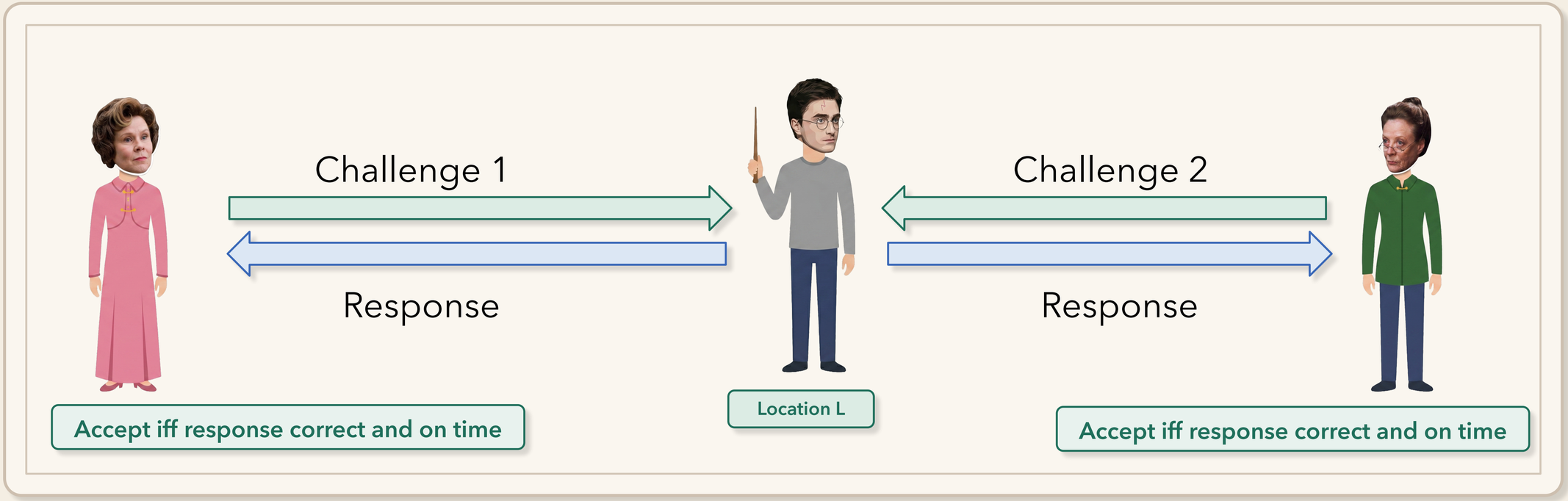

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What if there are two colluding spoofers?

Location L

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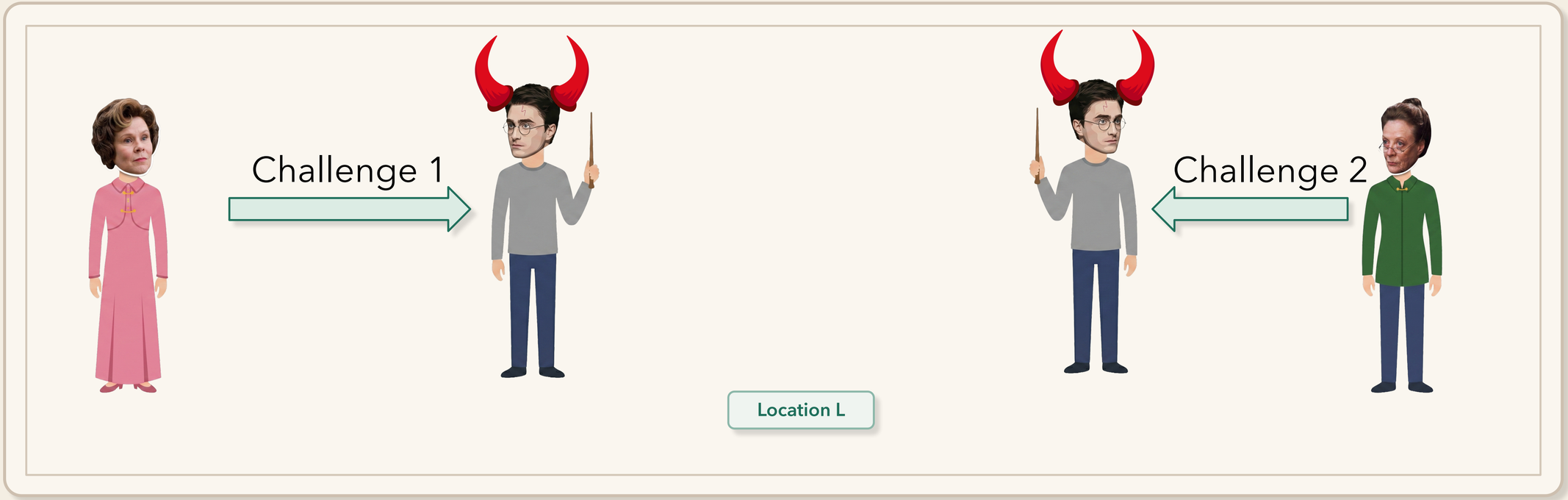

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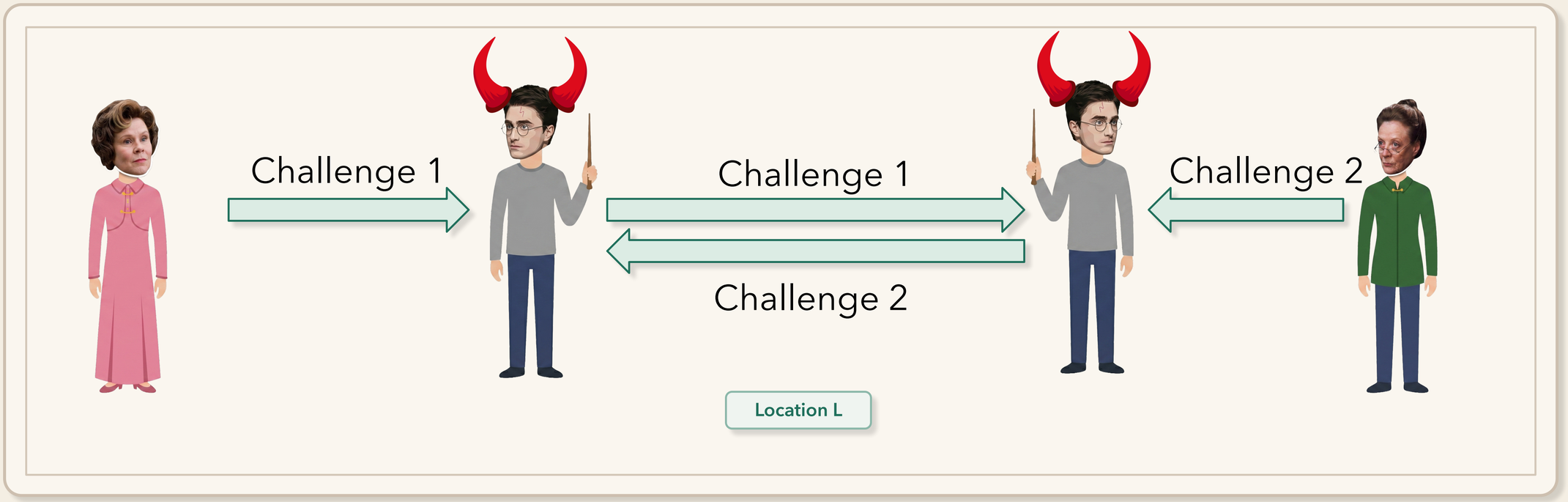

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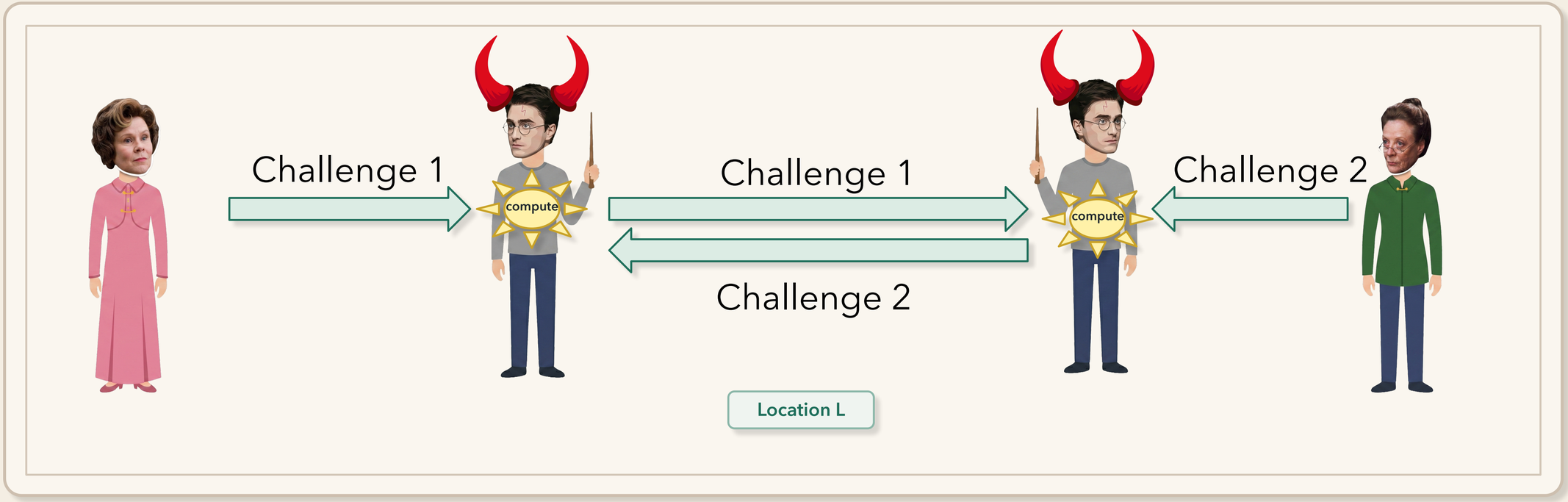

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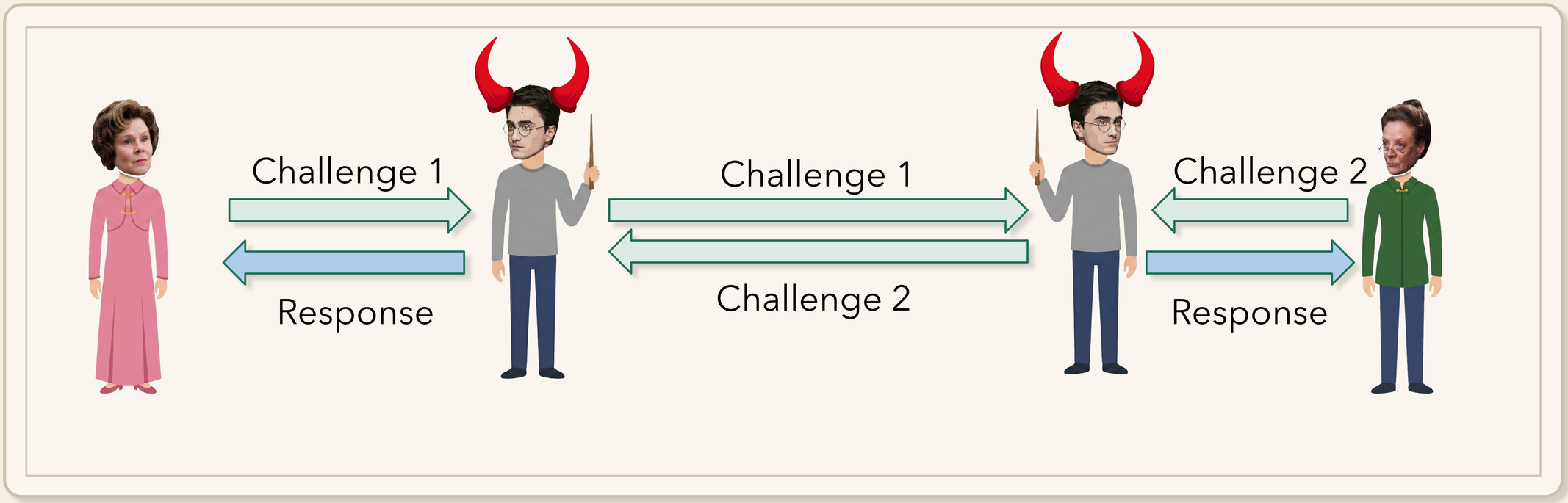

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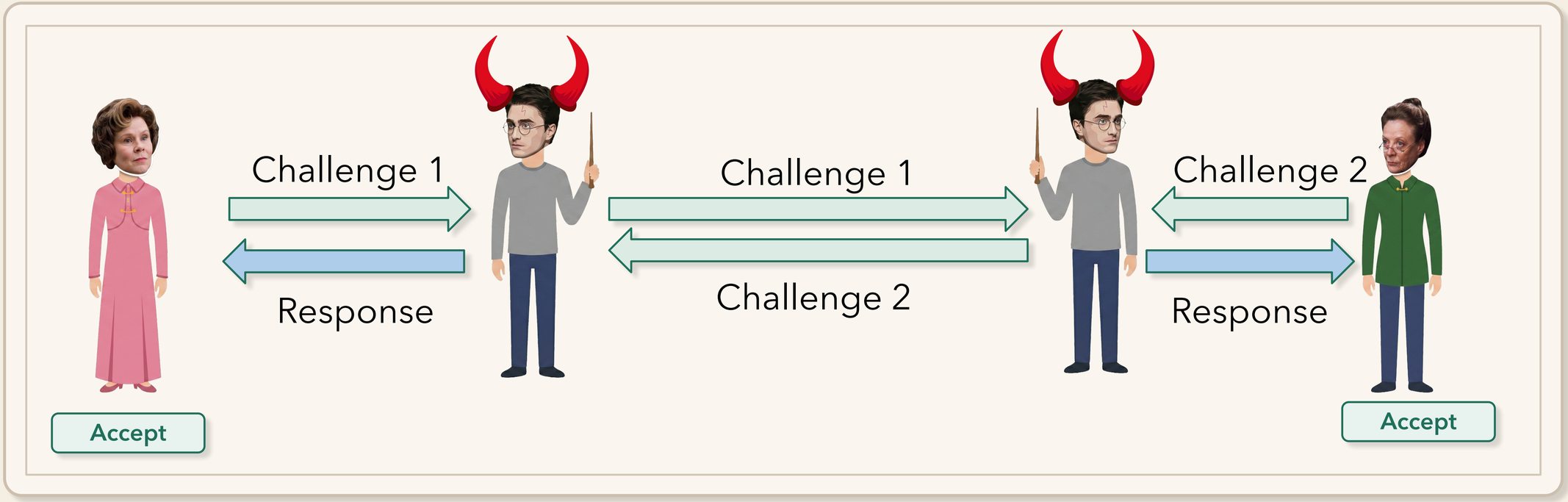

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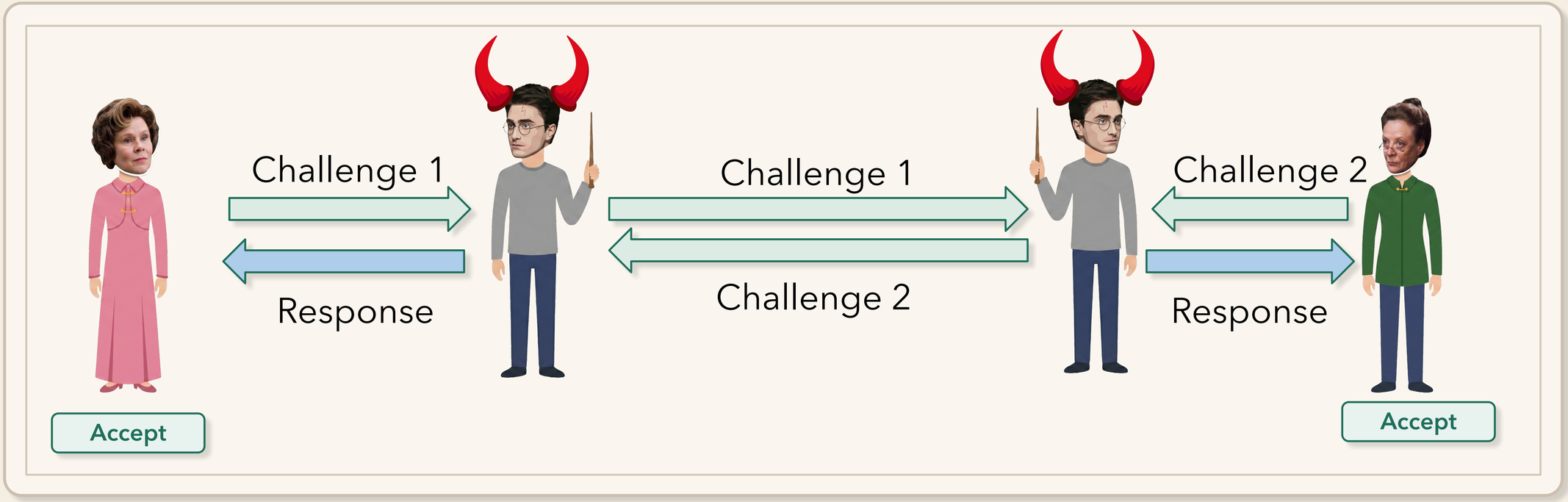
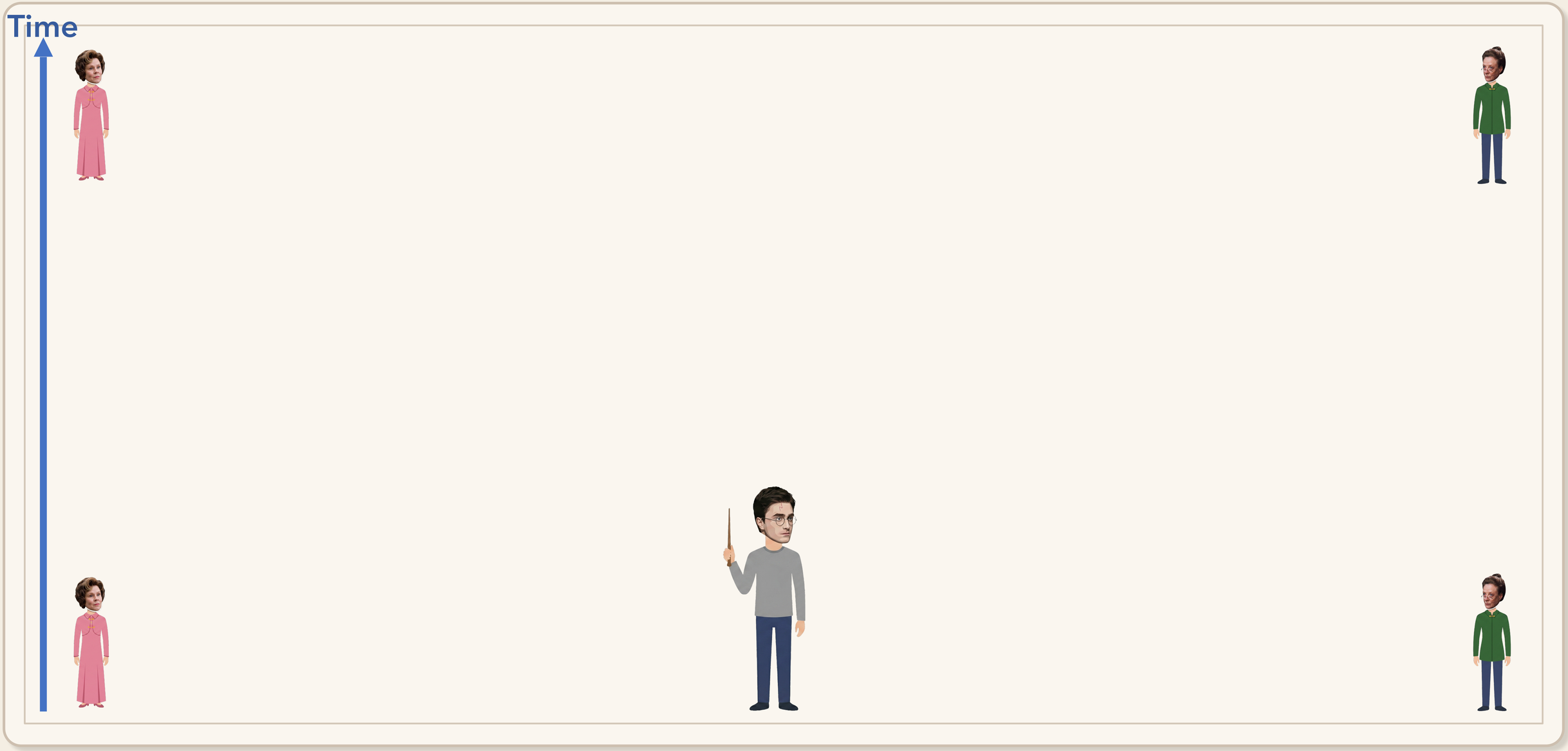


Classically, secure schemes don't exist ([CGMO'09]).
But what about quantum schemes?

A Quantum Position Verification Scheme



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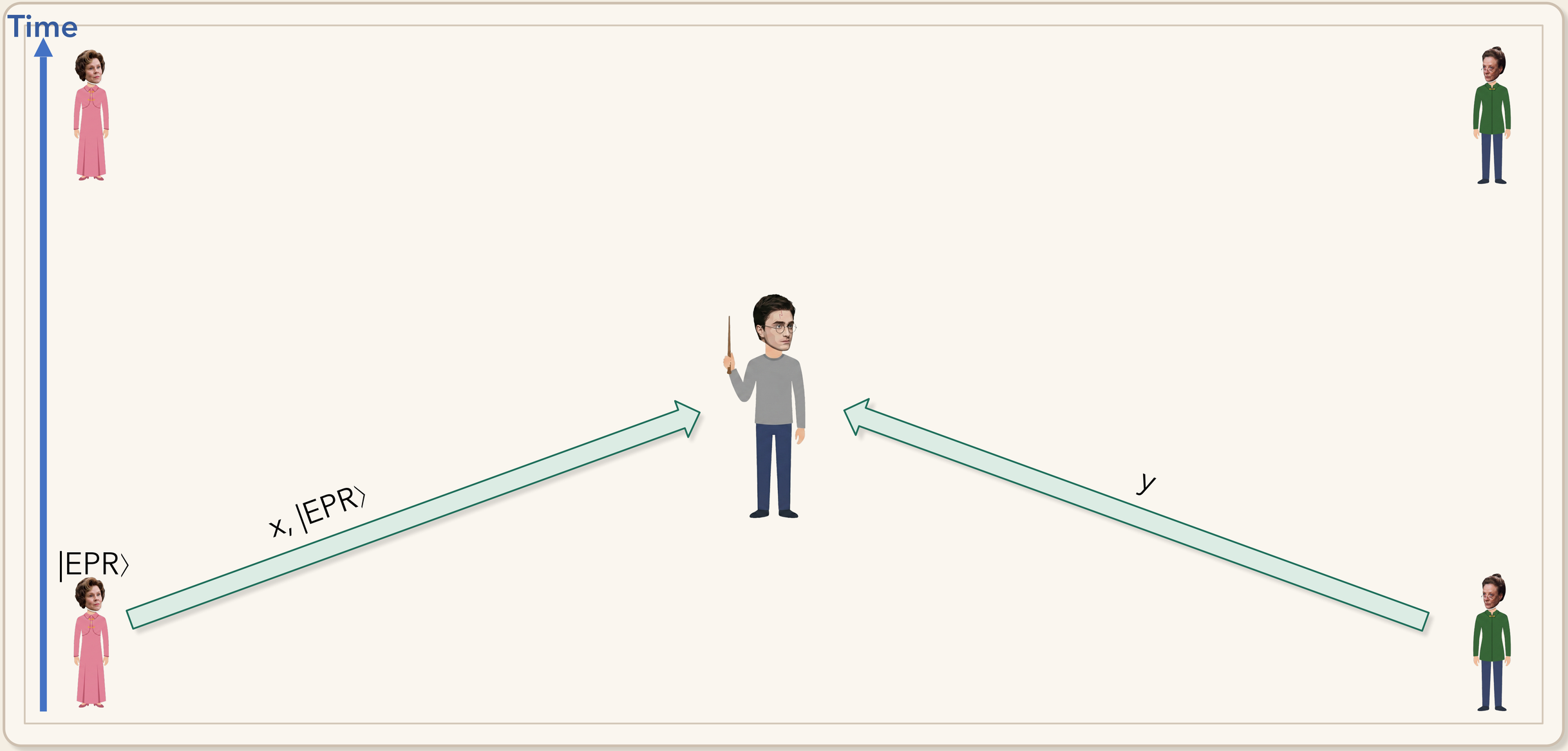

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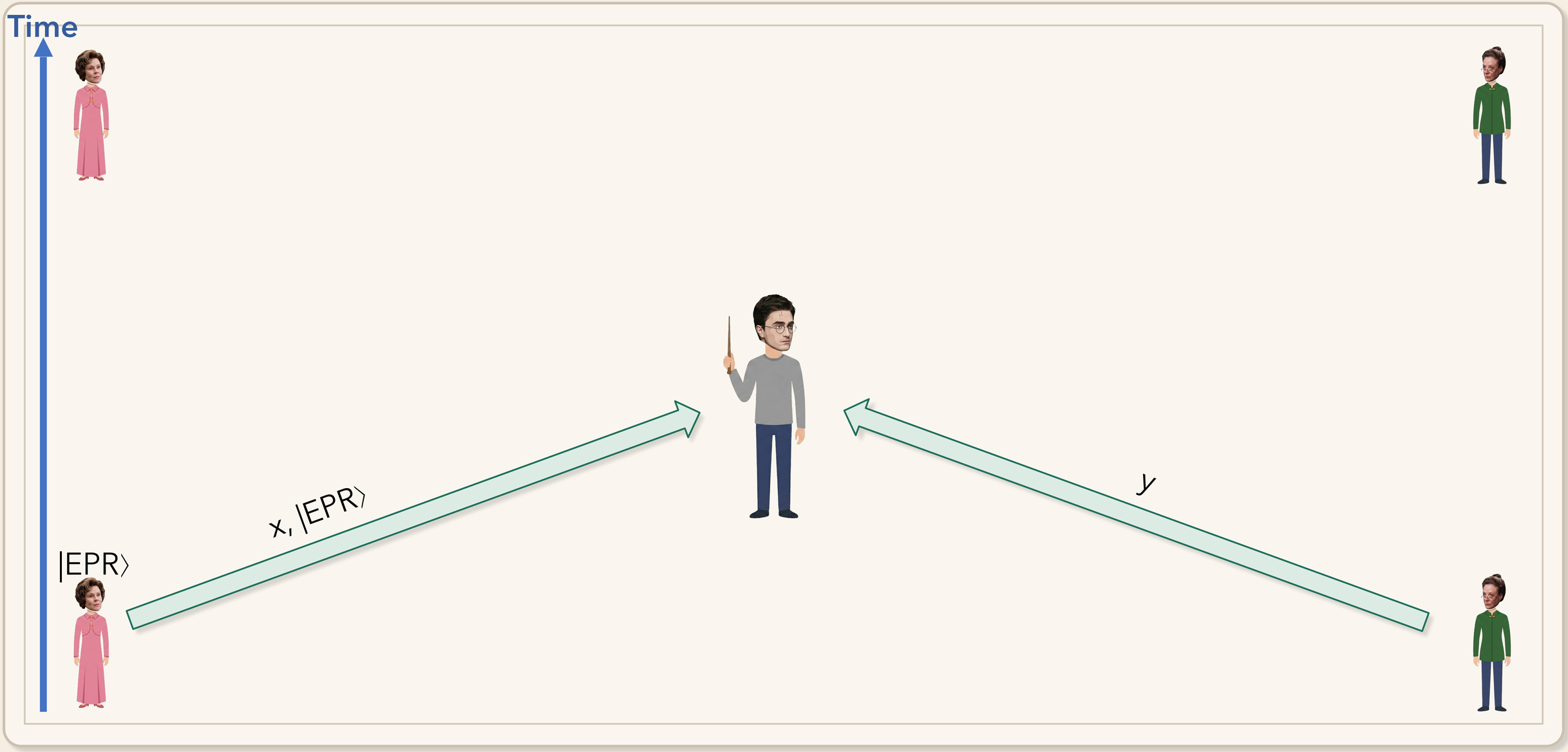

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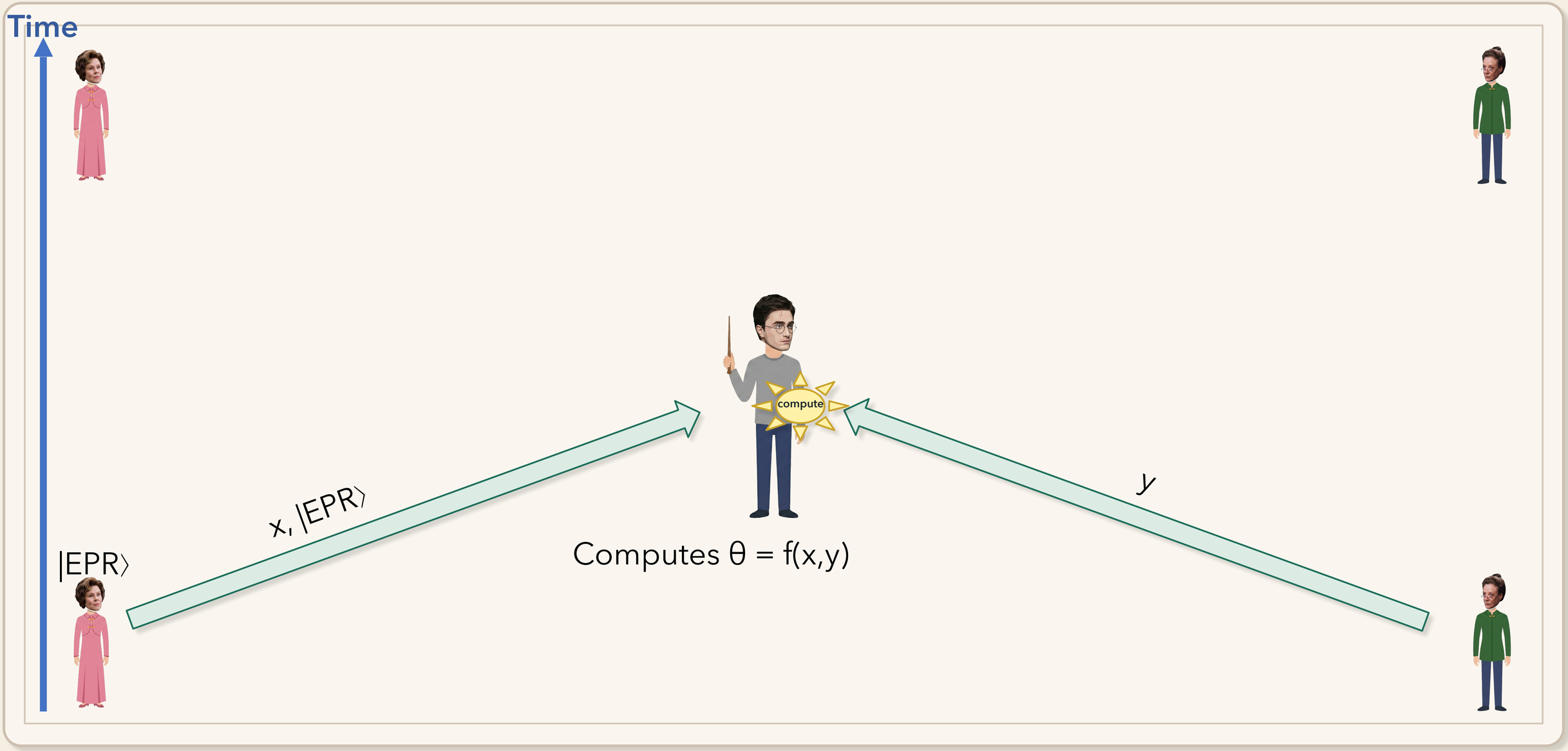

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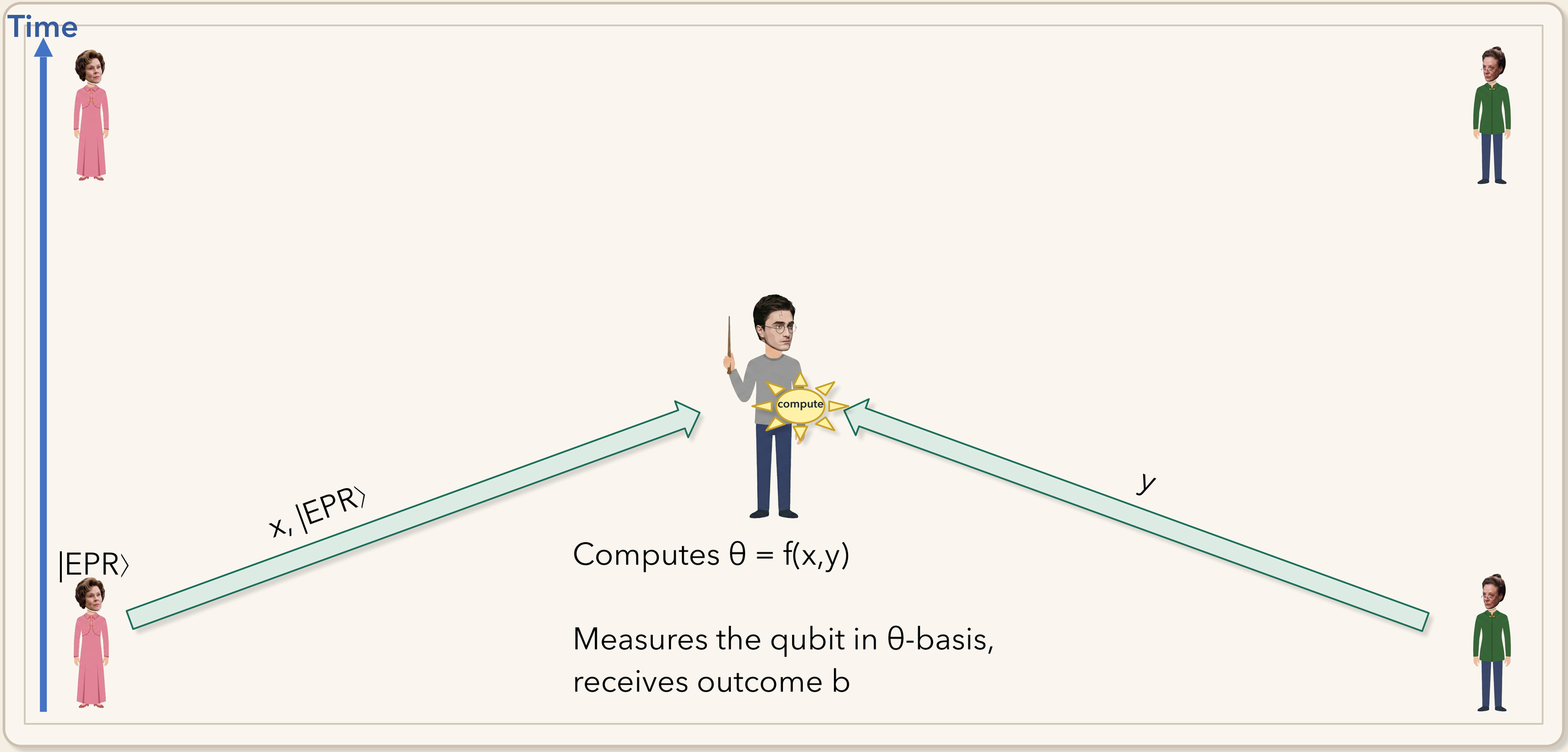

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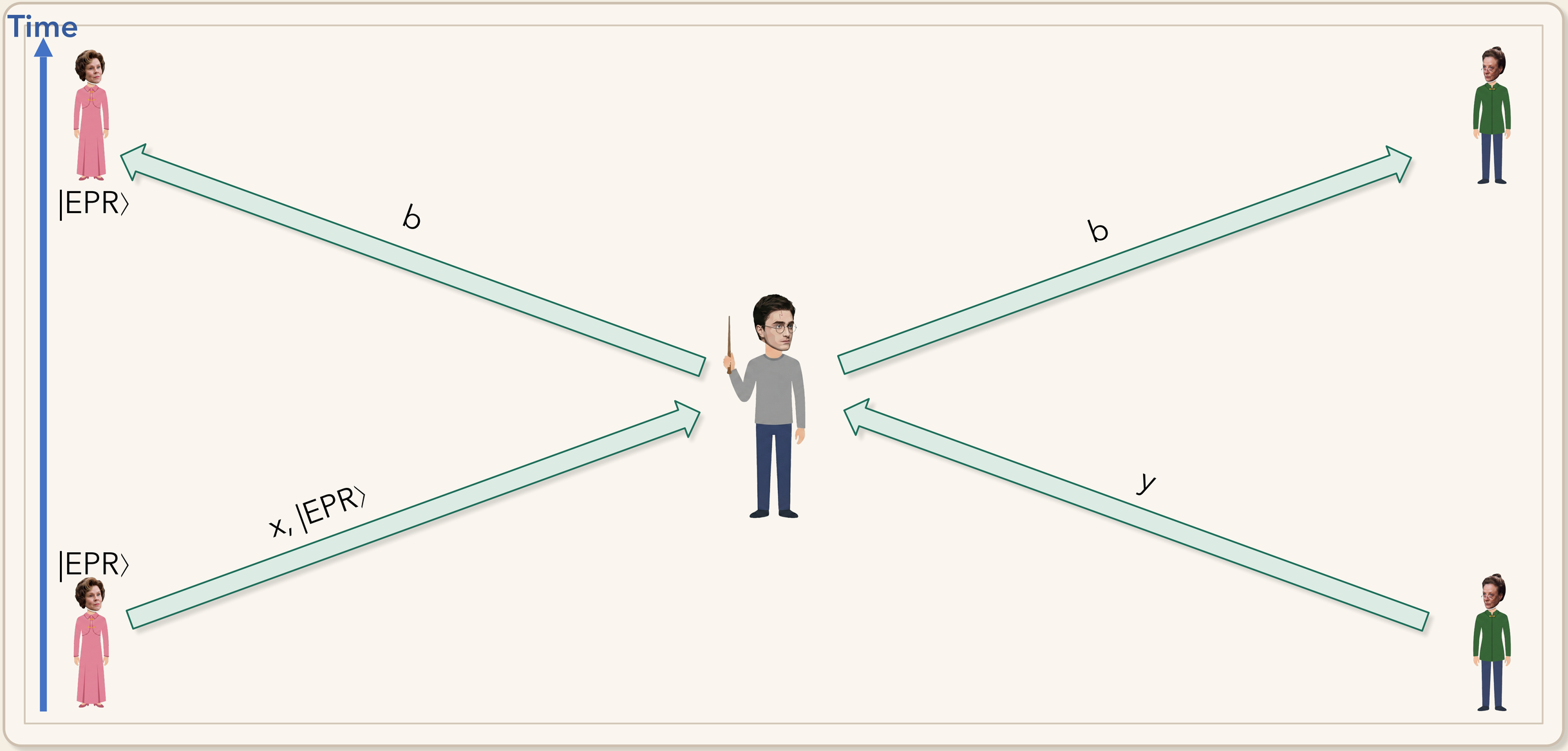

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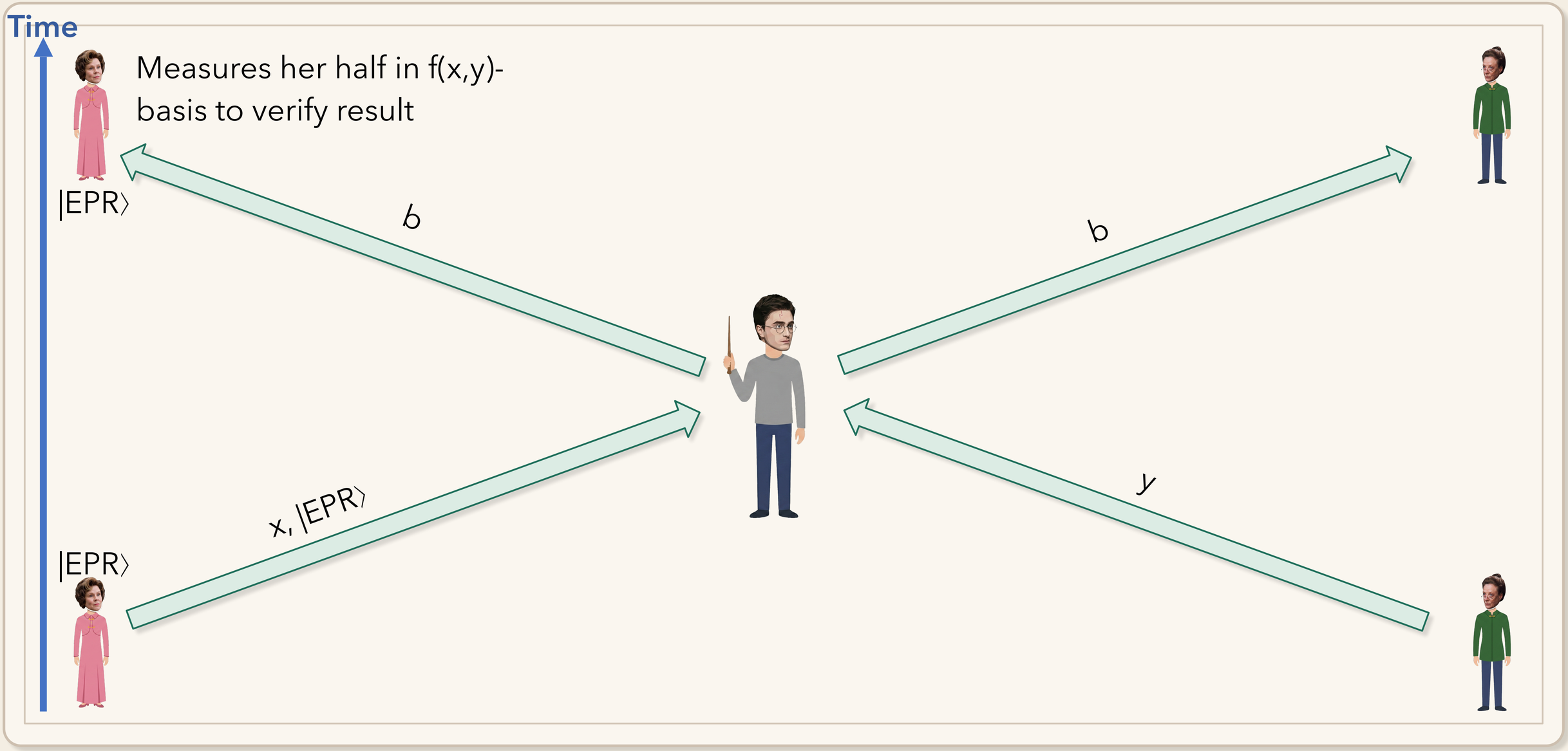

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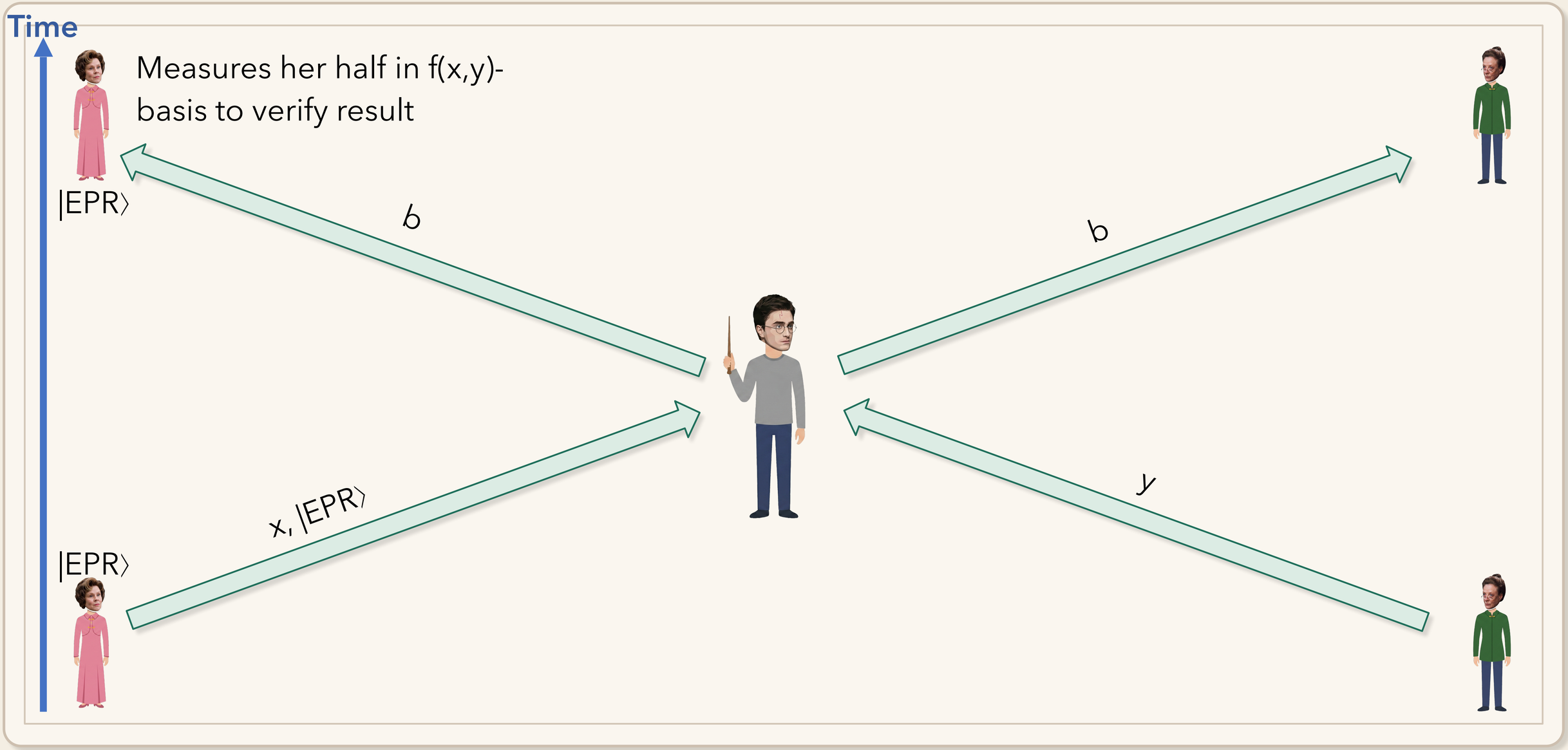

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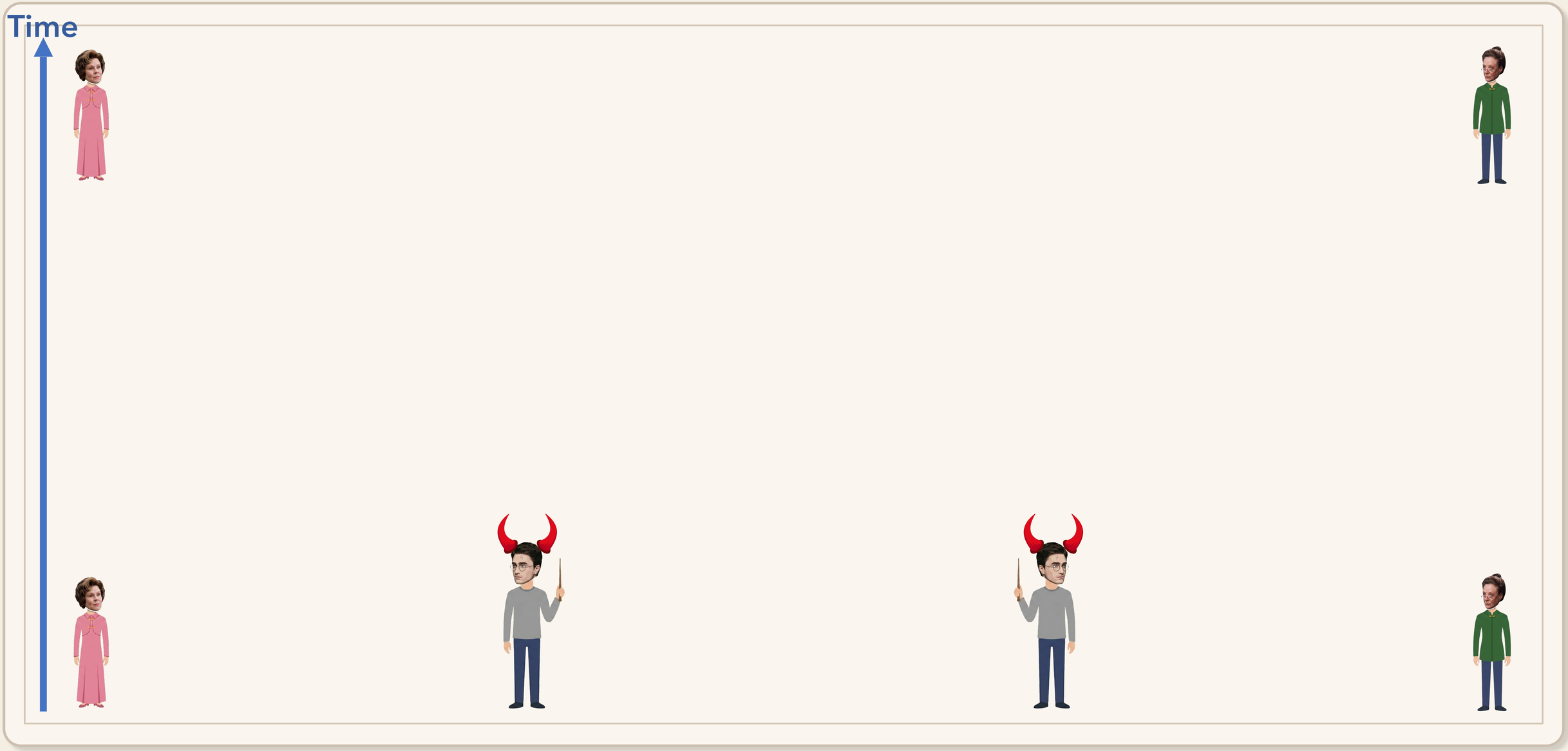

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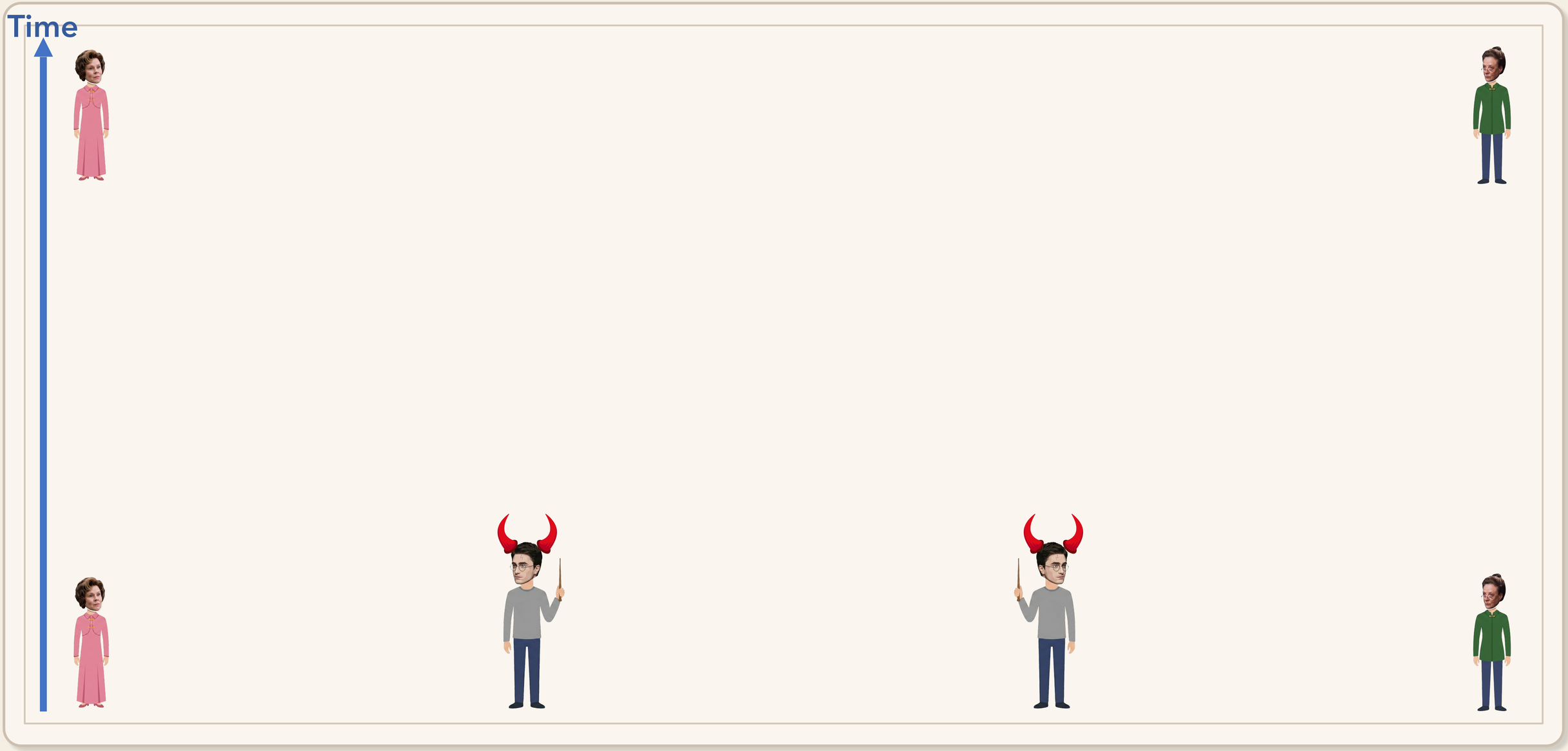

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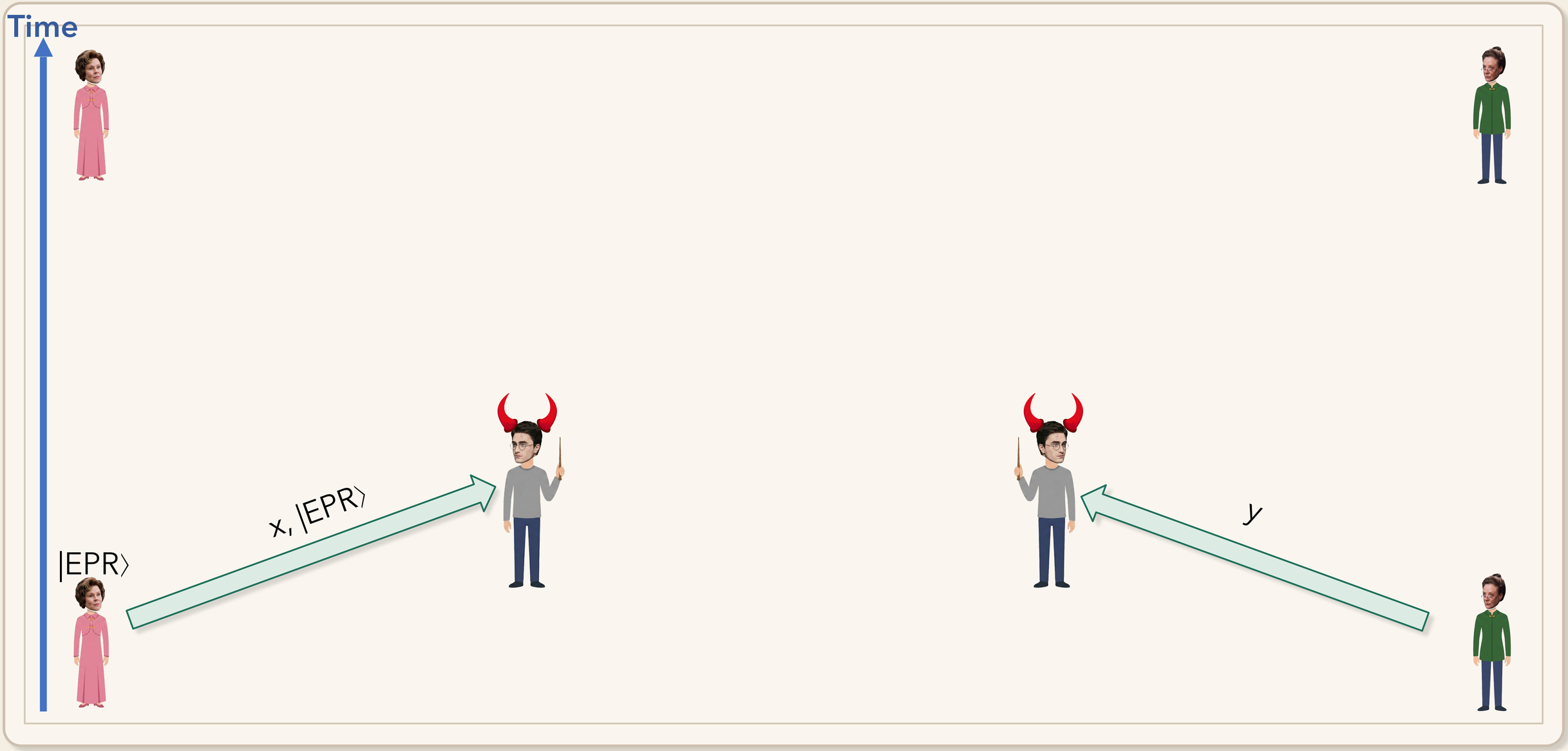

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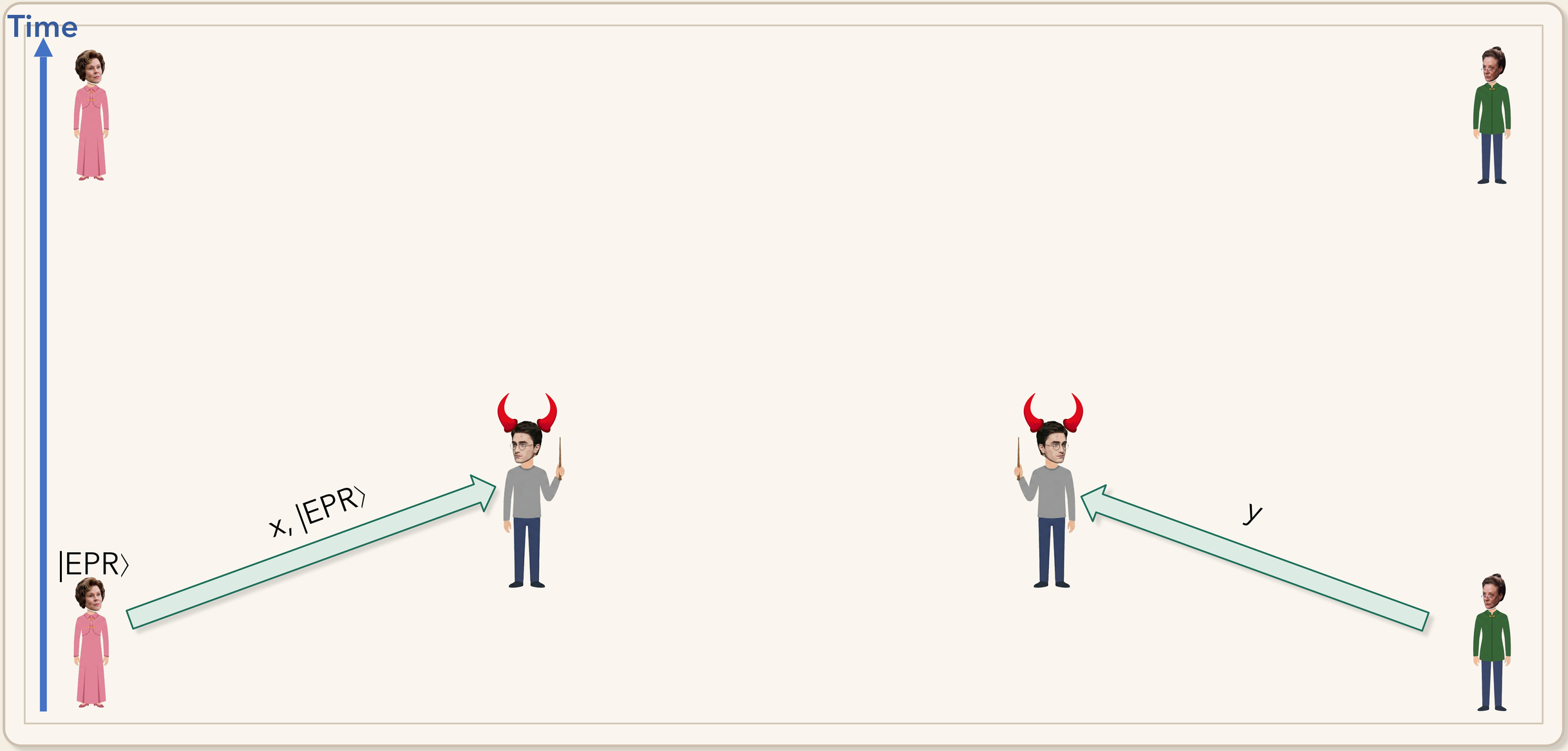

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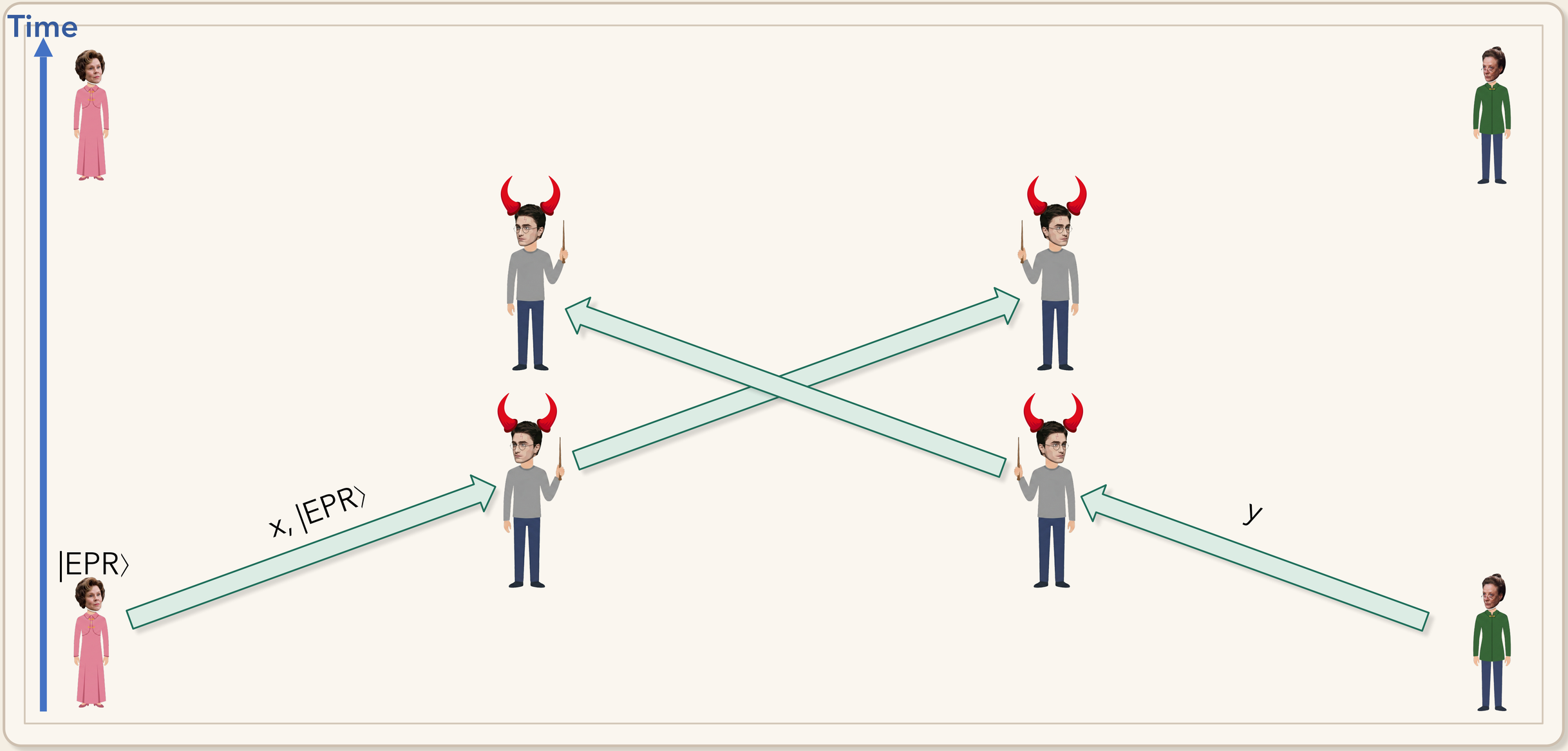

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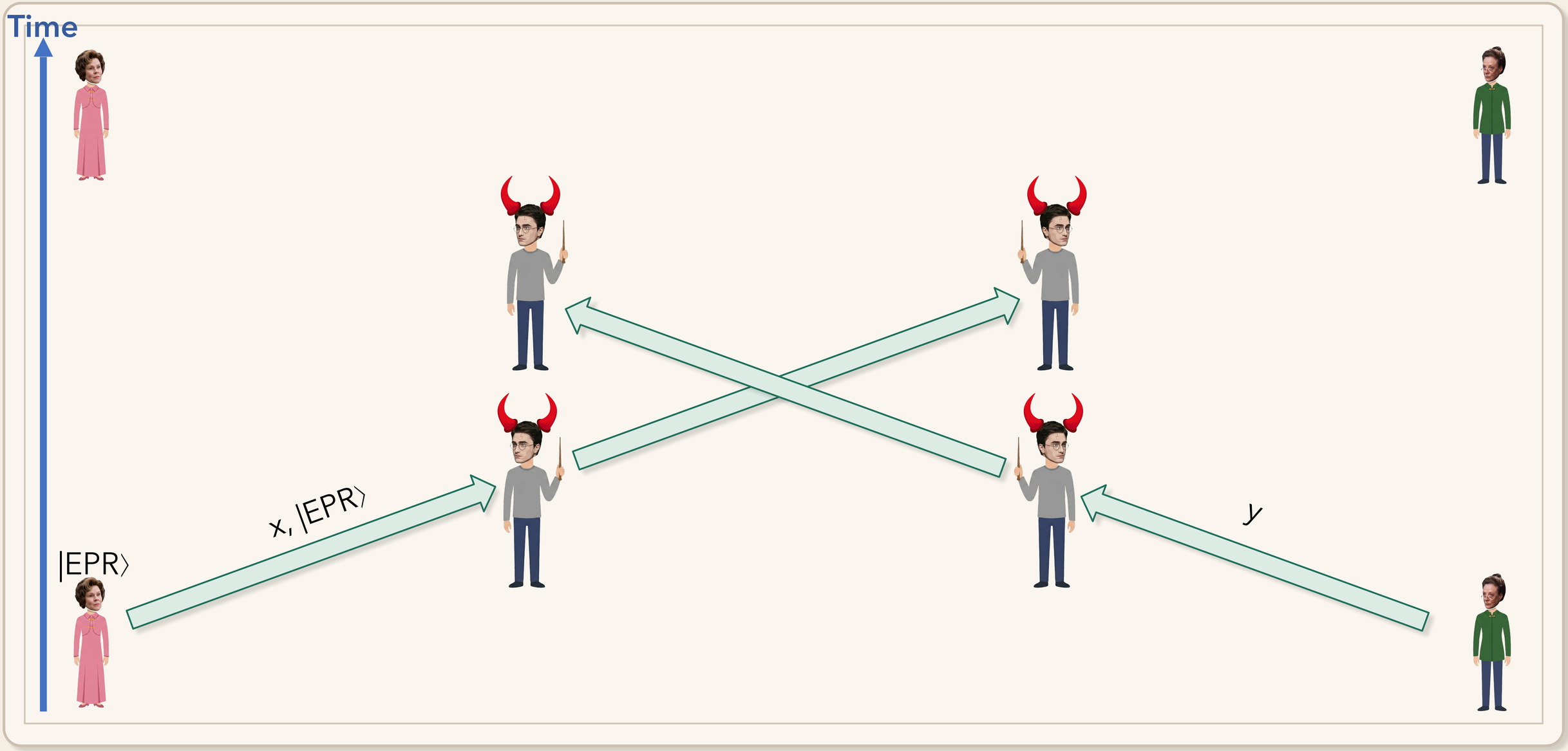

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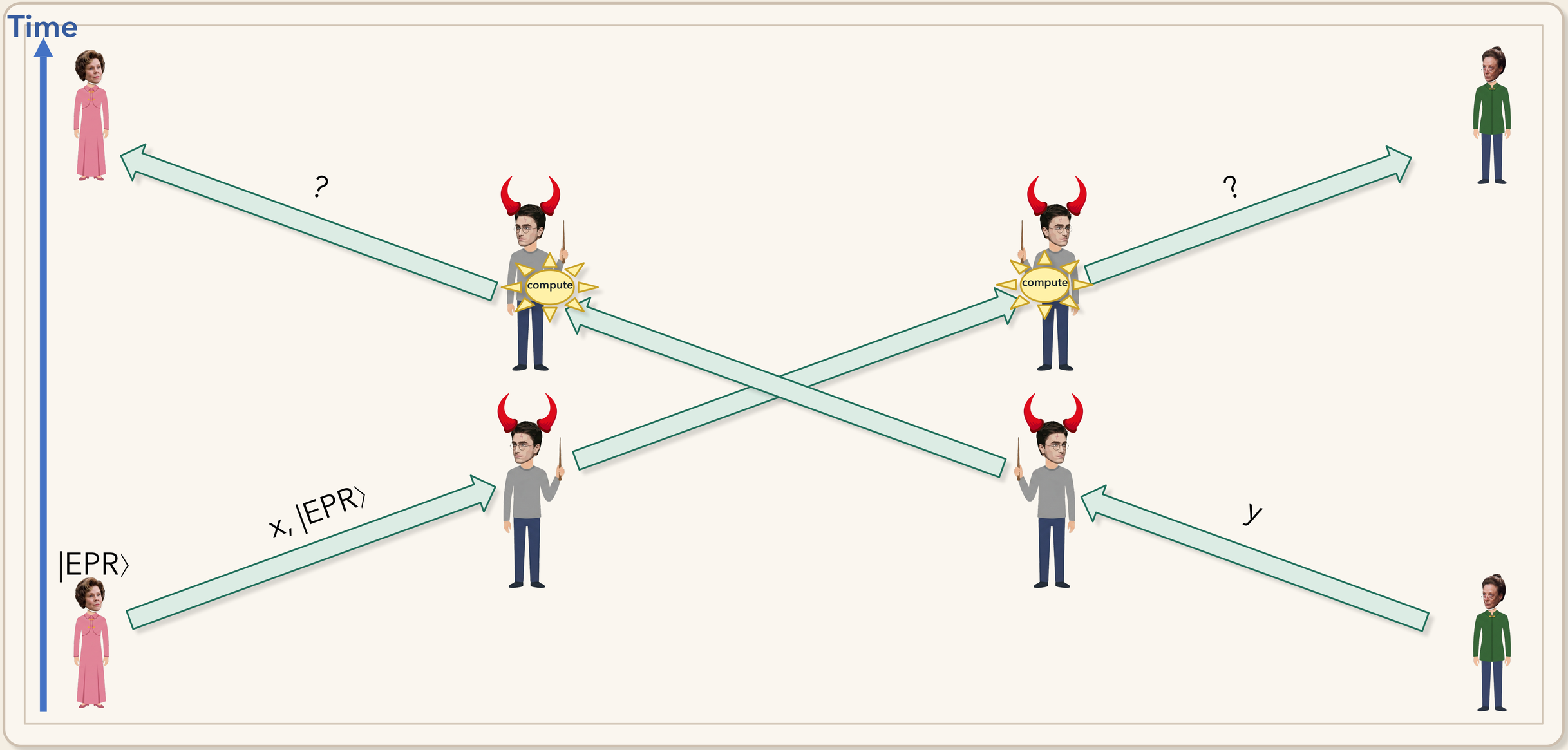

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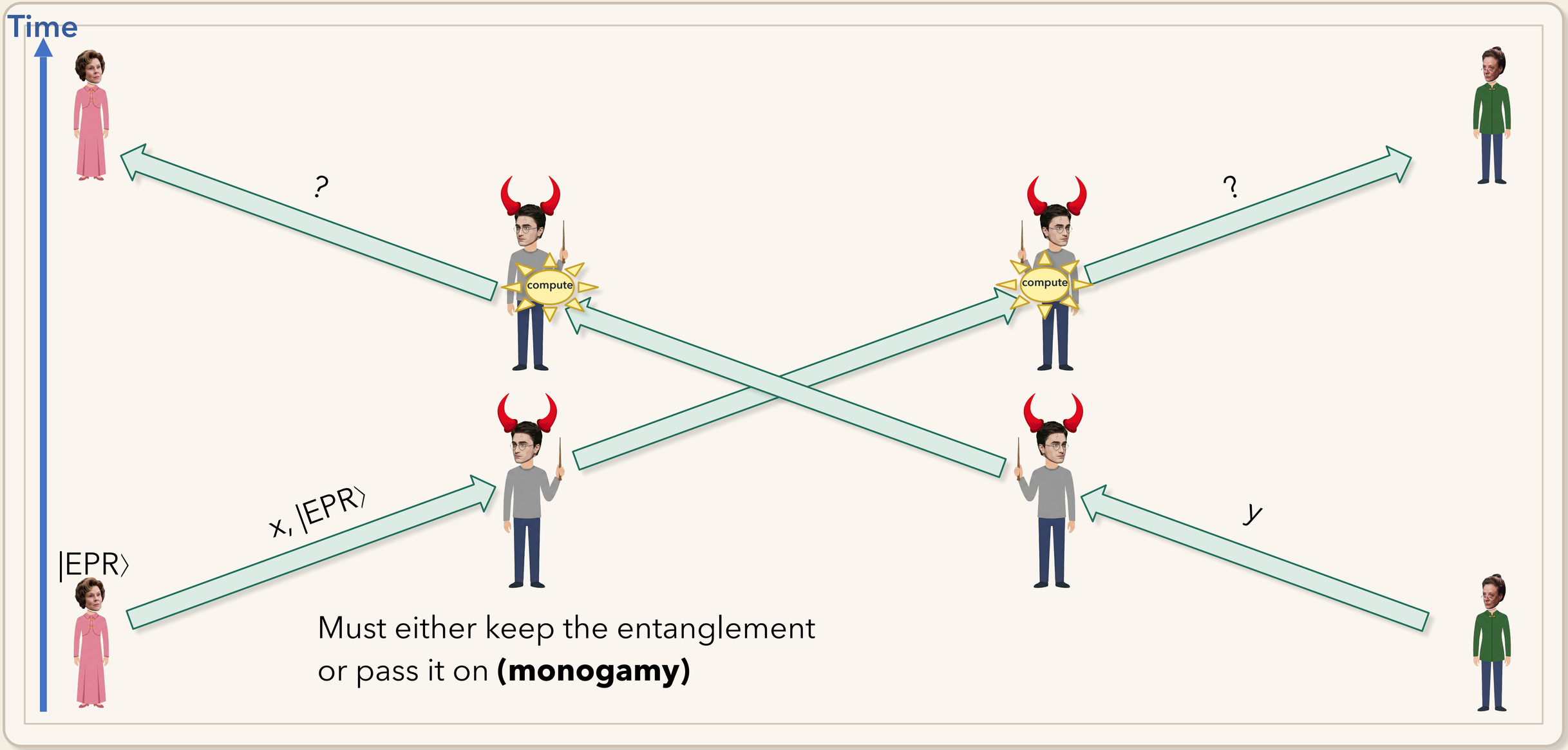

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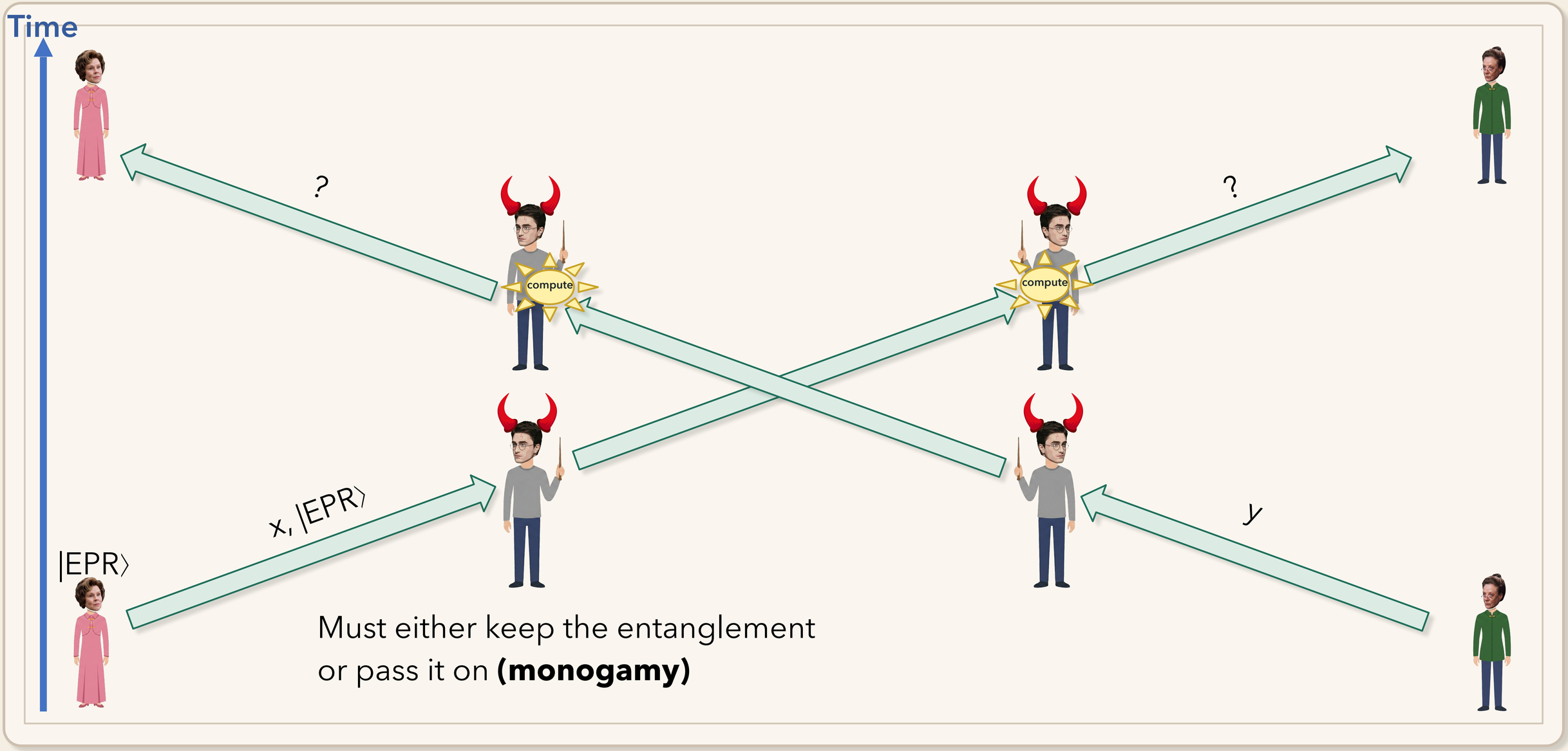

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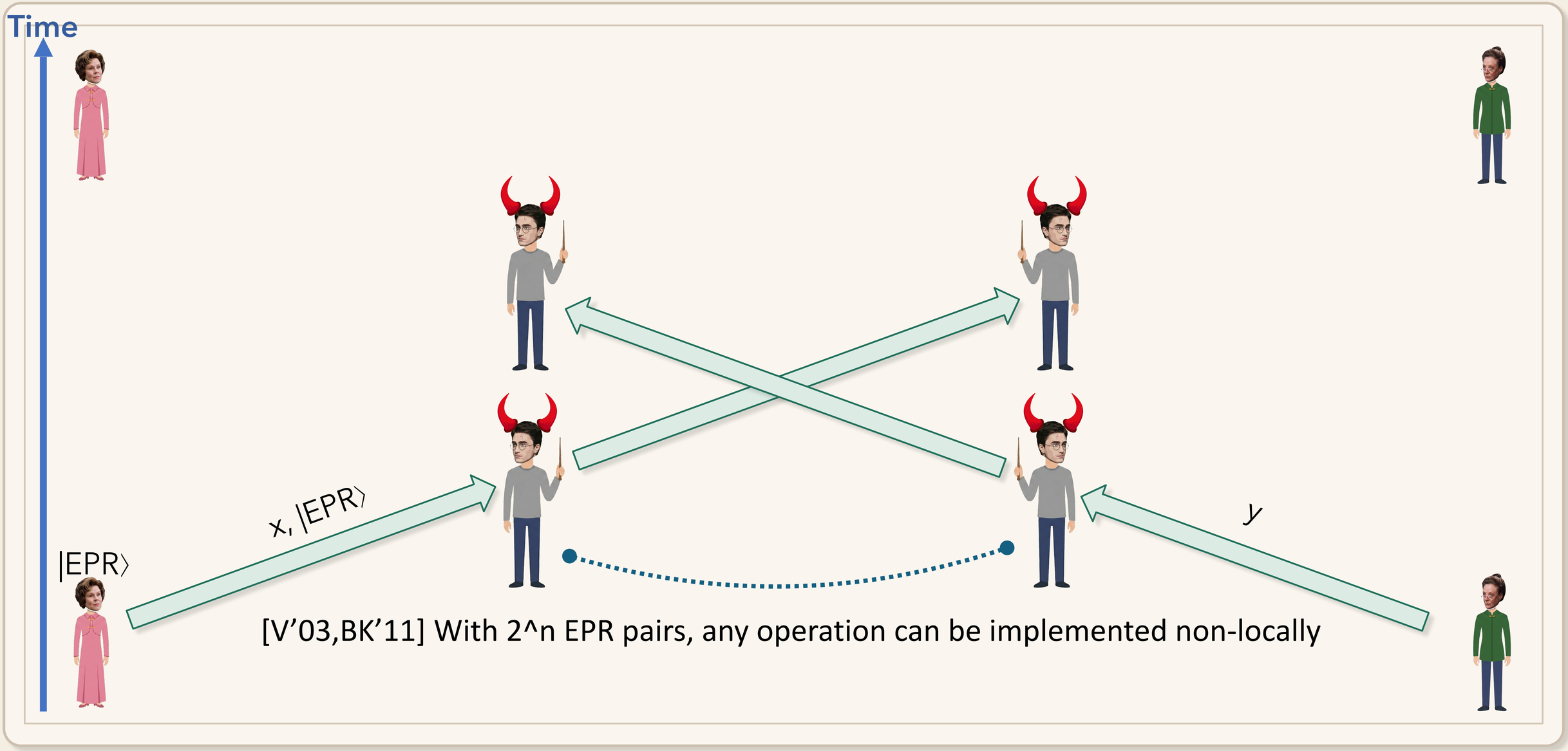

[V'03,BK'11] With 2^n EPR pairs, any operation can be implemented non-locally

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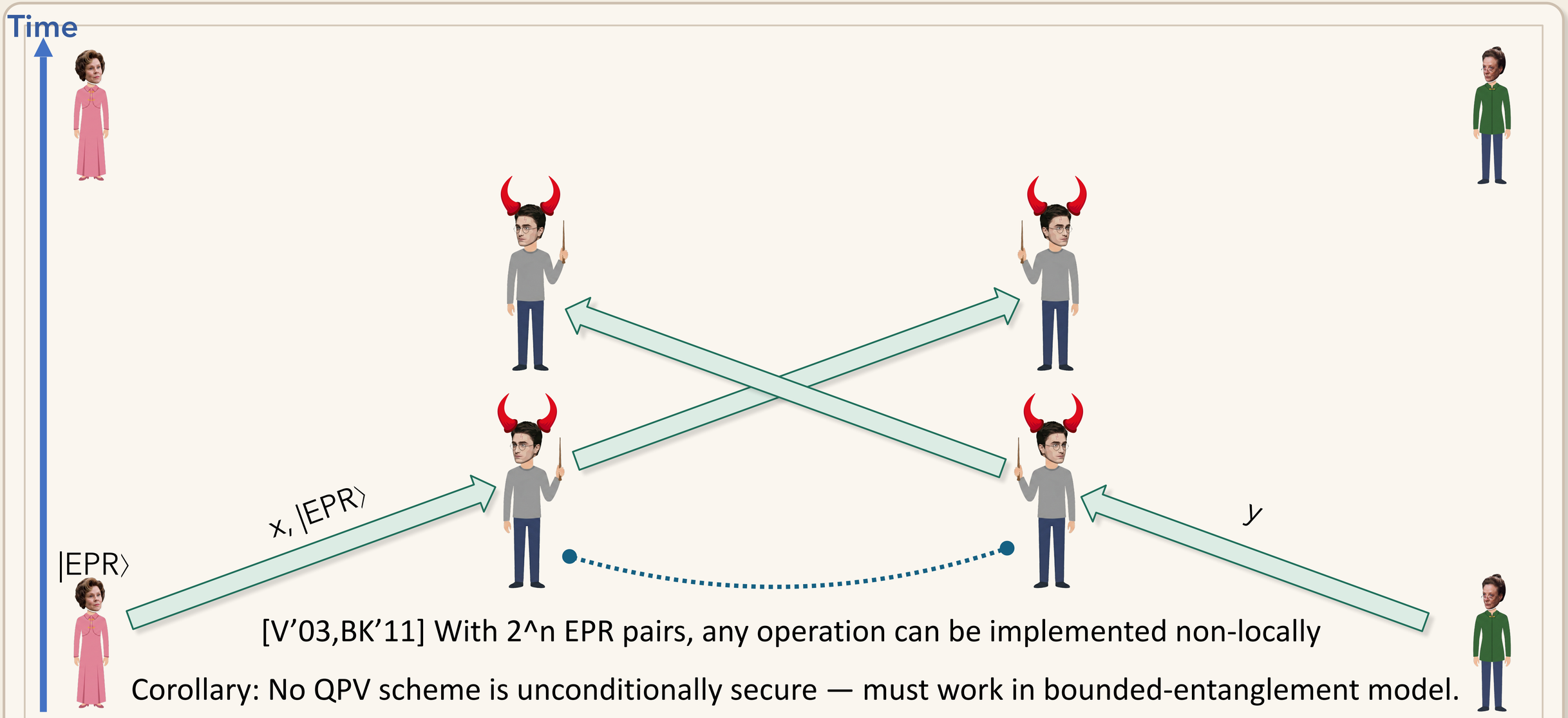

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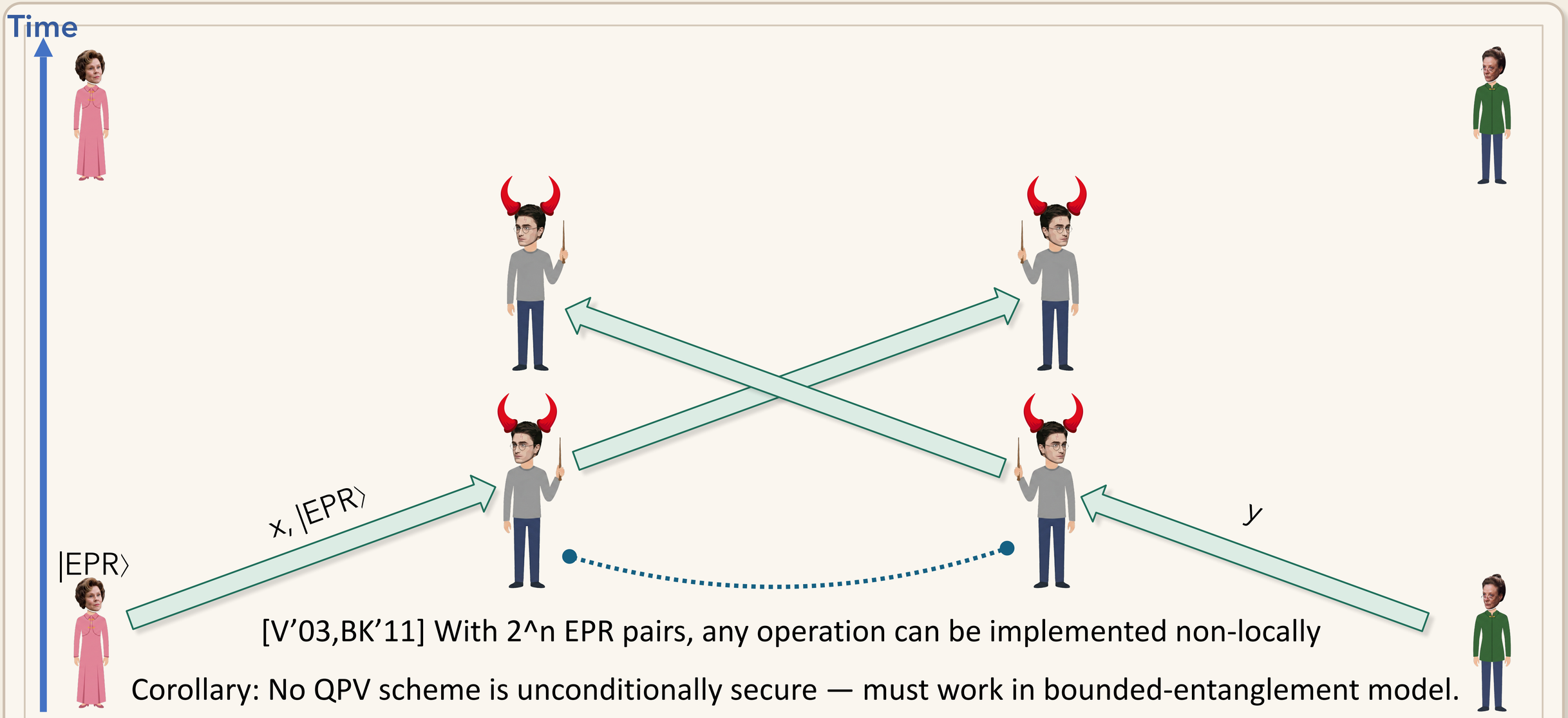
A Quantum Position Verification Scheme

"Purified f-measure/f-BB84"



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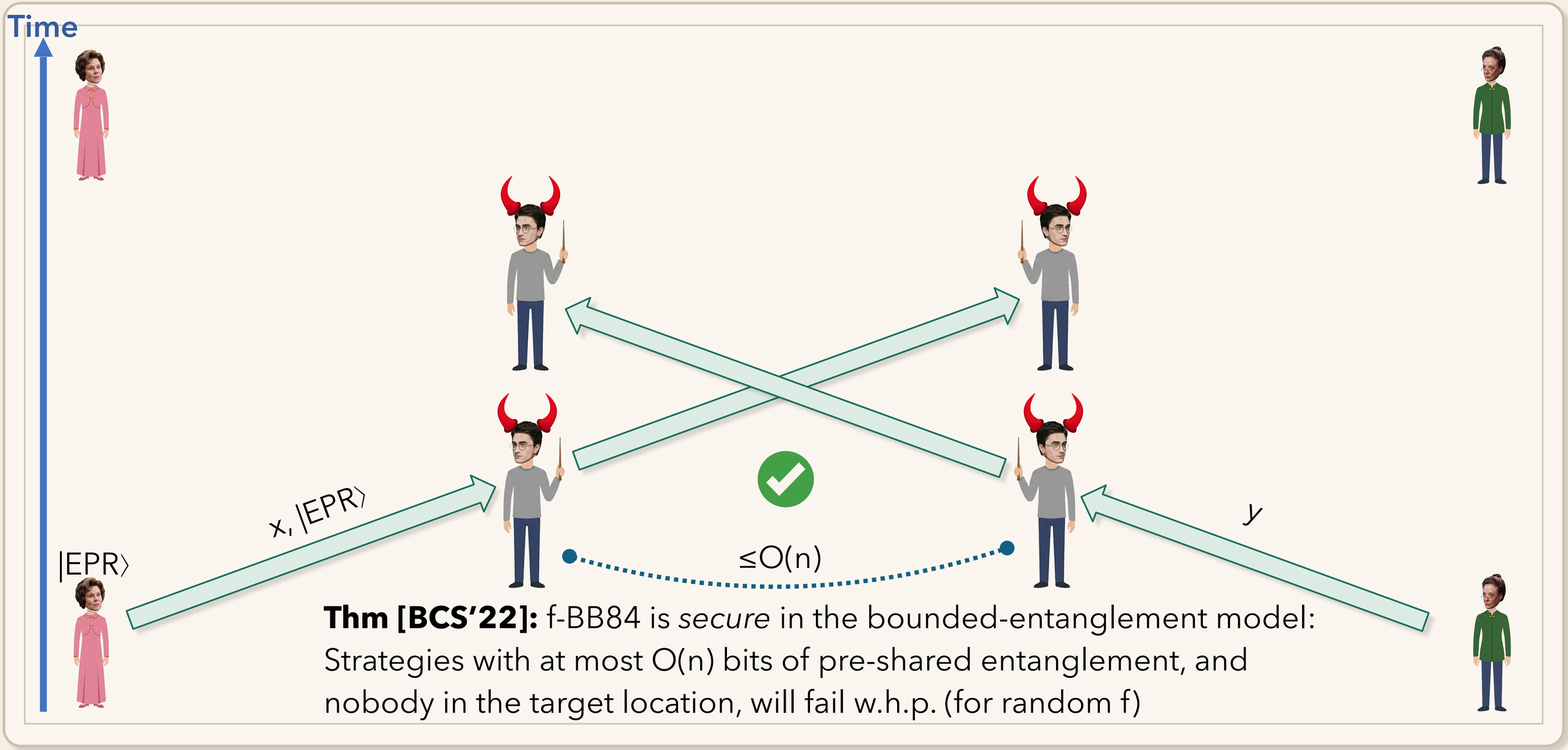

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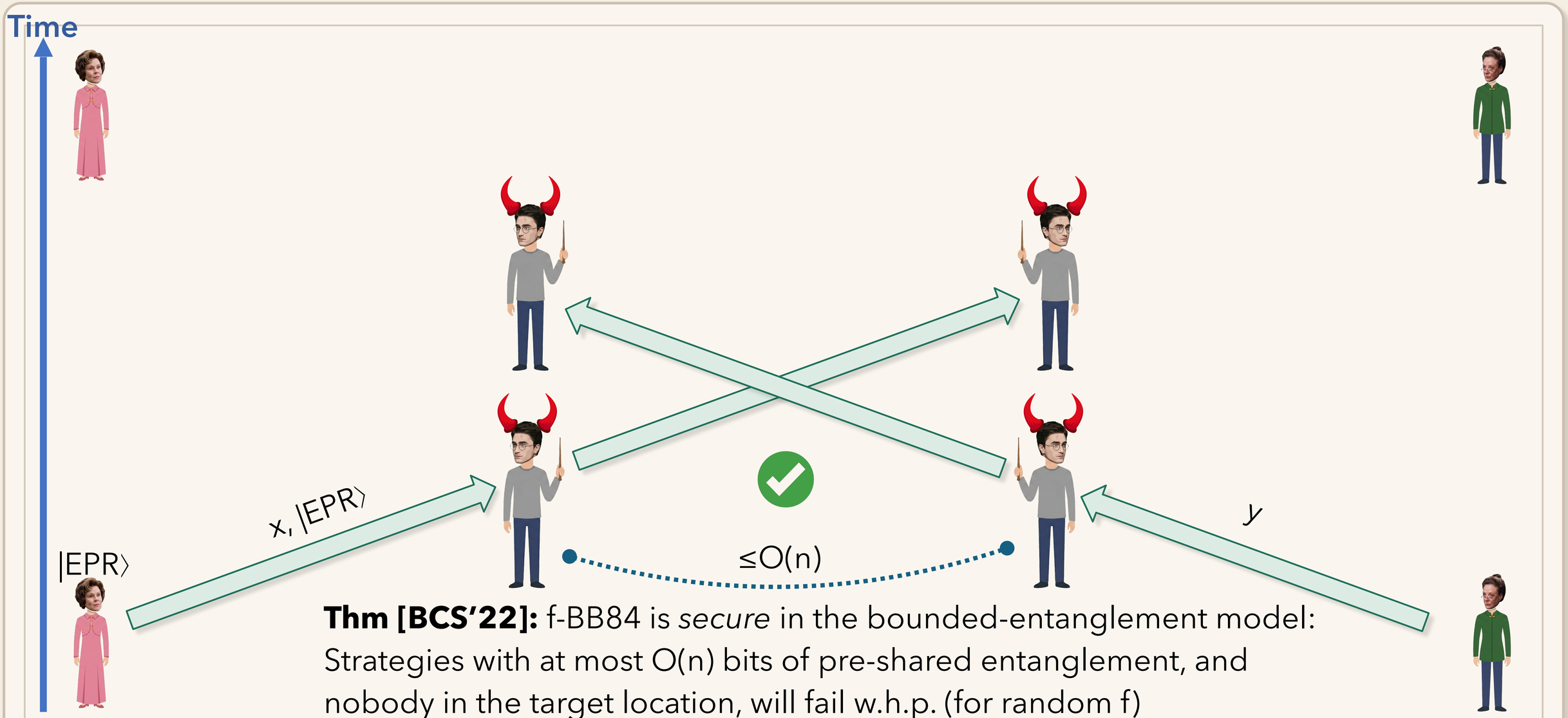

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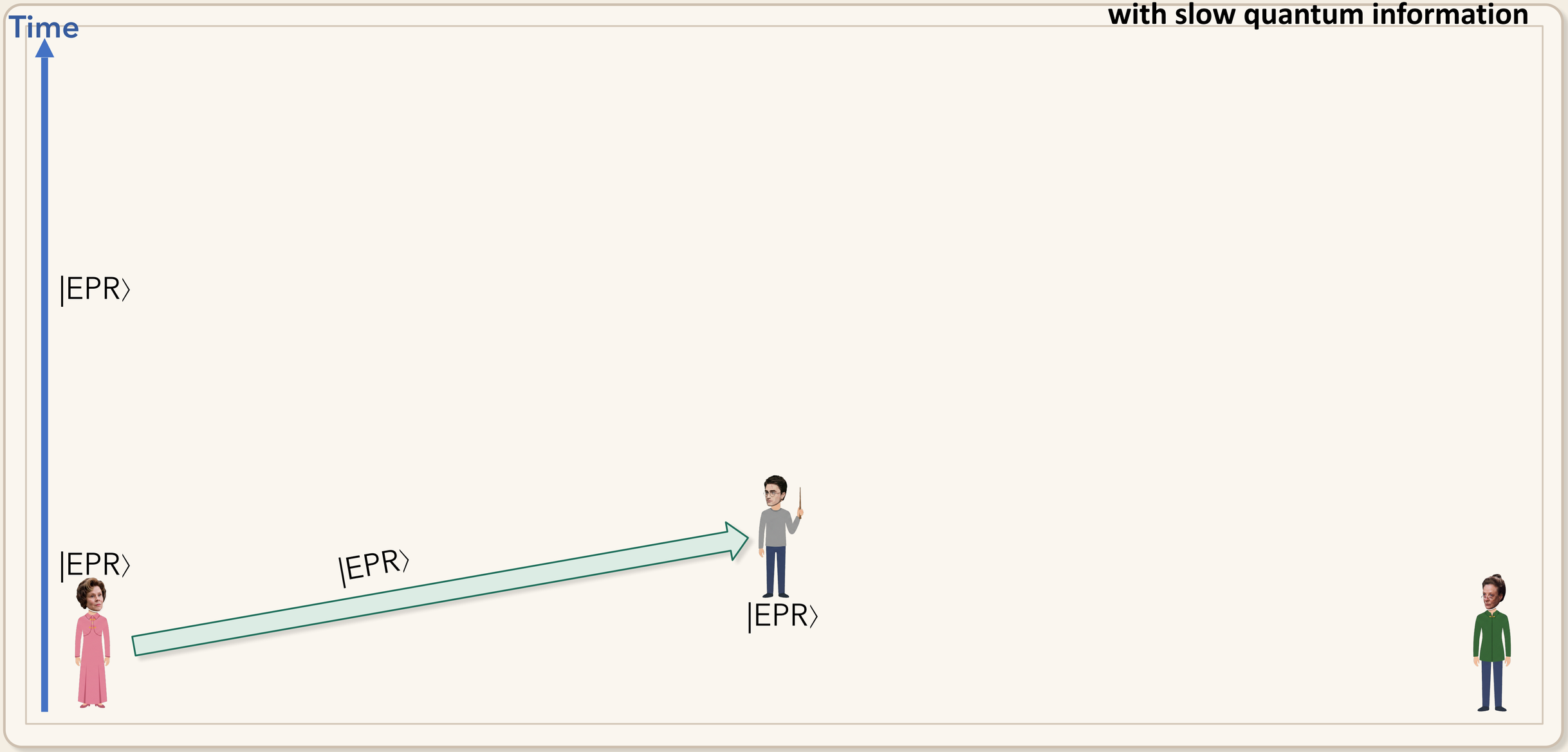

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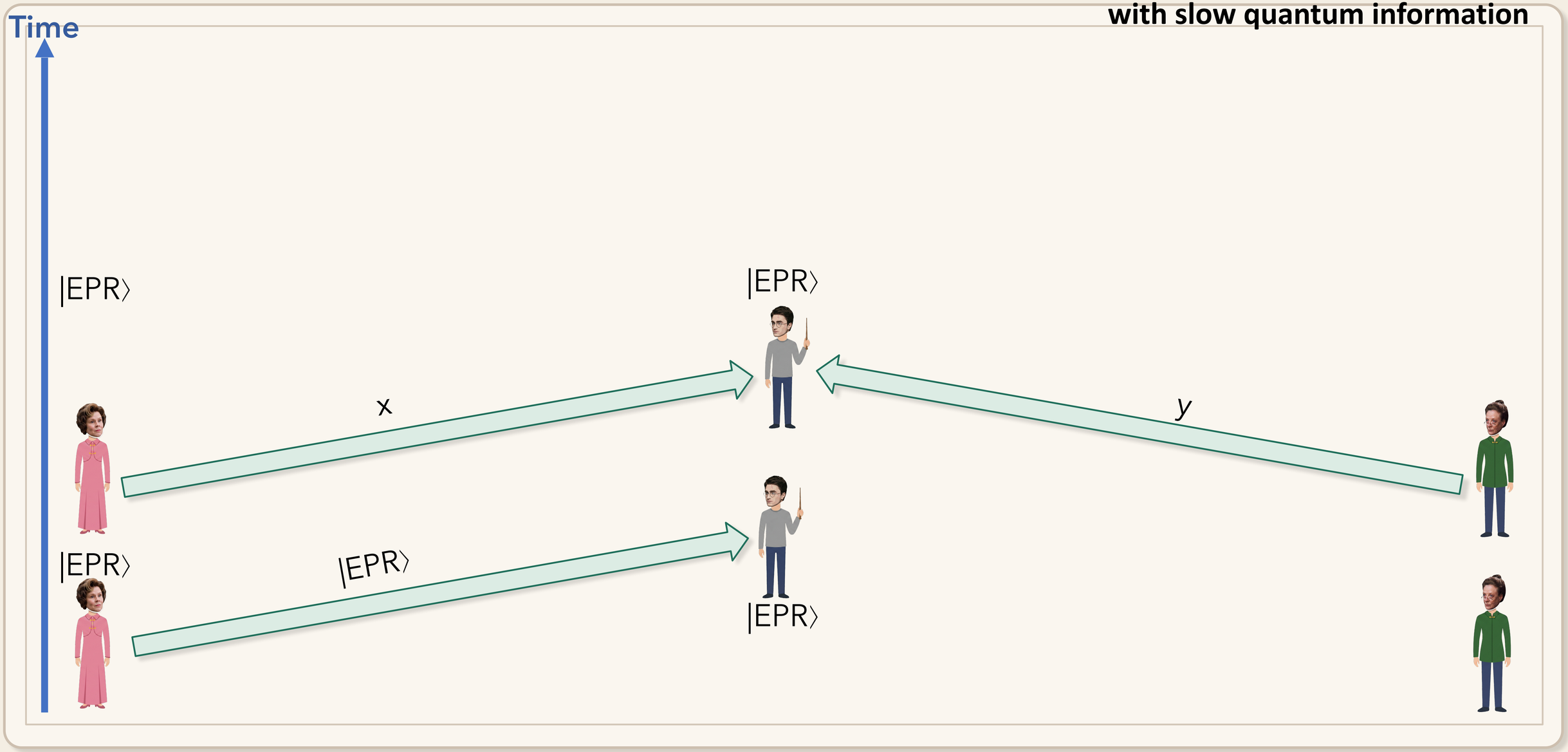

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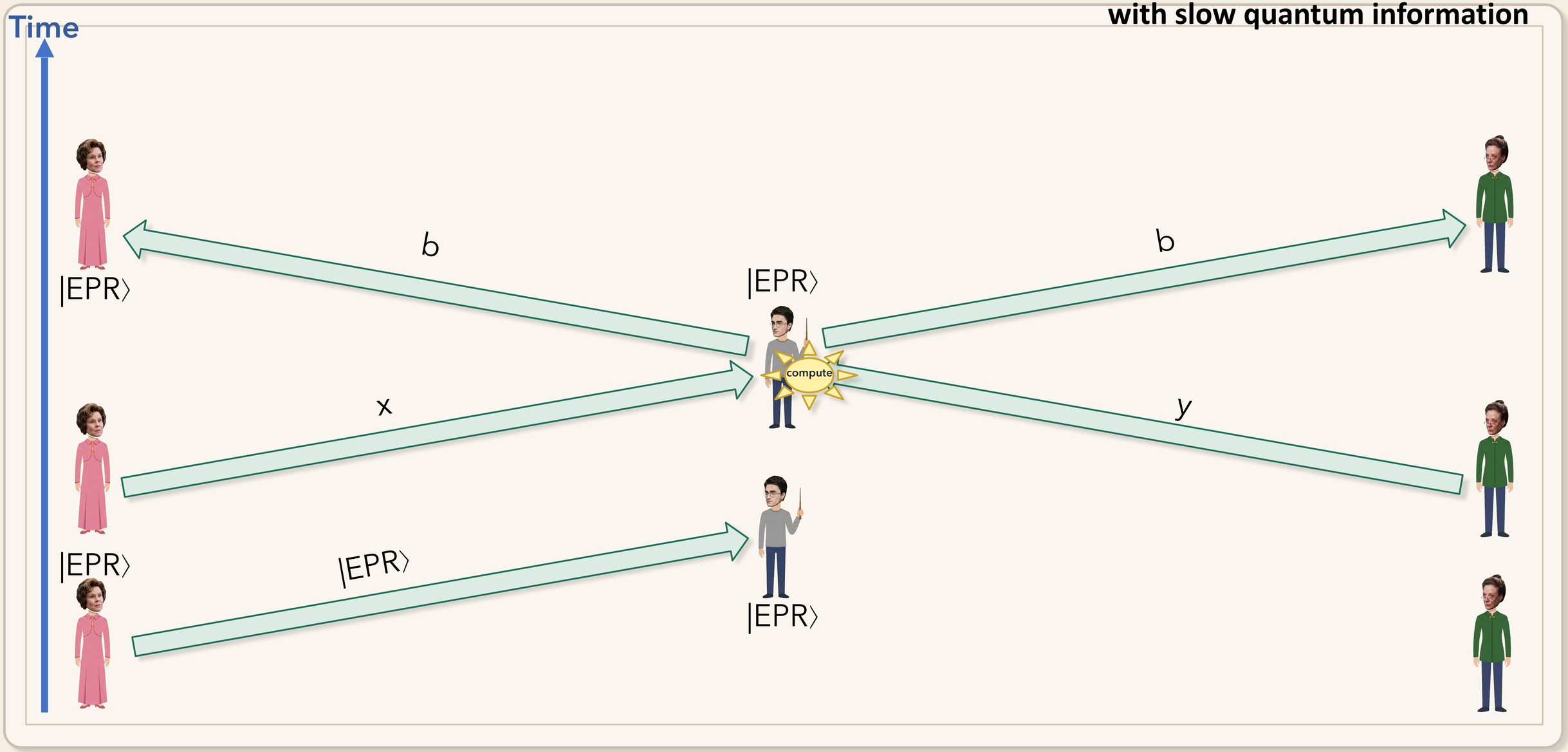

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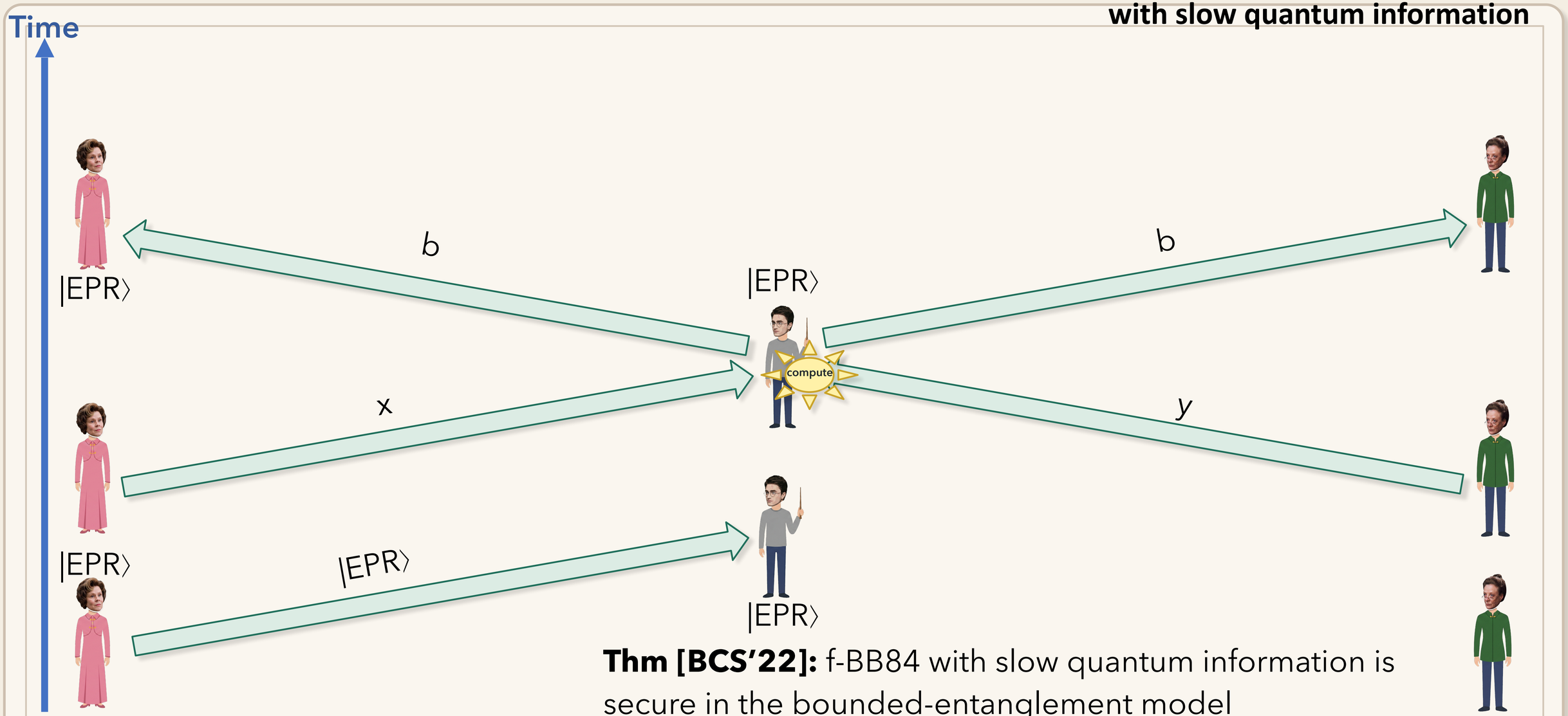

Thm [BCS'22]: f-BB84 with slow quantum information is secure in the bounded-entanglement model

Visualizing PV in Spacetime

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Secure: "Strategies with nobody in the middle should fail with high probability."
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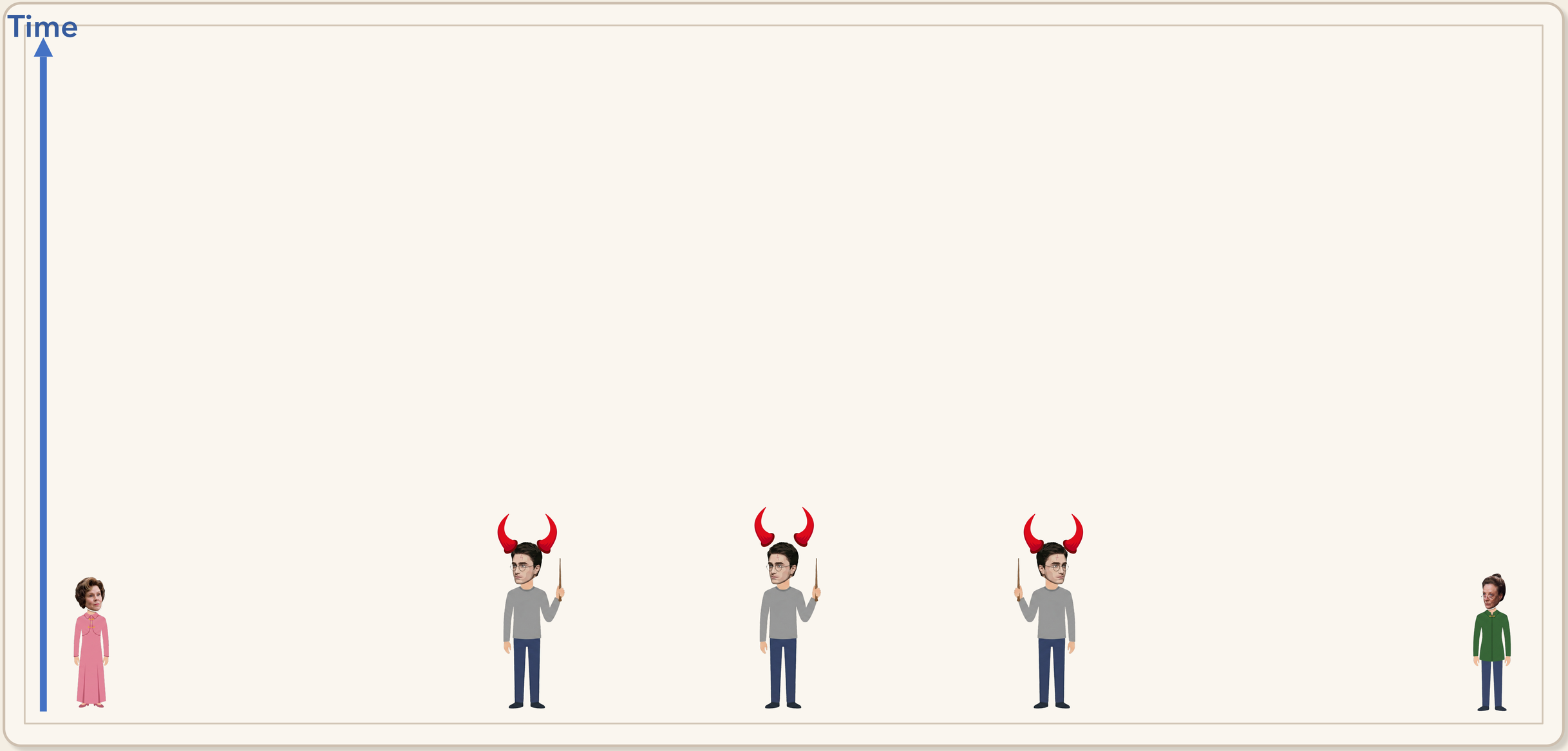
But we'd like to know *more* about what's happening at the middle.

Can we say, e.g., that the qubit must be measured in the middle?

Non-Localizability of f-BB84



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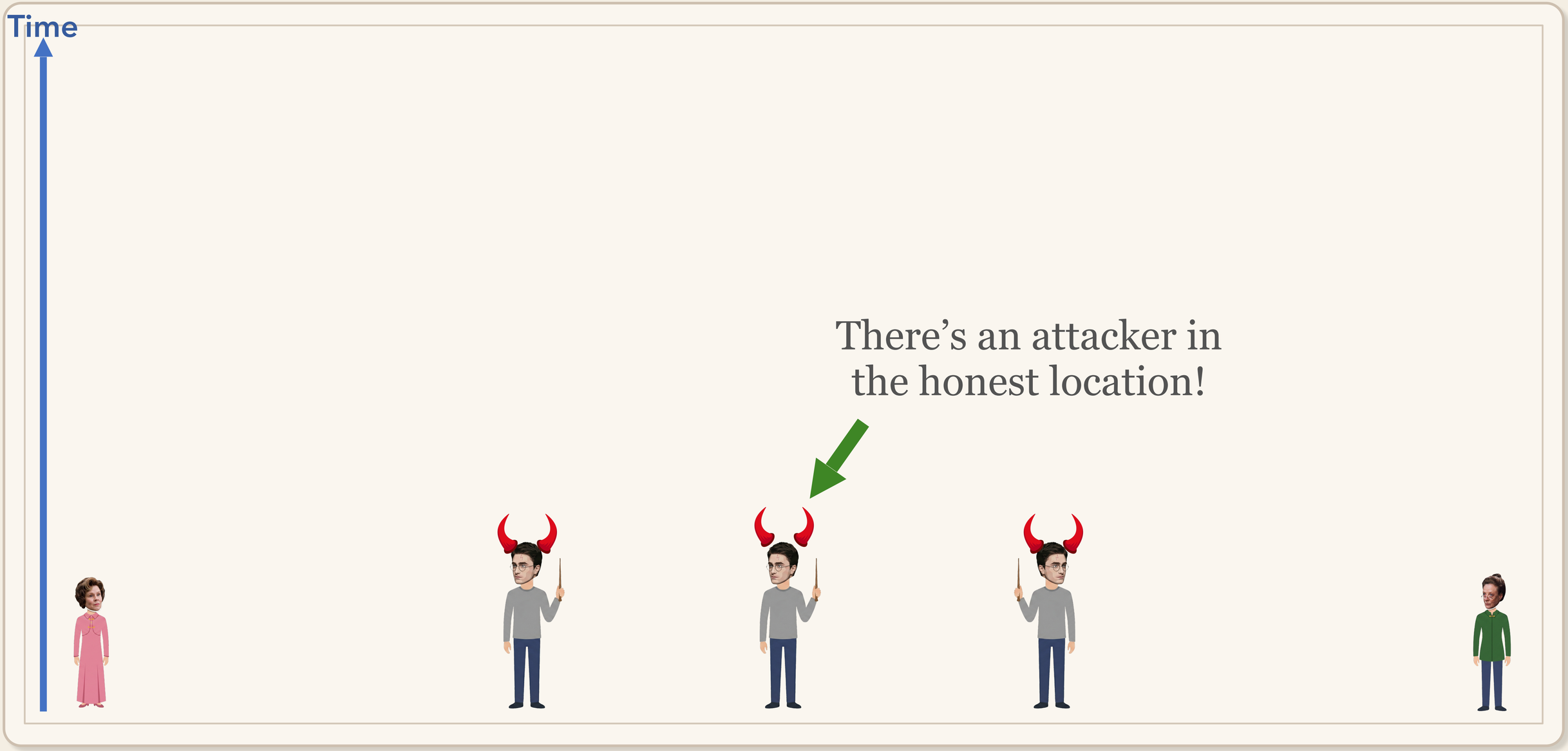

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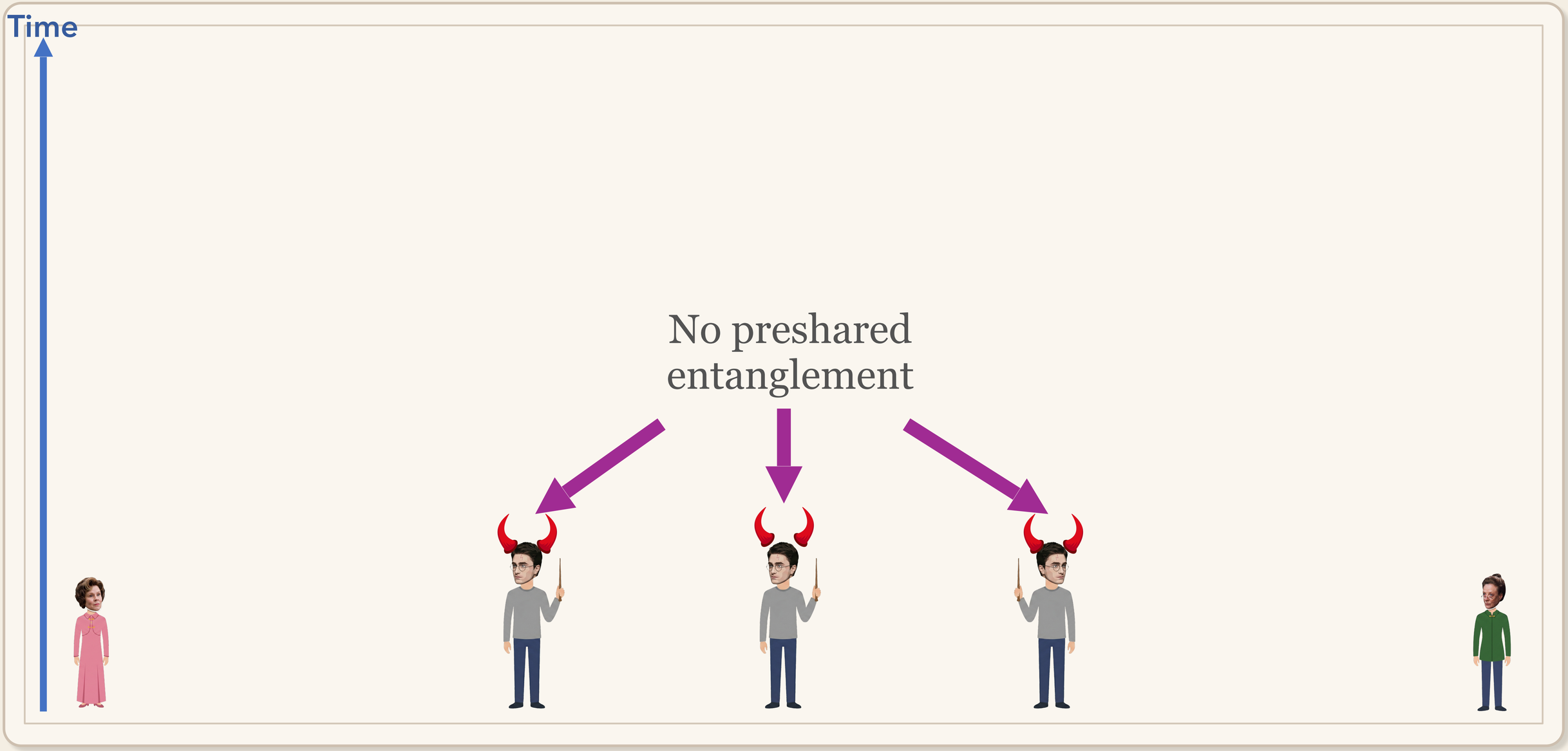

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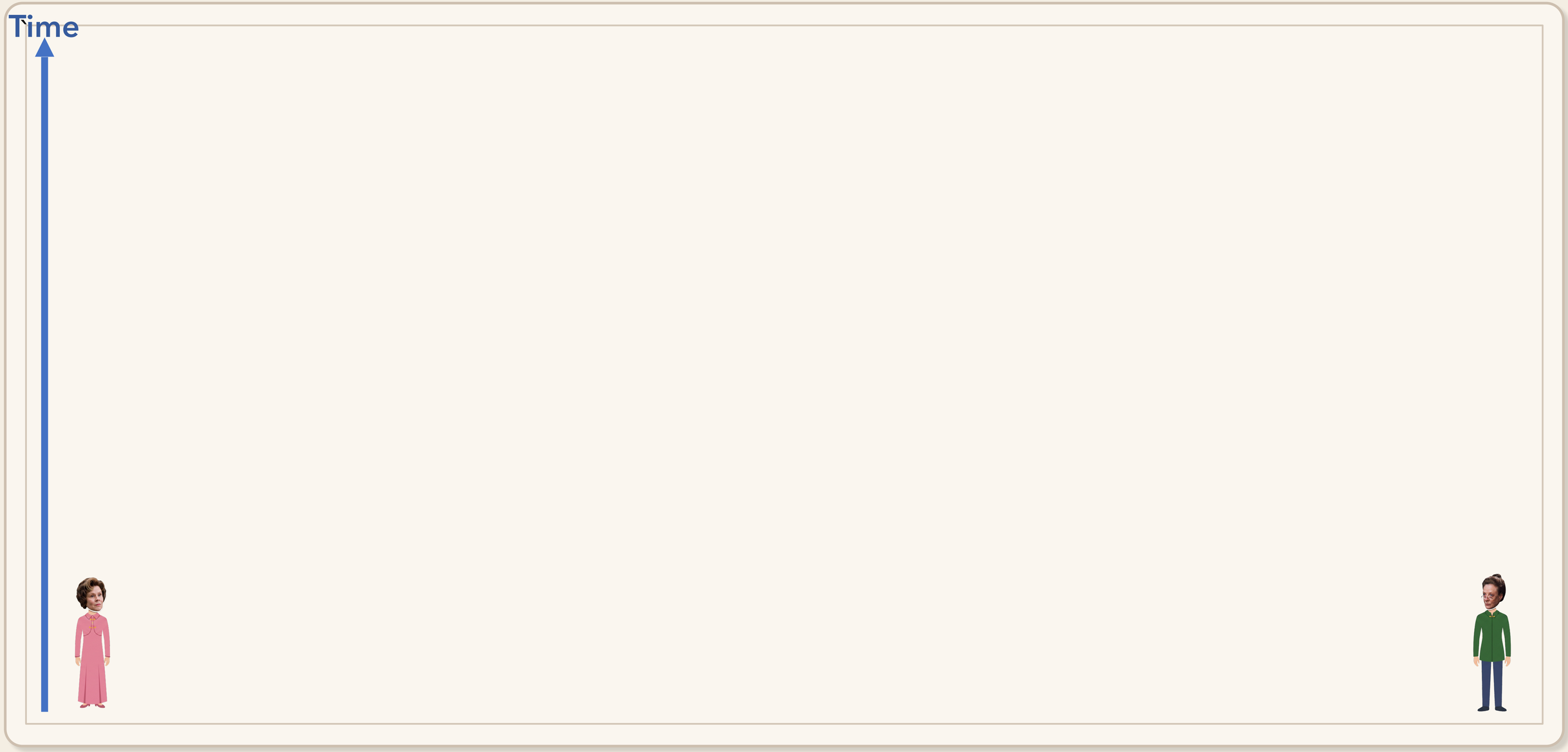

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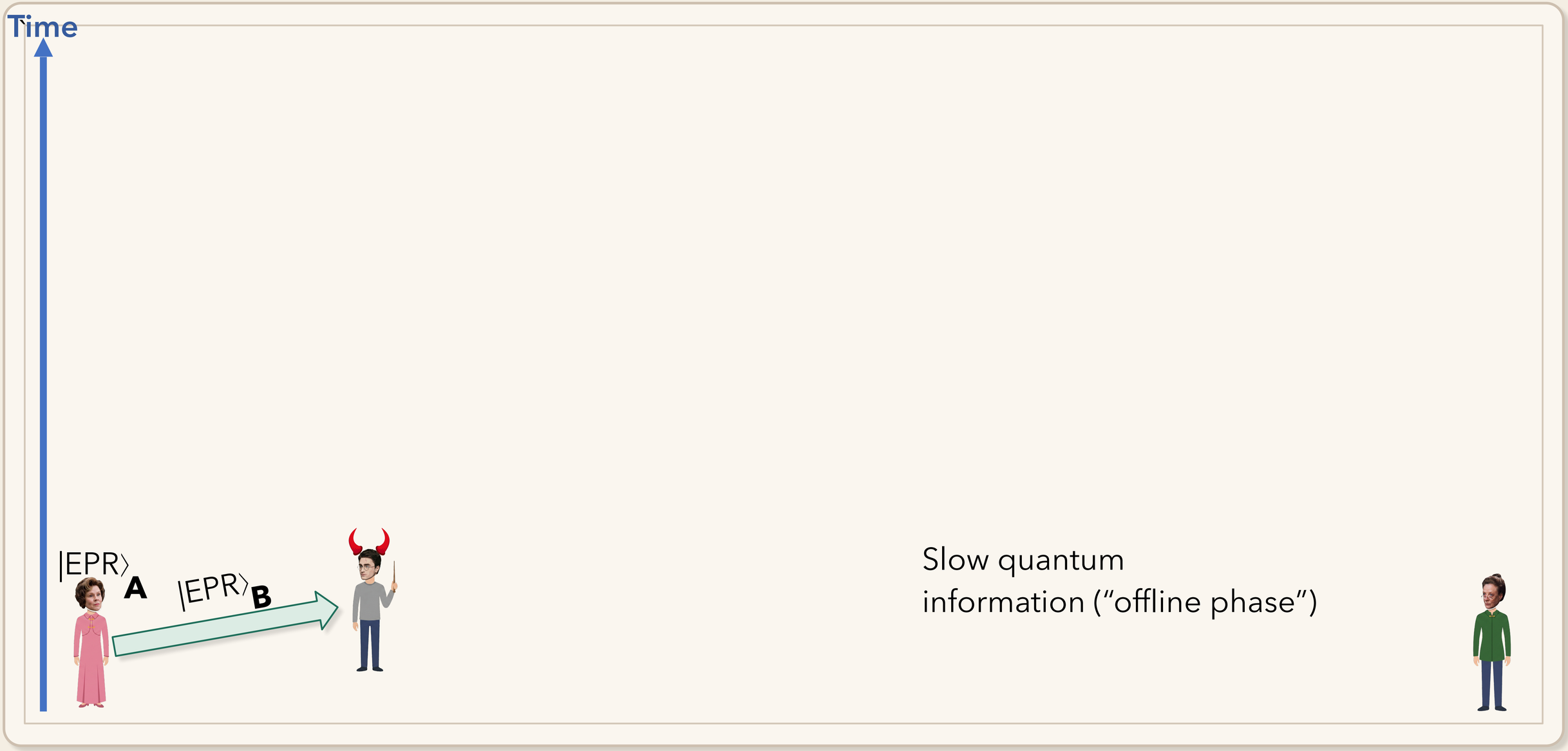

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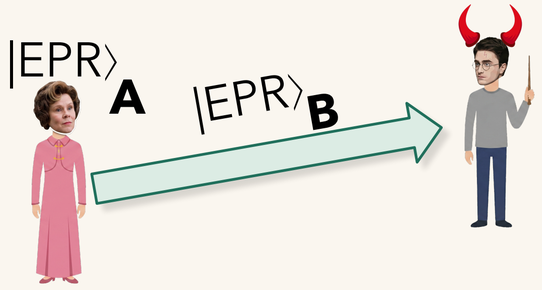
Time



Slow quantum
information (“offline phase”)

$$|x\rangle_{\mathbf{B}} \to \frac{1}{\sqrt{2}} \sum_{b\in\{0,1\}} |b\rangle_{\mathbf{L}} |b\rangle_{\mathbf{M}_0} |b\oplus x\rangle_{\mathbf{M}_1} |b\oplus x\rangle_{\mathbf{R}}$$

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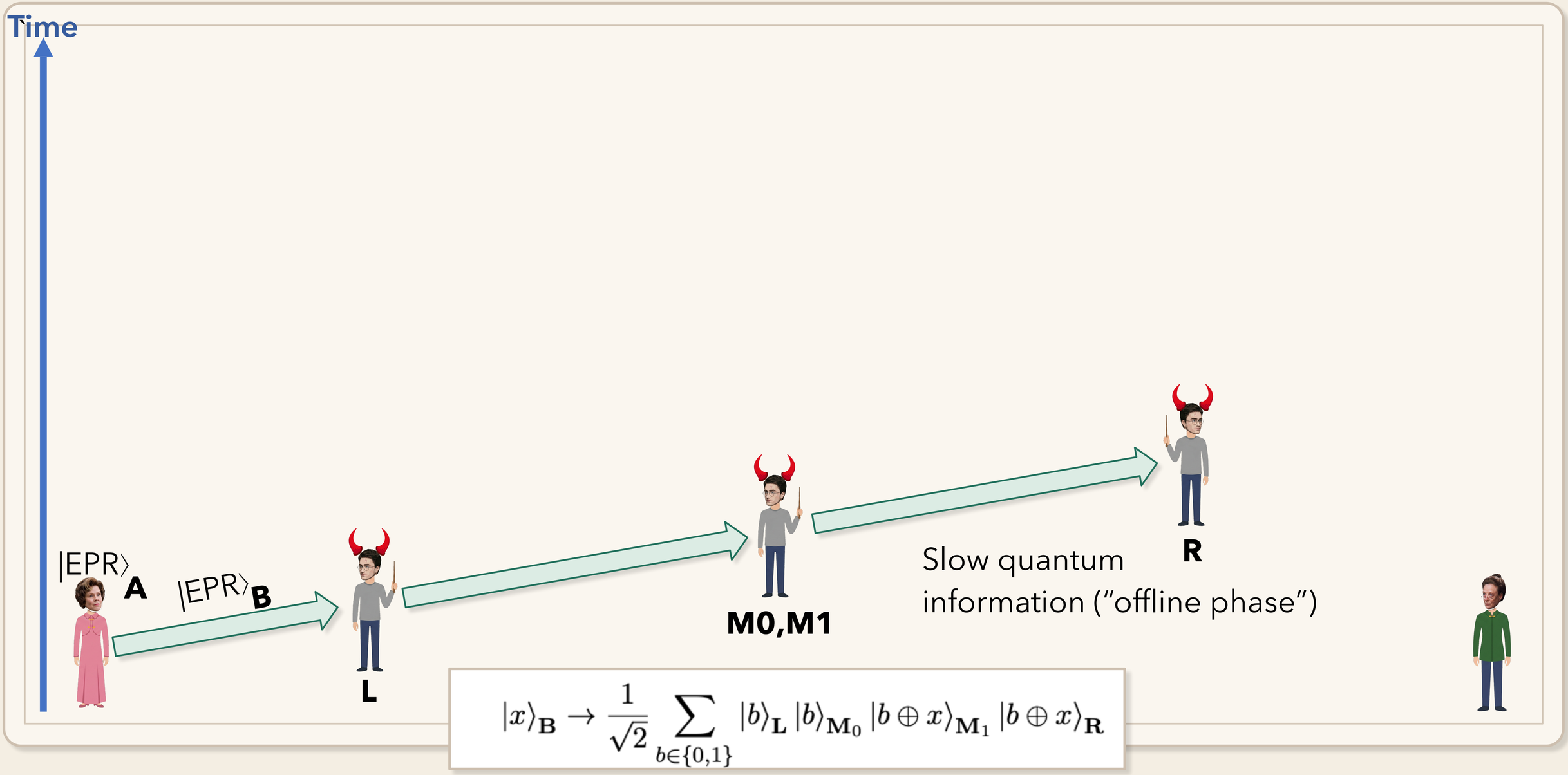

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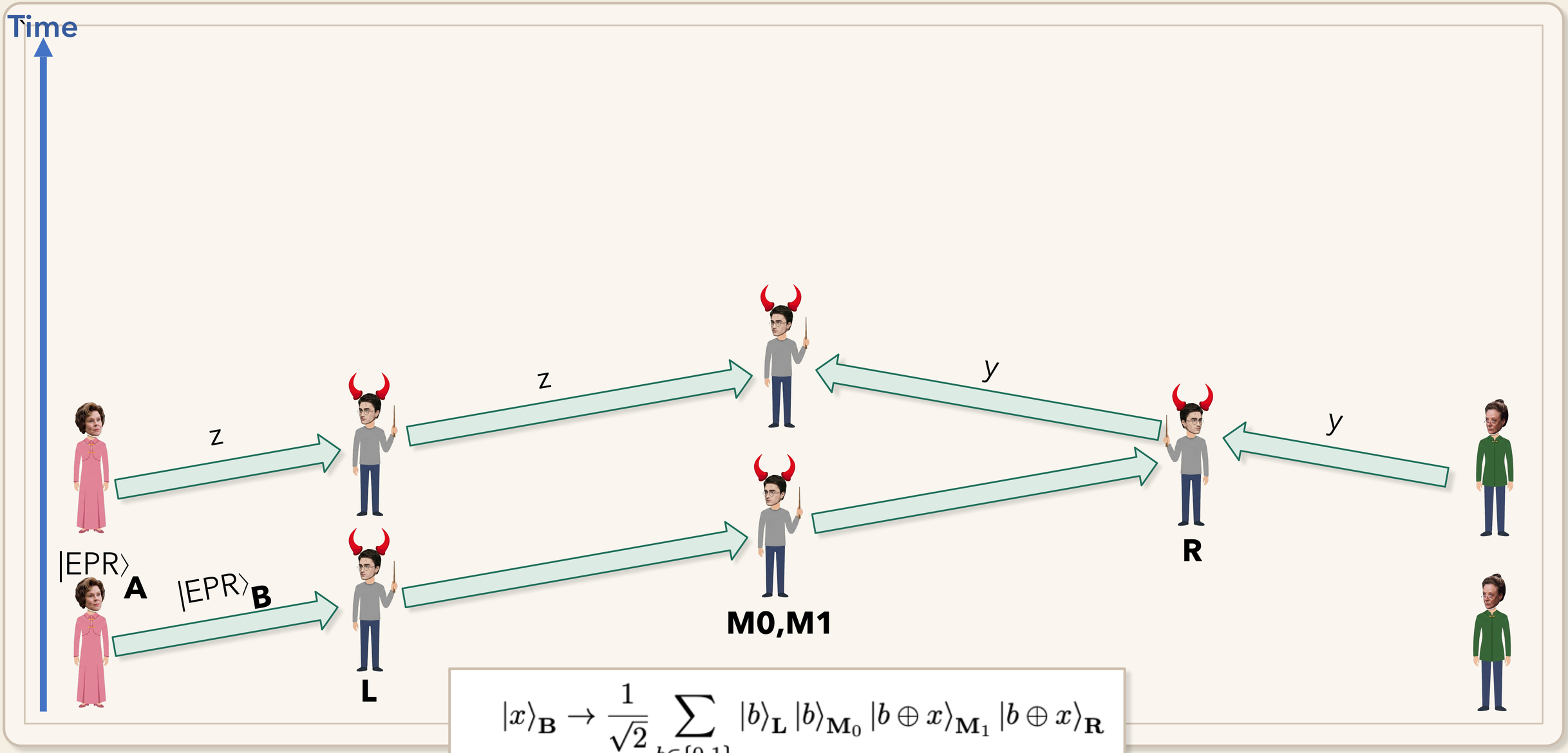

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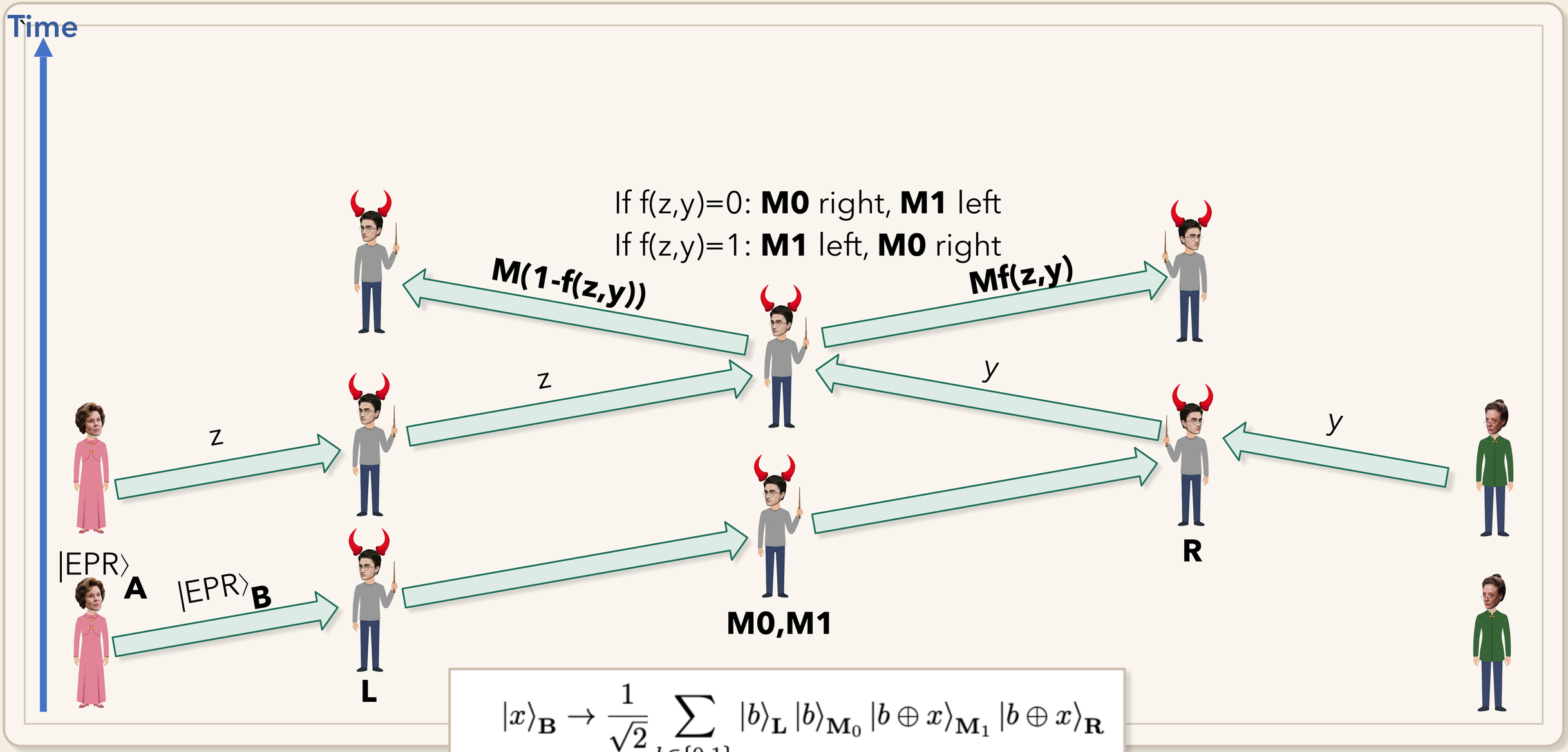

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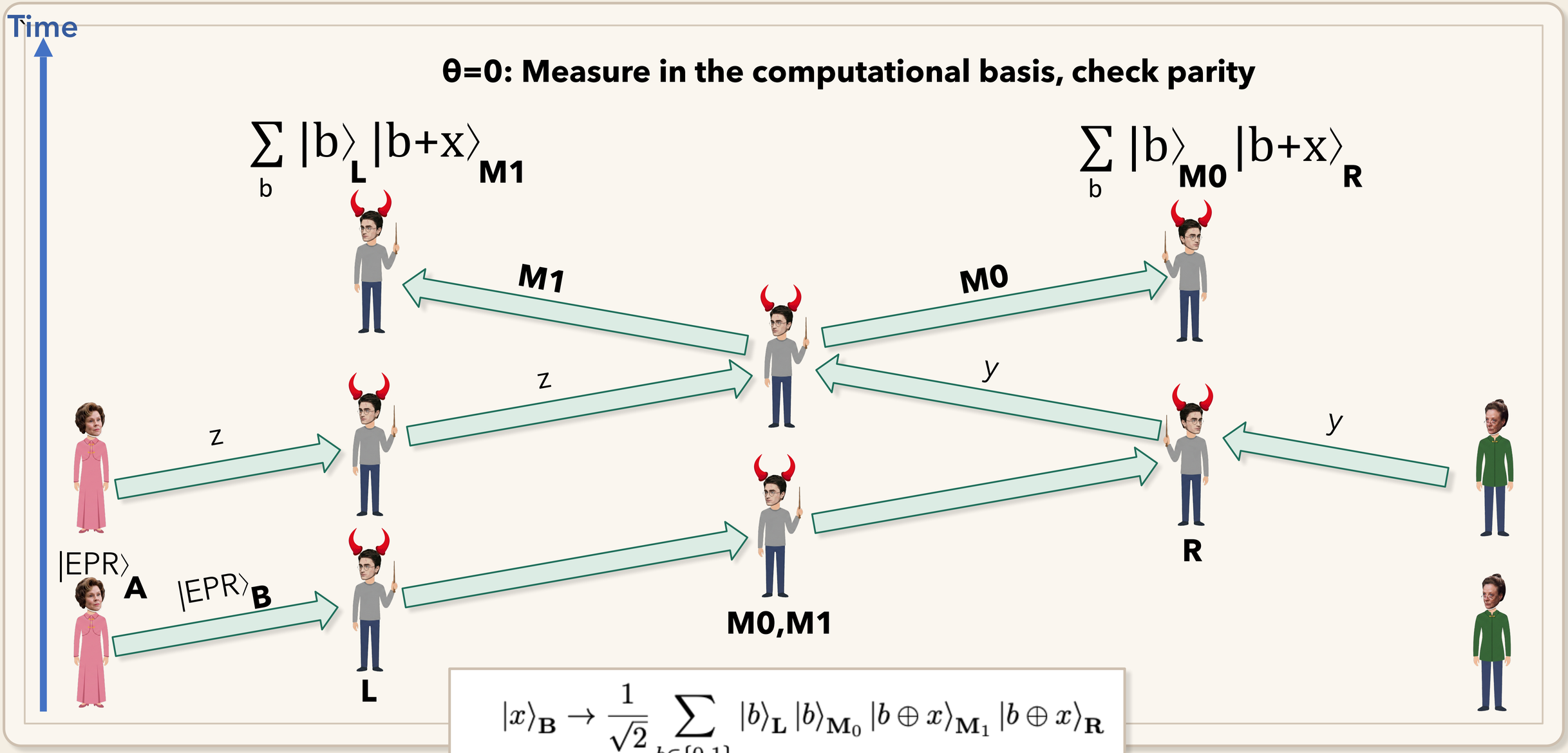

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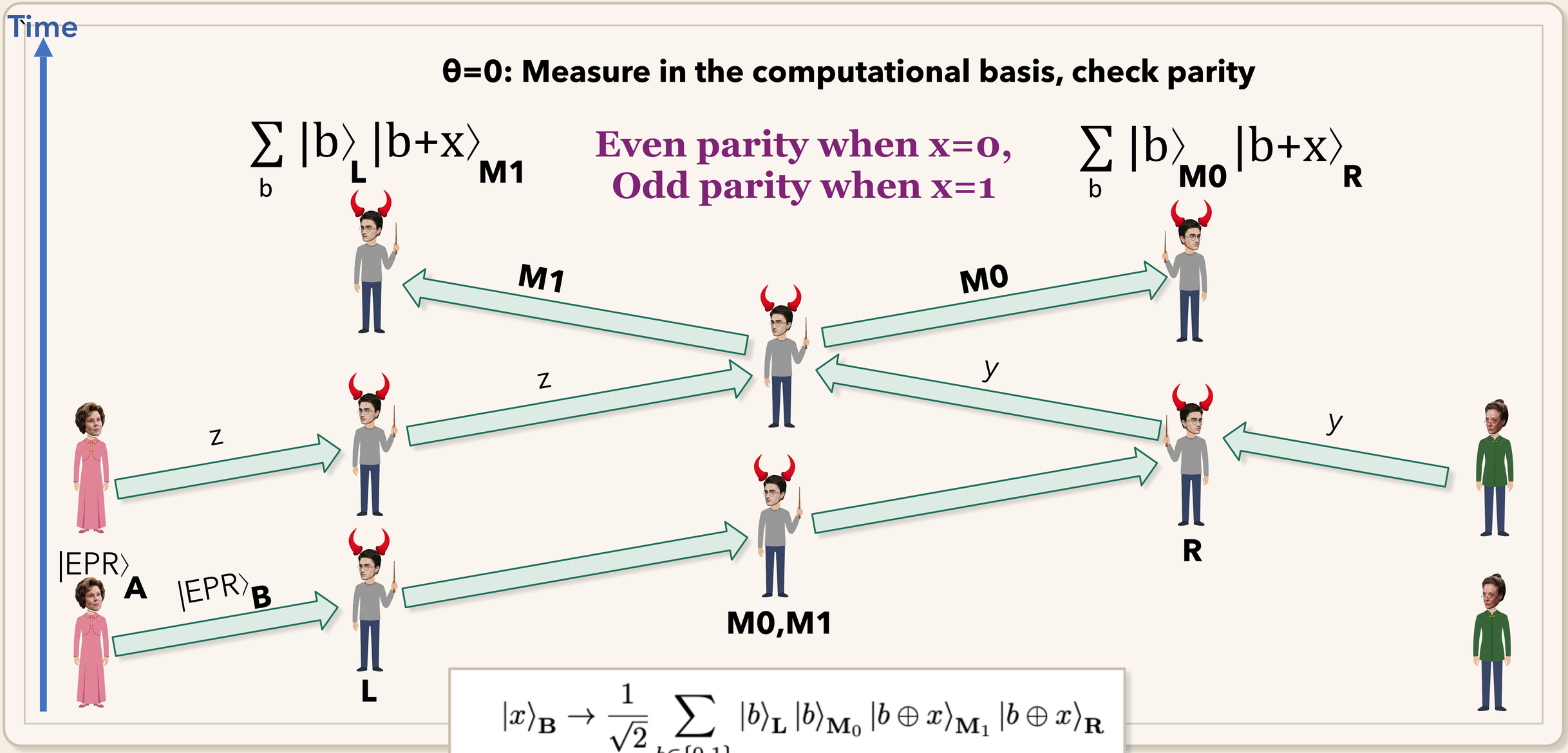

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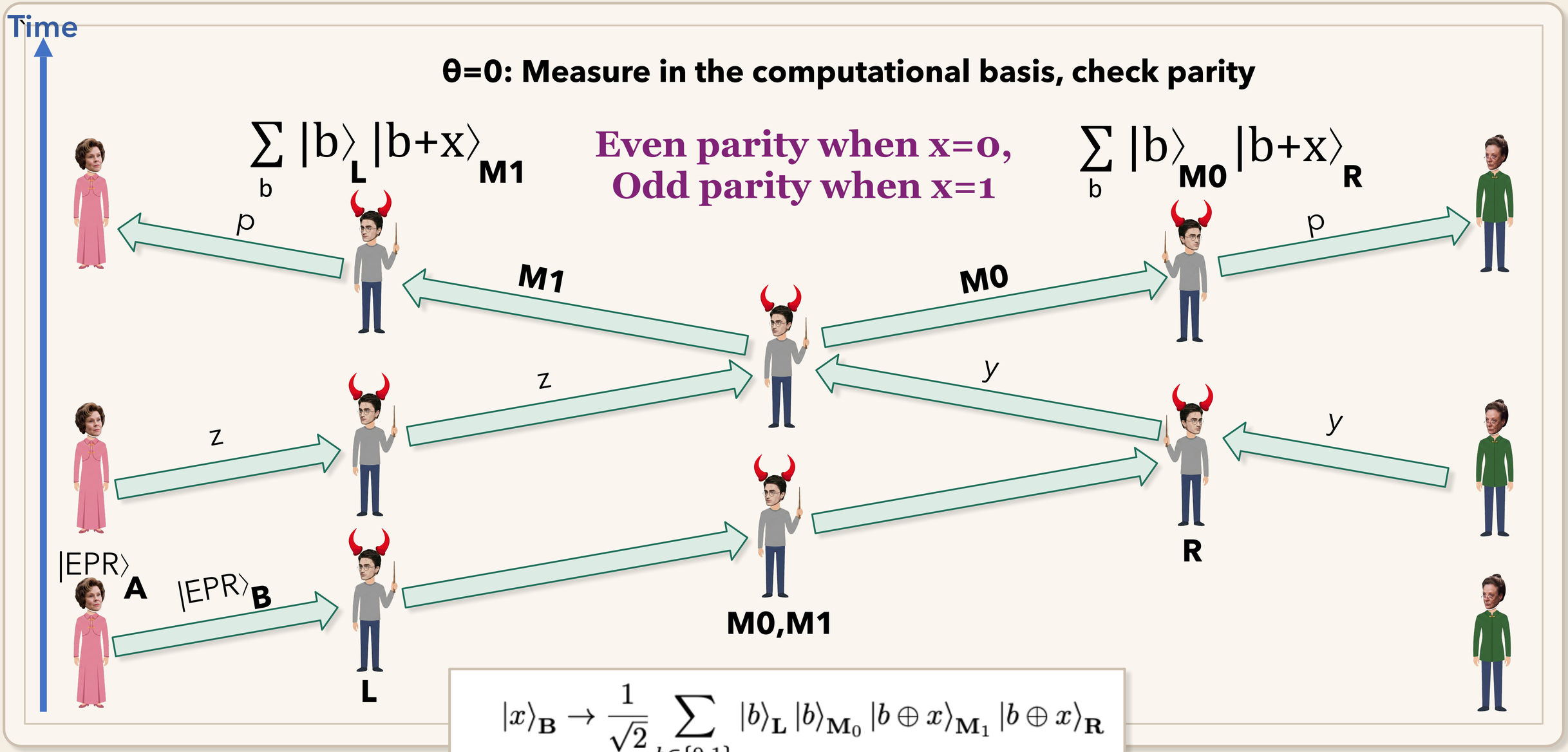
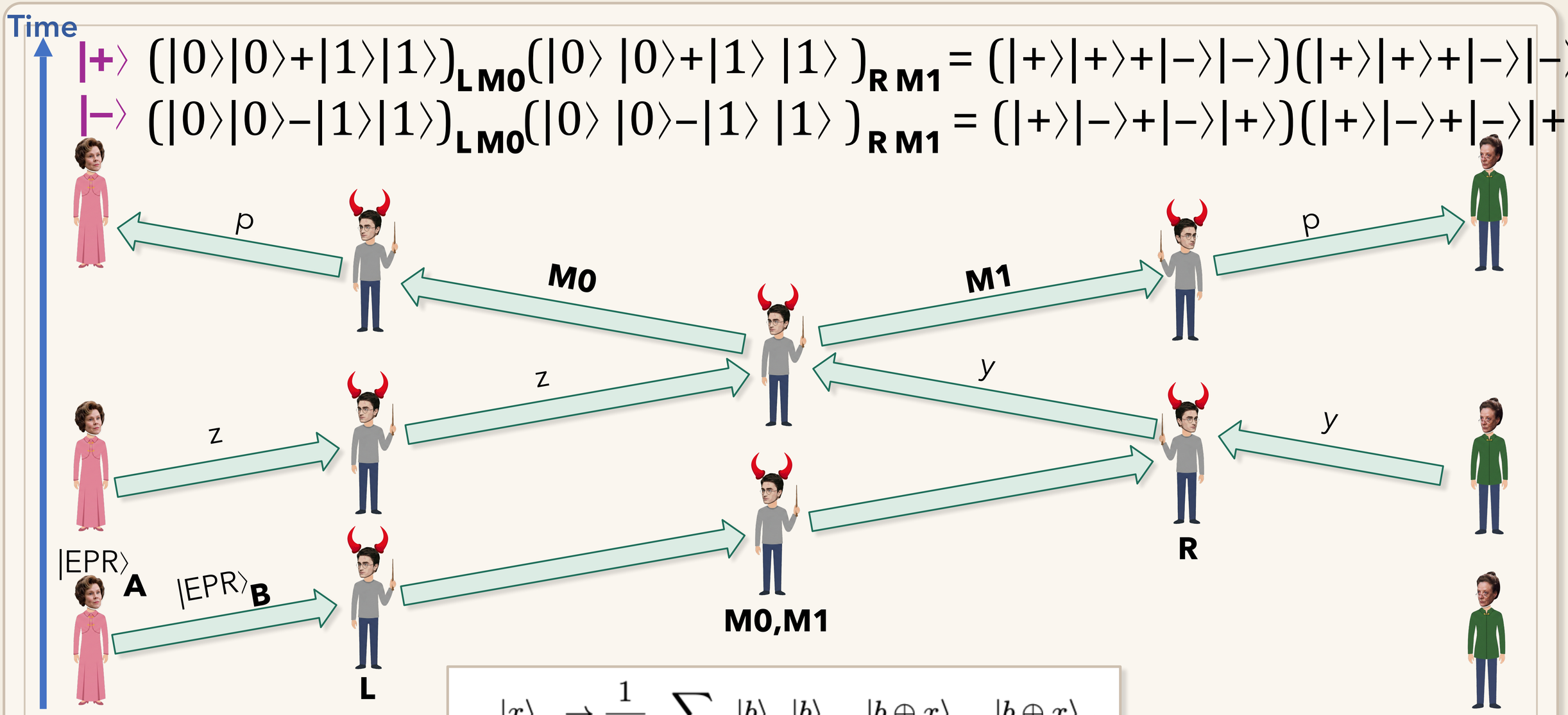


Non-Localizability of f-BB84

θ=1: Measure in the Hadamard basis, check parity



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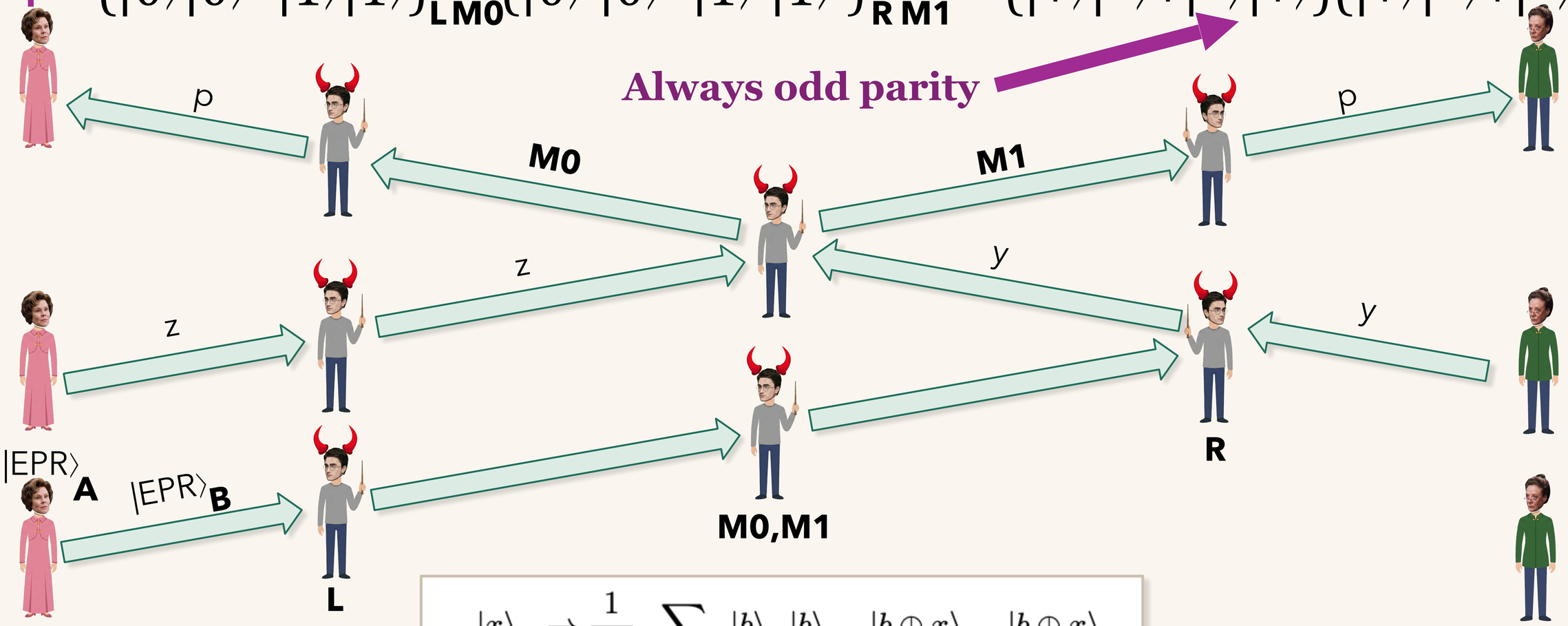

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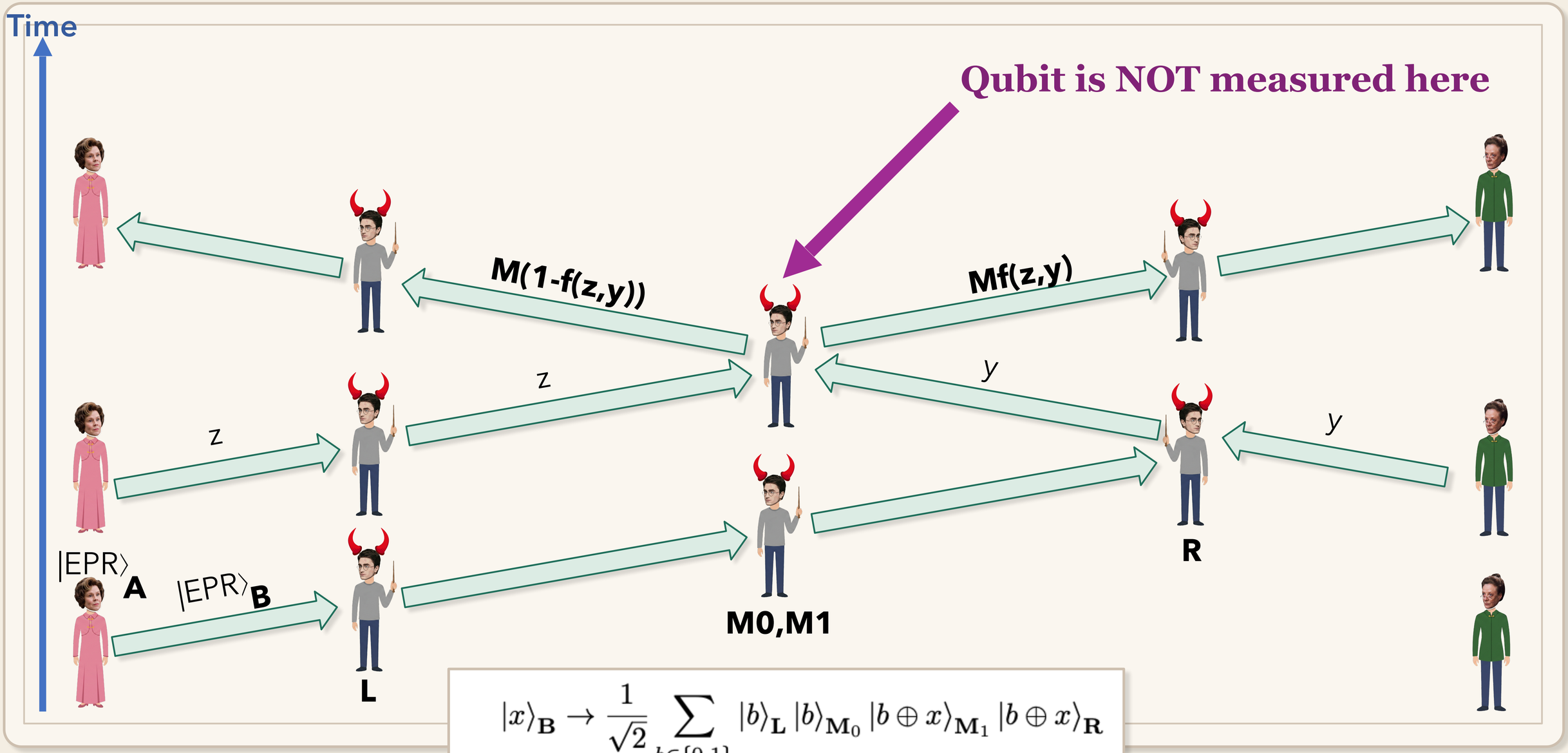

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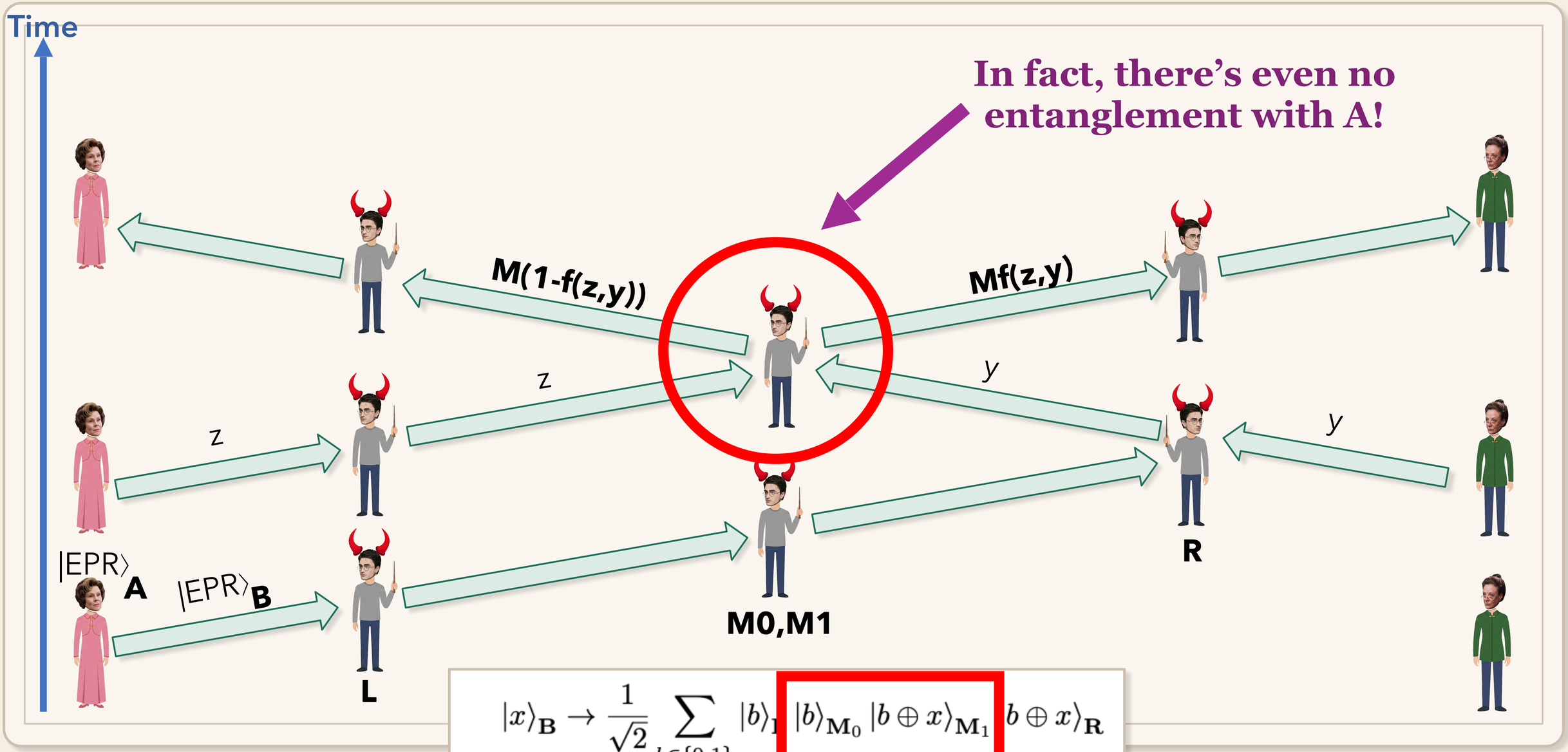

Security Guarantees for Position-Based Crypto

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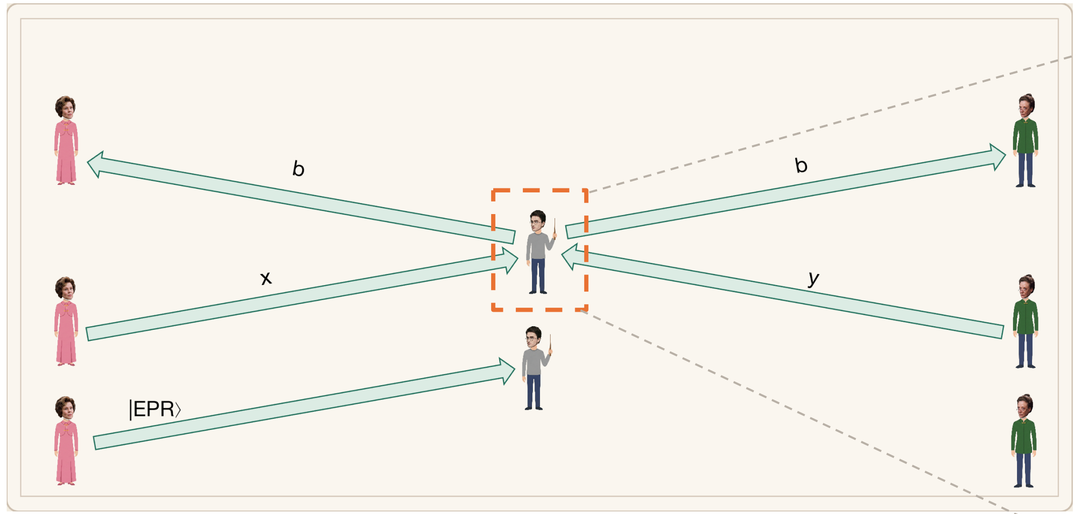
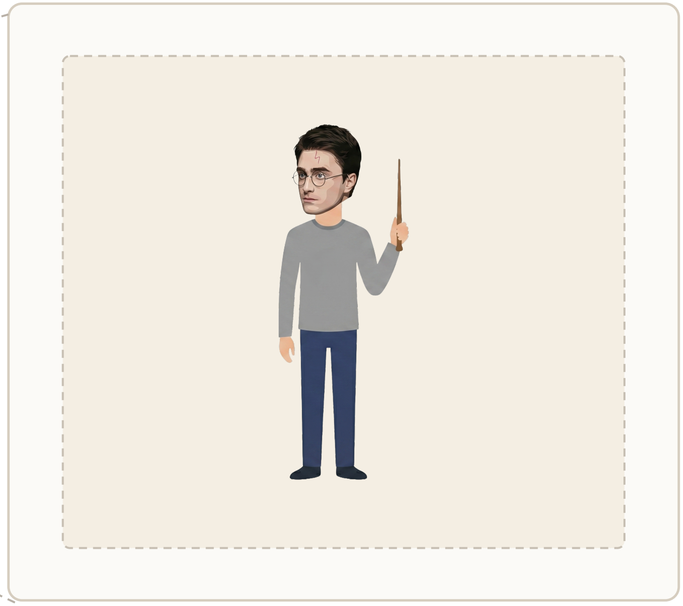
Finding an Entity Somewhere

> “Zooming in” on a region of space to see what it contains





Given all of the quantum registers in [ ], can we reconstruct  ?

Finding a Unique Quantum State

But again, even if we find something,
how do we *identify* it?

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For any pair of *disjoint* spatial regions A and B, and any time t, a physically unique object can be "found" within region A at time t only if it *cannot* be "found" within region B at time t.

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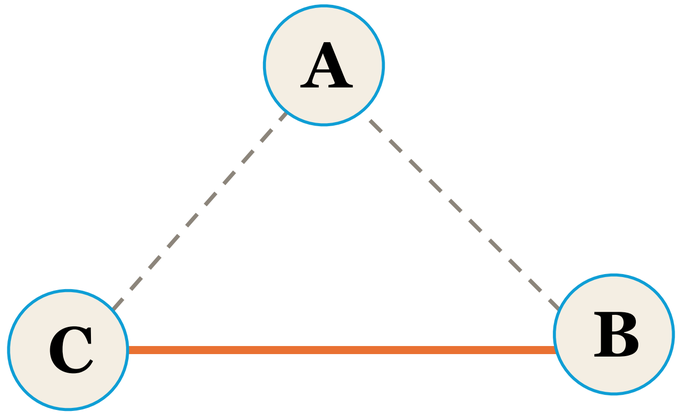
Note: unique => unclonable,
but not vice versa! e.g. BB84 states

Physically Unique Resource — Entanglement!

Monogamy-of-entanglement is precisely a physical uniqueness property



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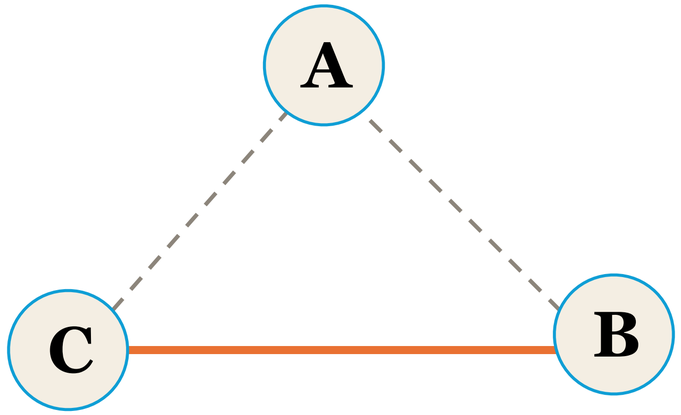
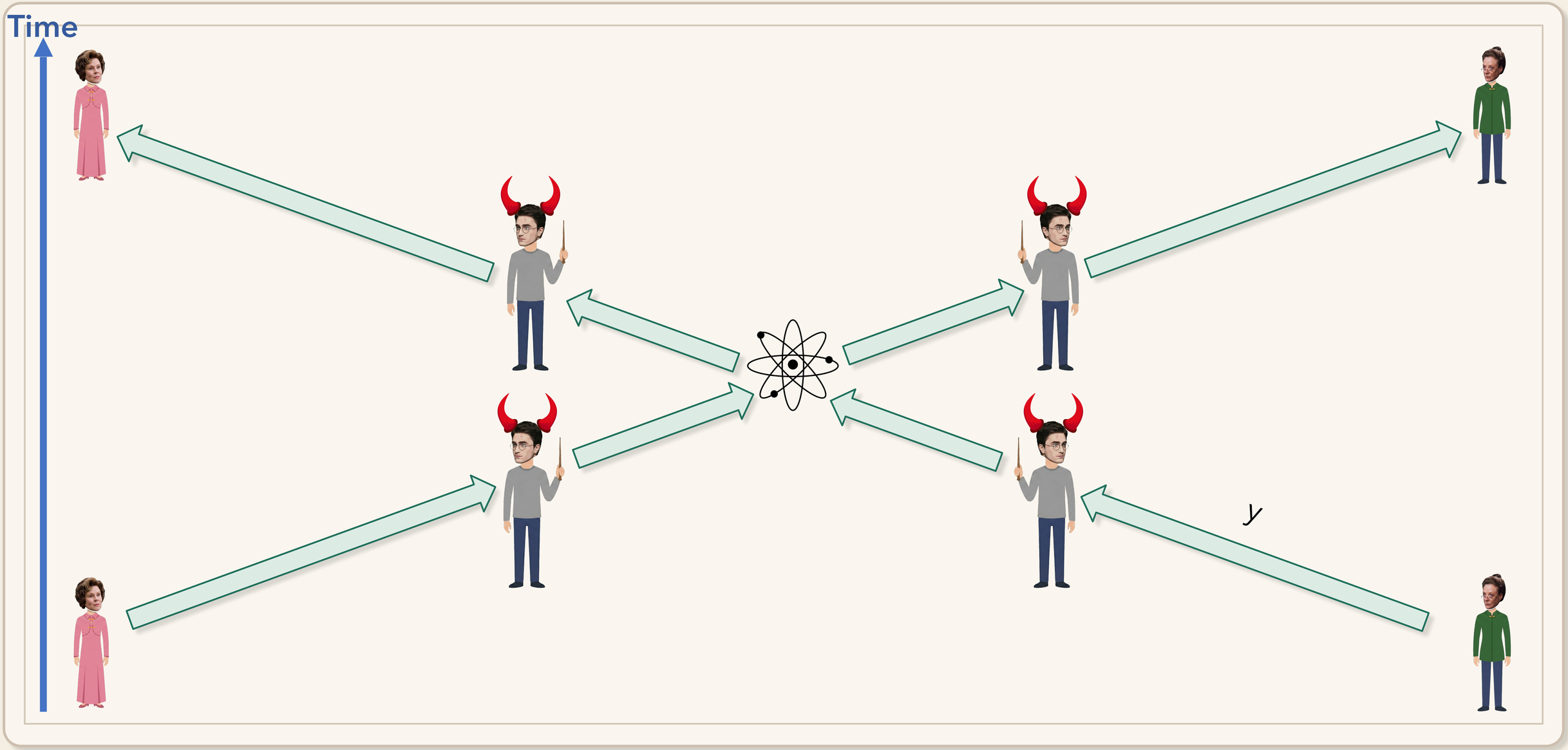


Uniqueness: If an EPR half can be found in one region, then it cannot be found in any disjoint region

Visualizing PV in Spacetime



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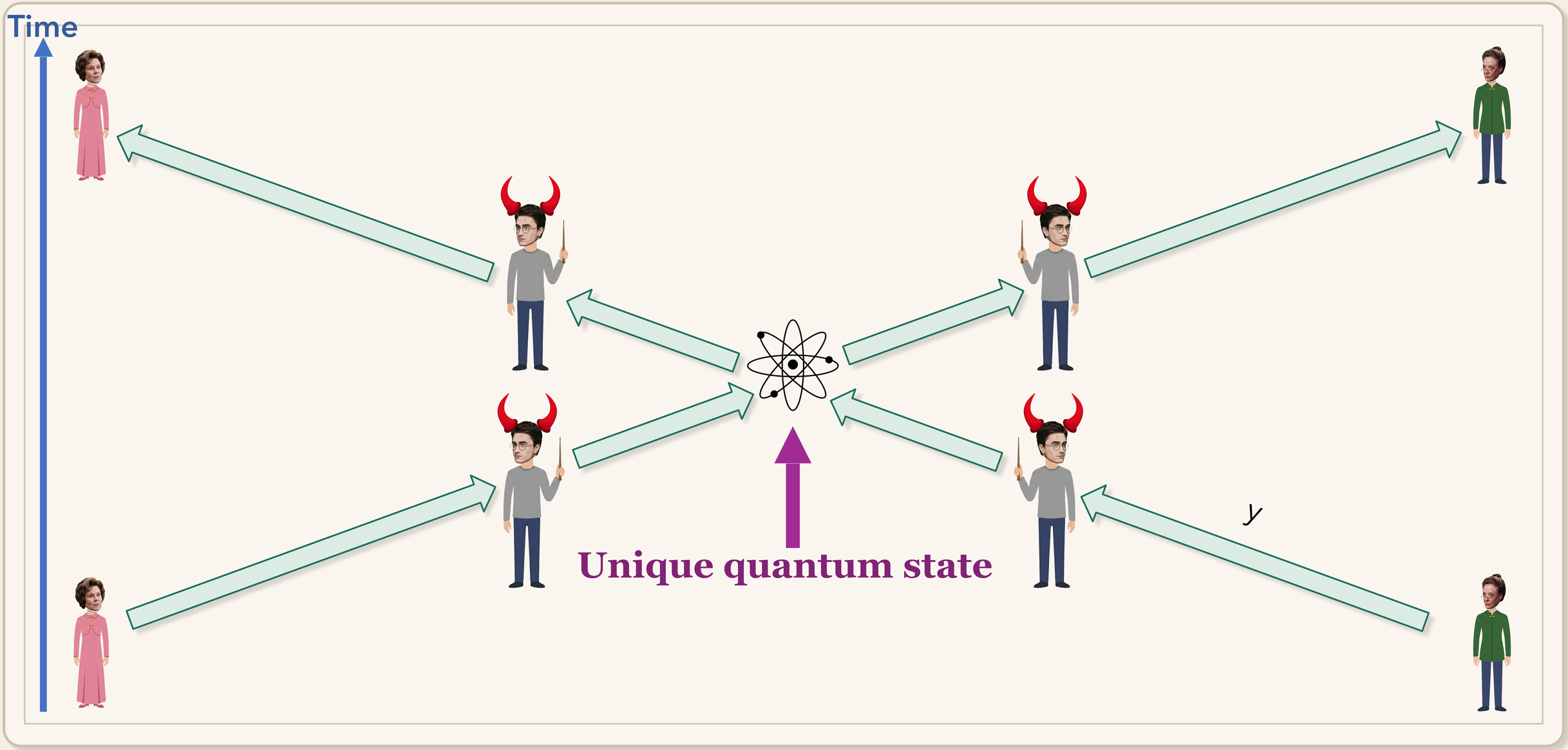

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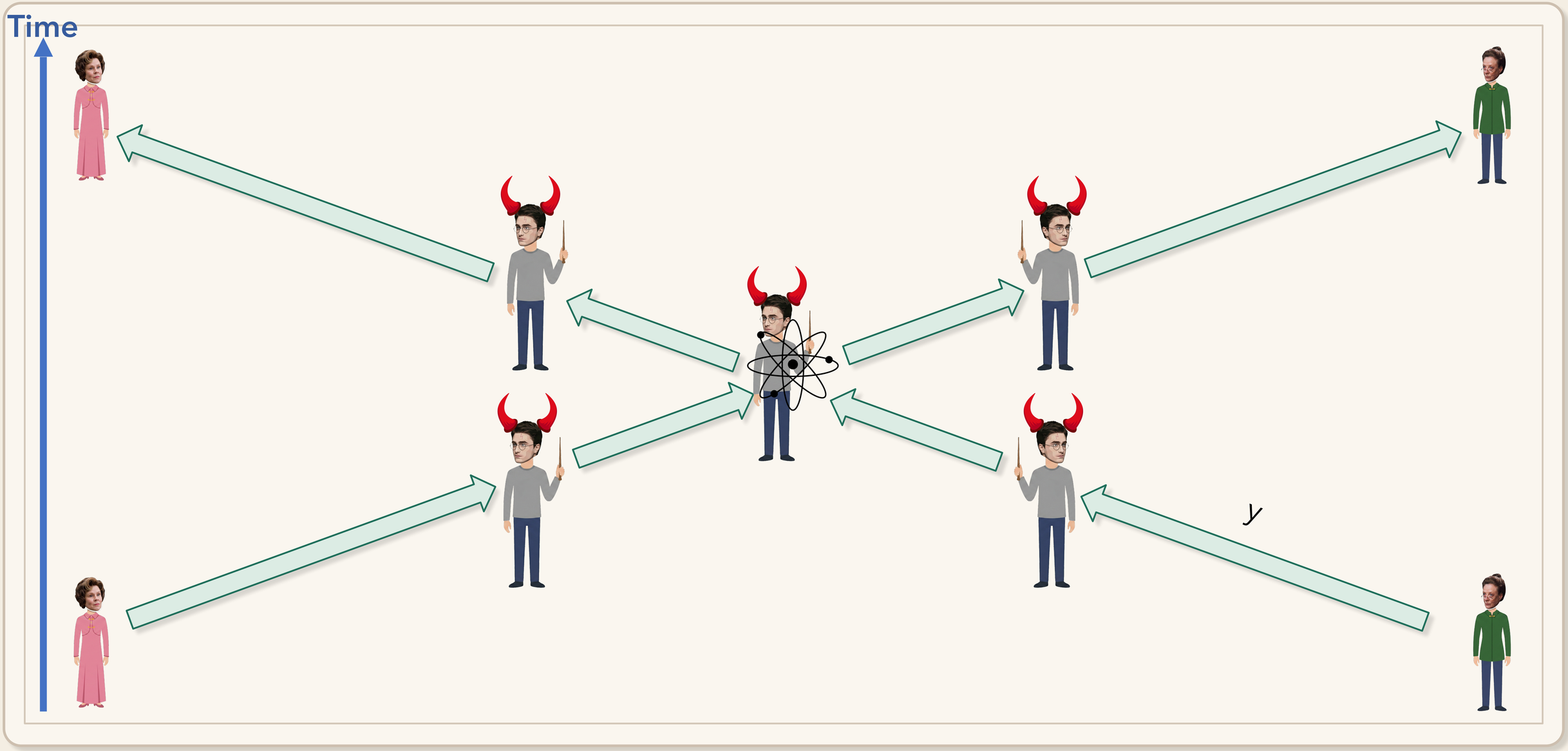

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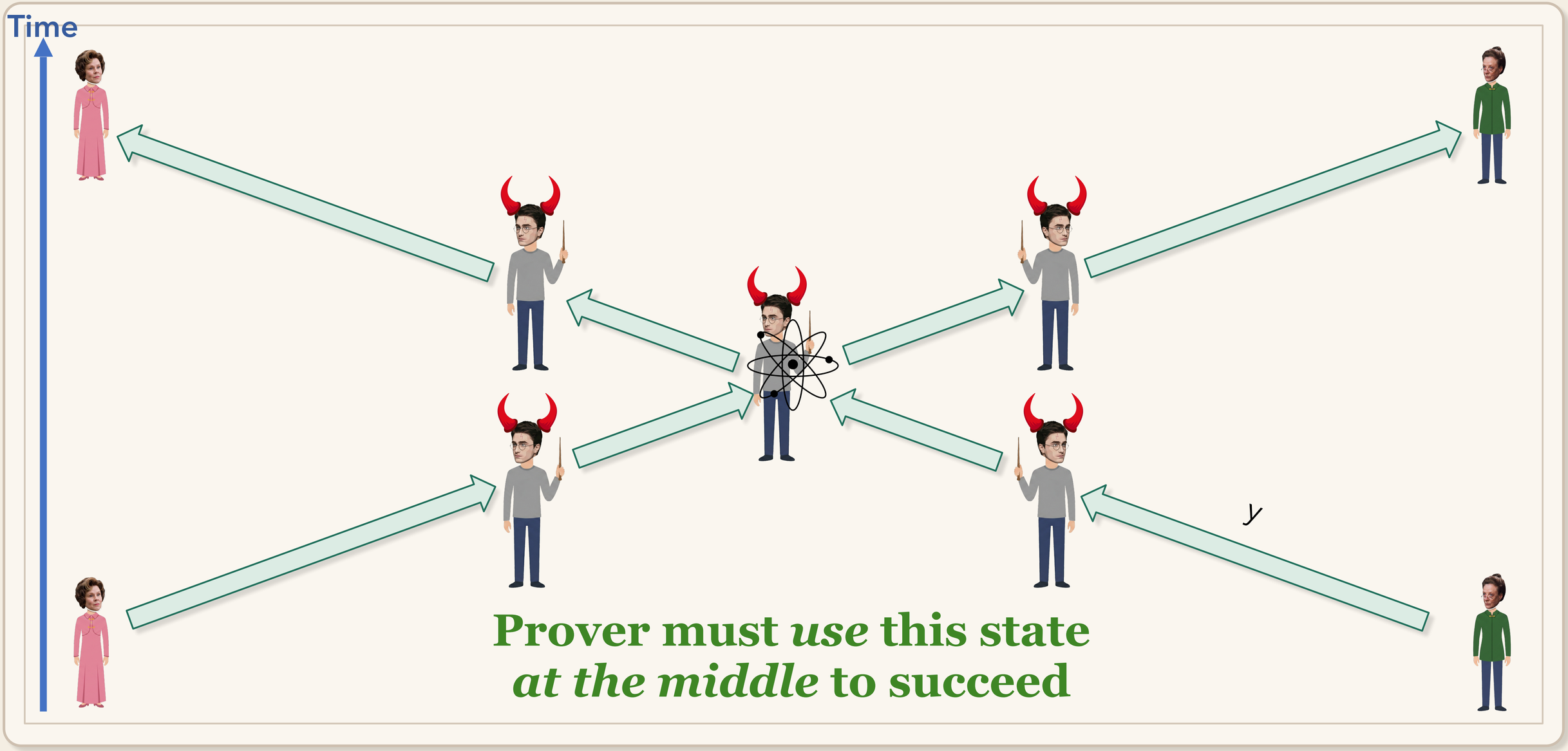

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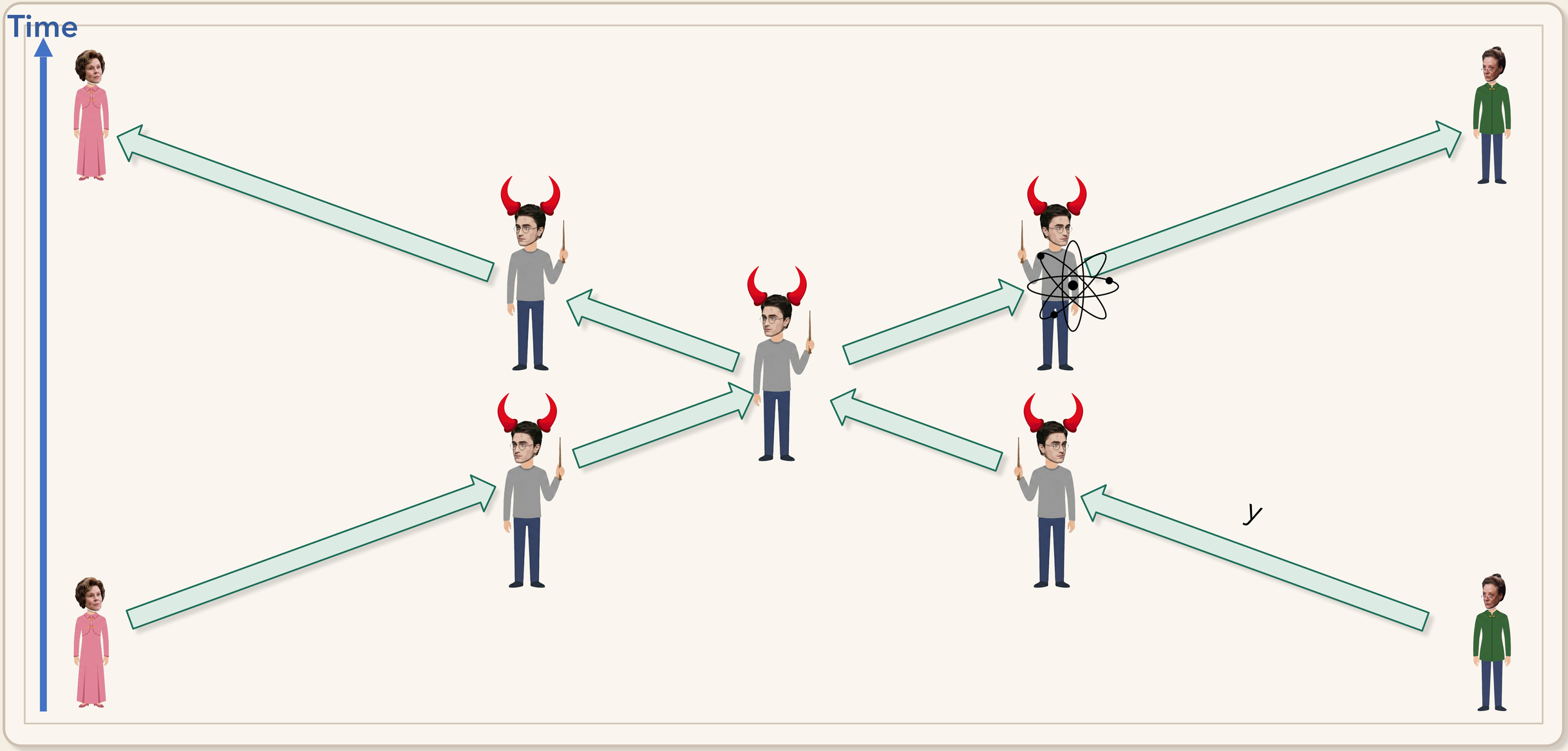

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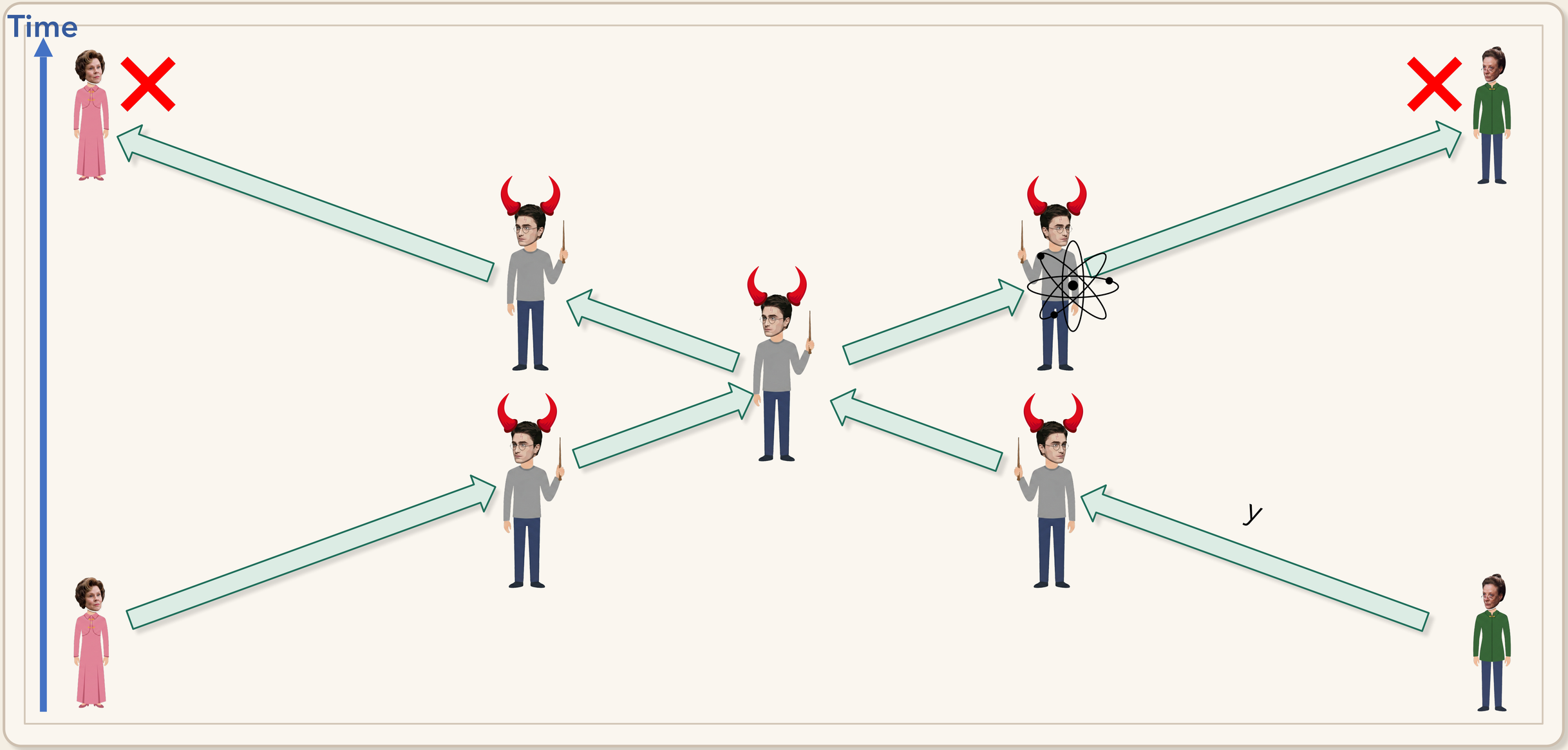
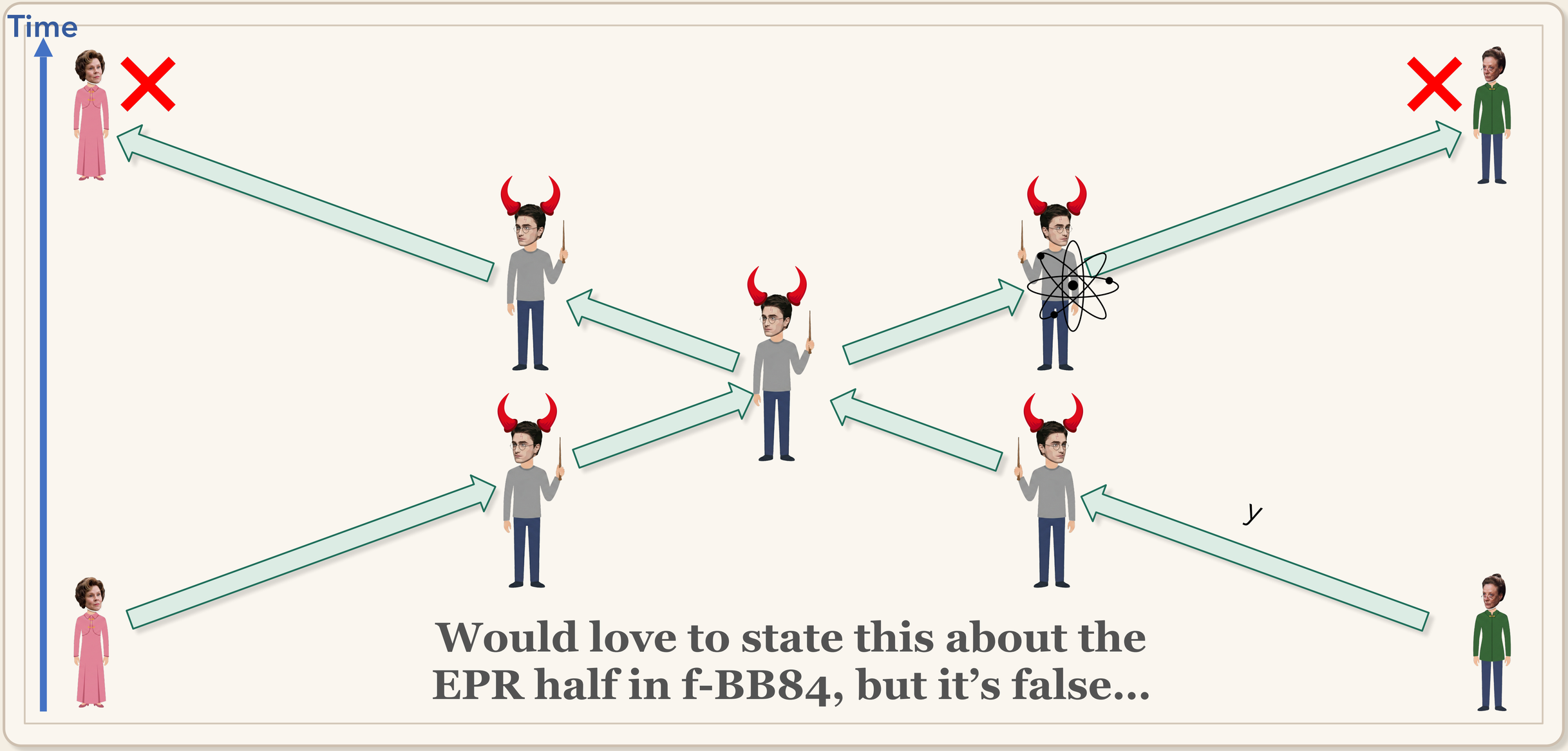


Visualizing PV in Spacetime



Would love to state this about the EPR half in f-BB84, but it's false…

Finding a Quantum State at a Location

Given a “snapshot” of a spatial region, can we **extract** some quantum state from it?

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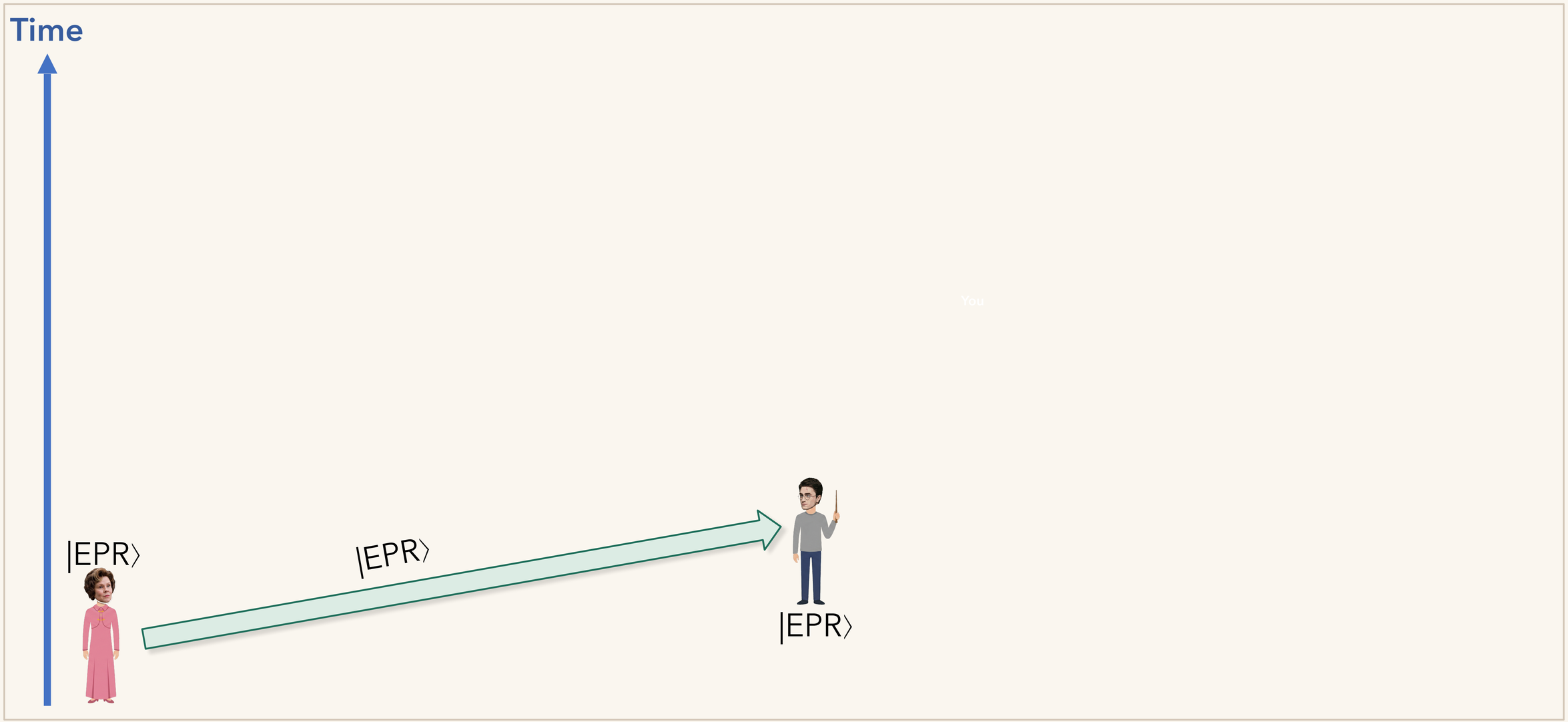
CPTP "Extractor"

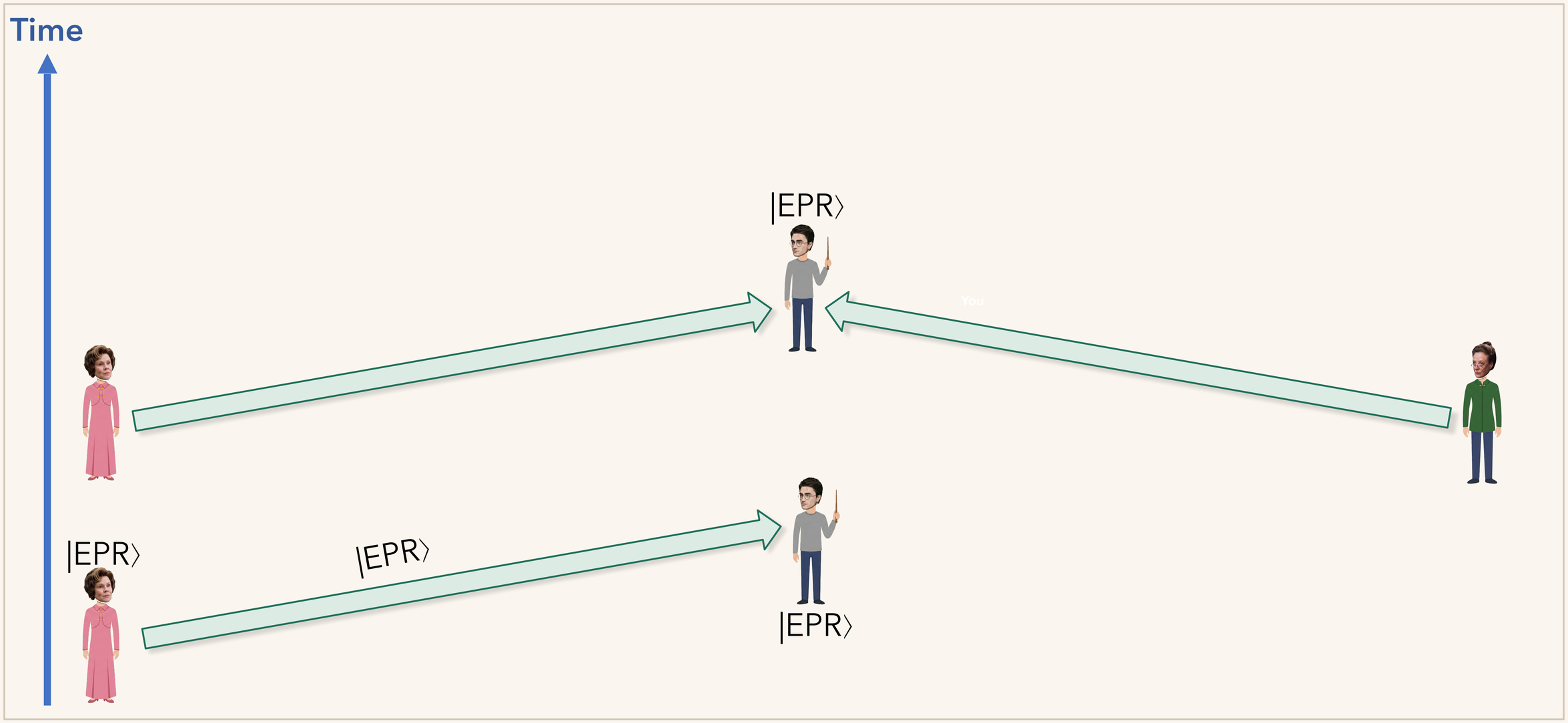
The state on "all registers of R" at the given point in time.

Really, w.h.p. output $|\phi\rangle$ having high fidelity with $|\psi\rangle$

Task: Entanglement Localization



Task: Entanglement Localization



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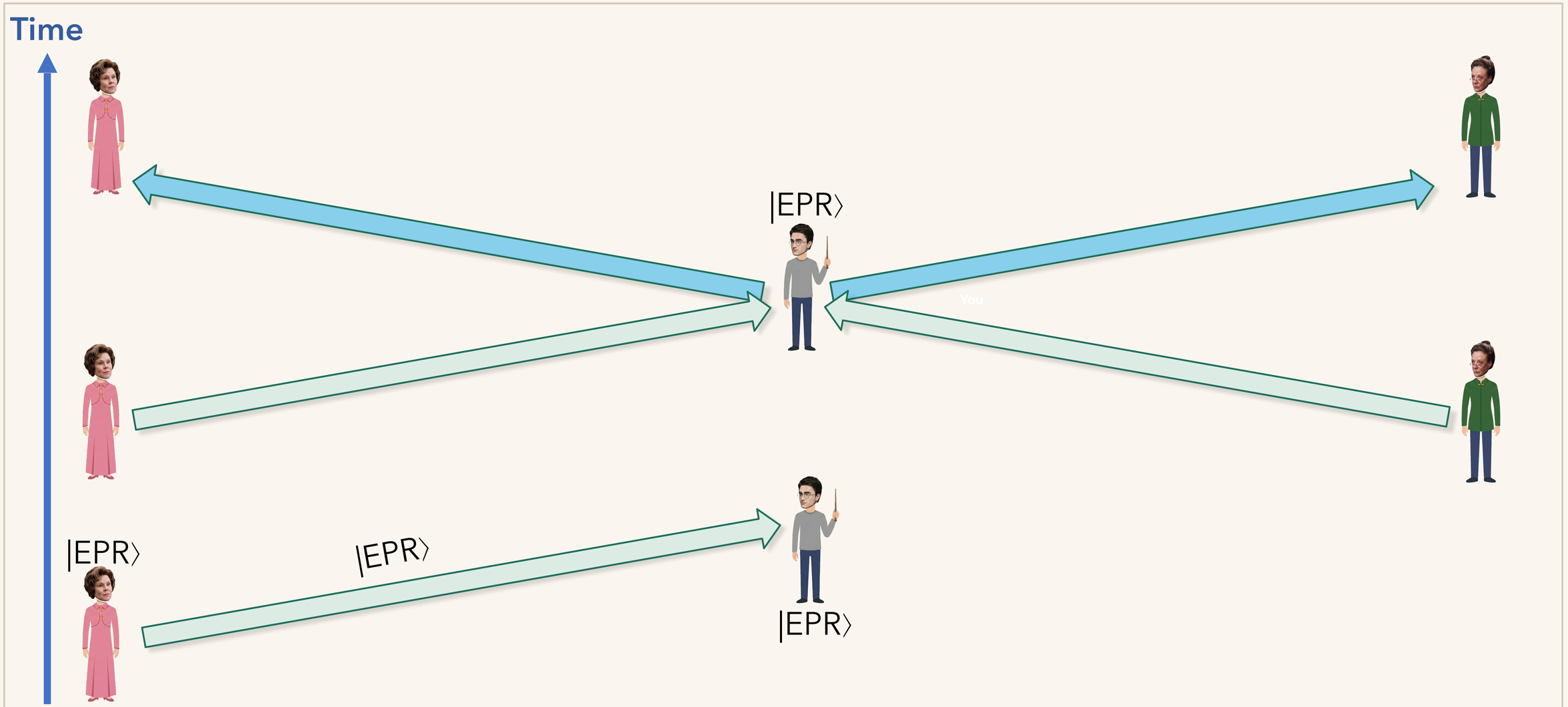
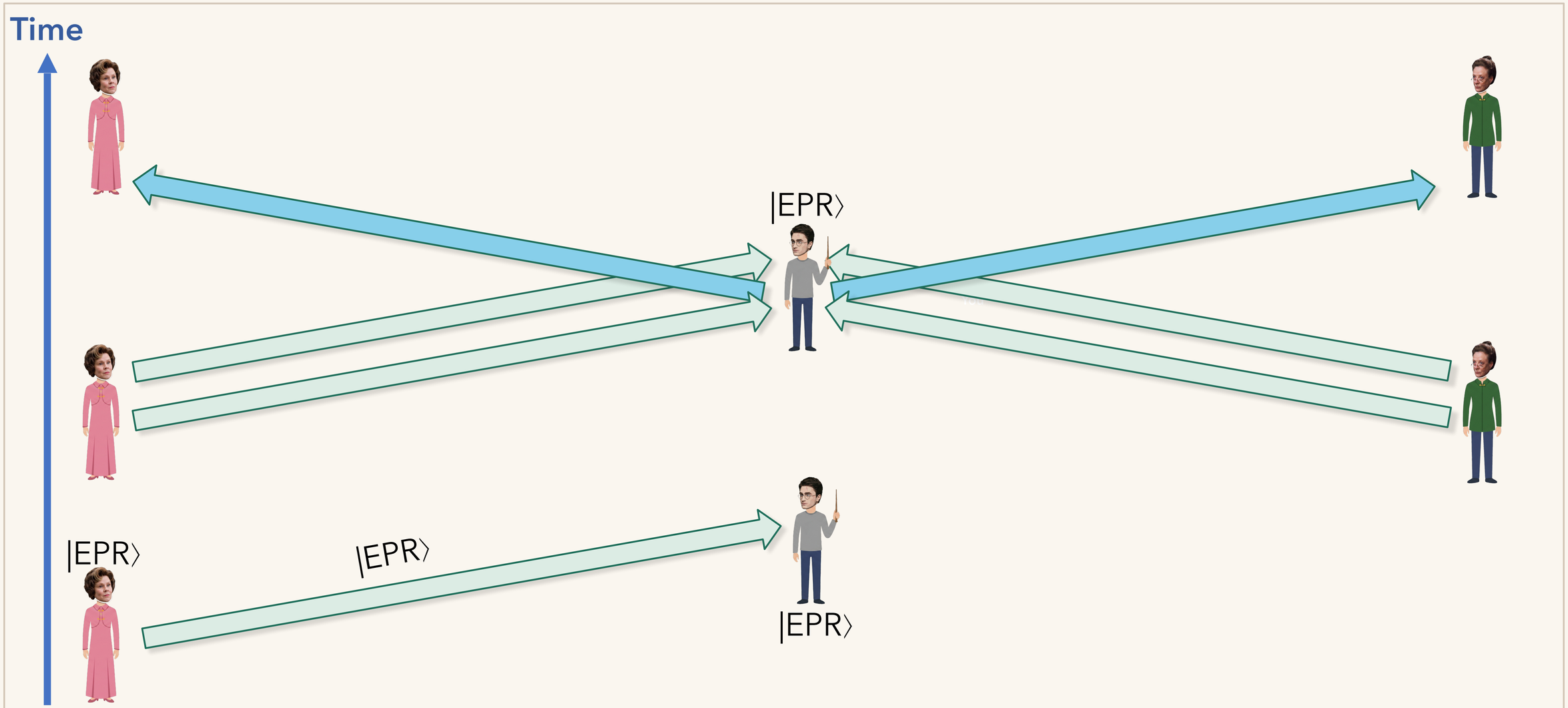
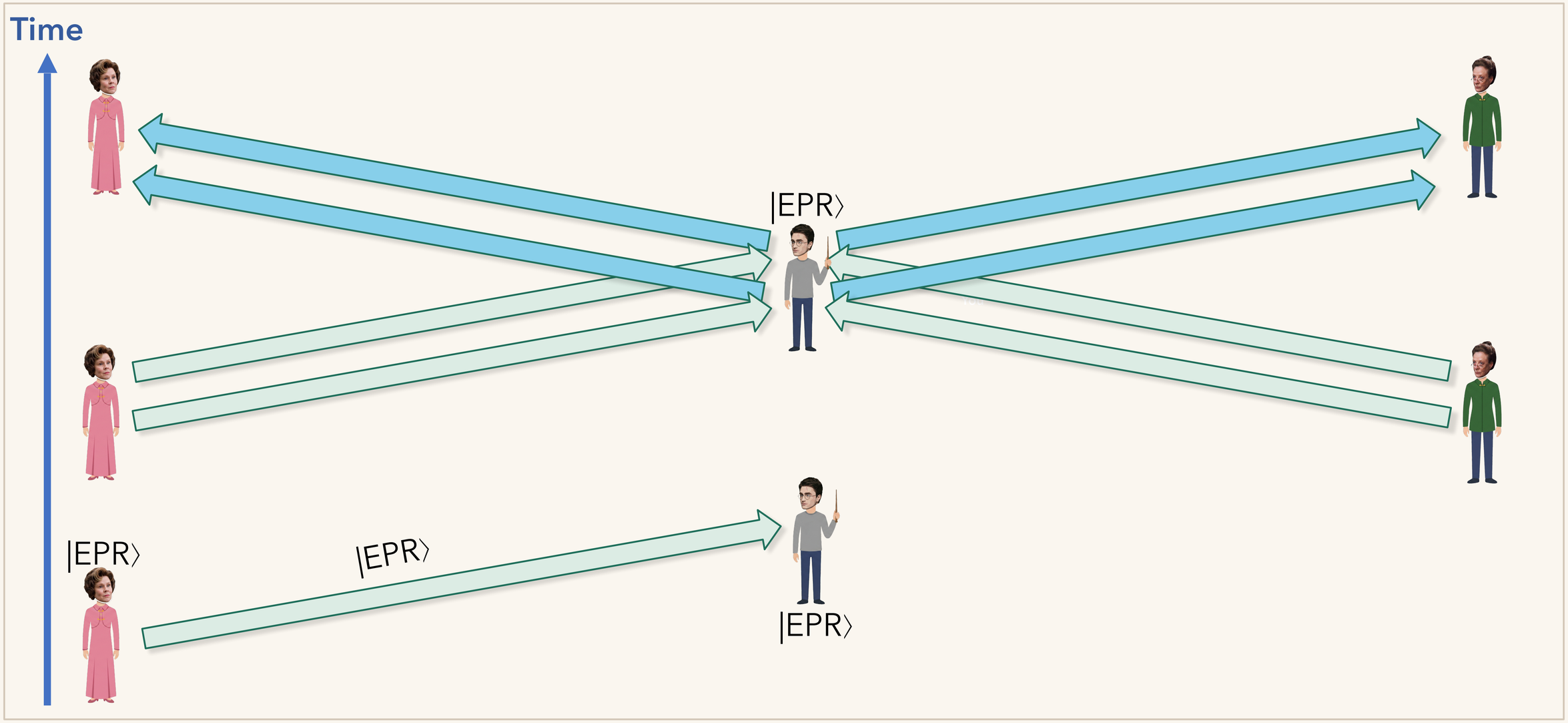


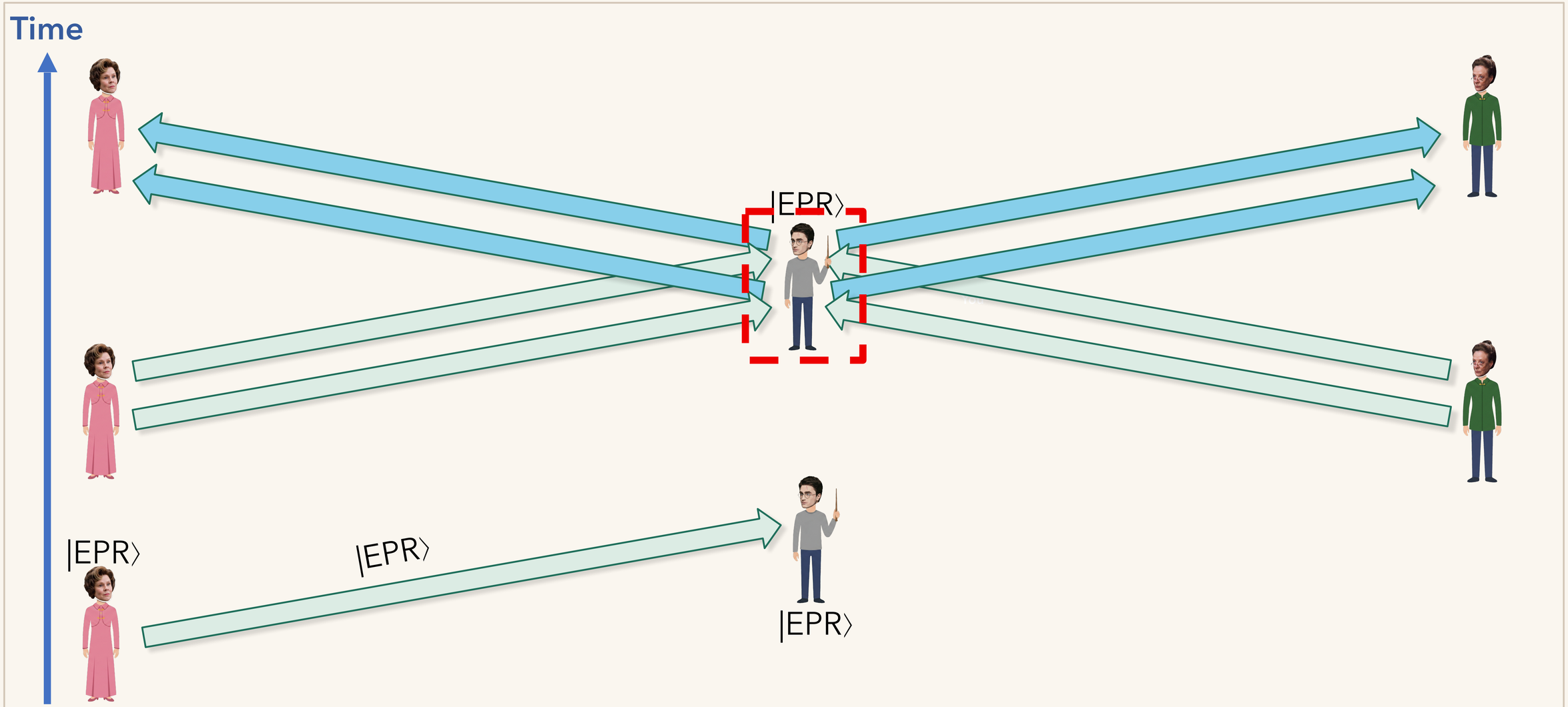
Task: Entanglement Localization



Task: Entanglement Localization



Task: Entanglement Localization



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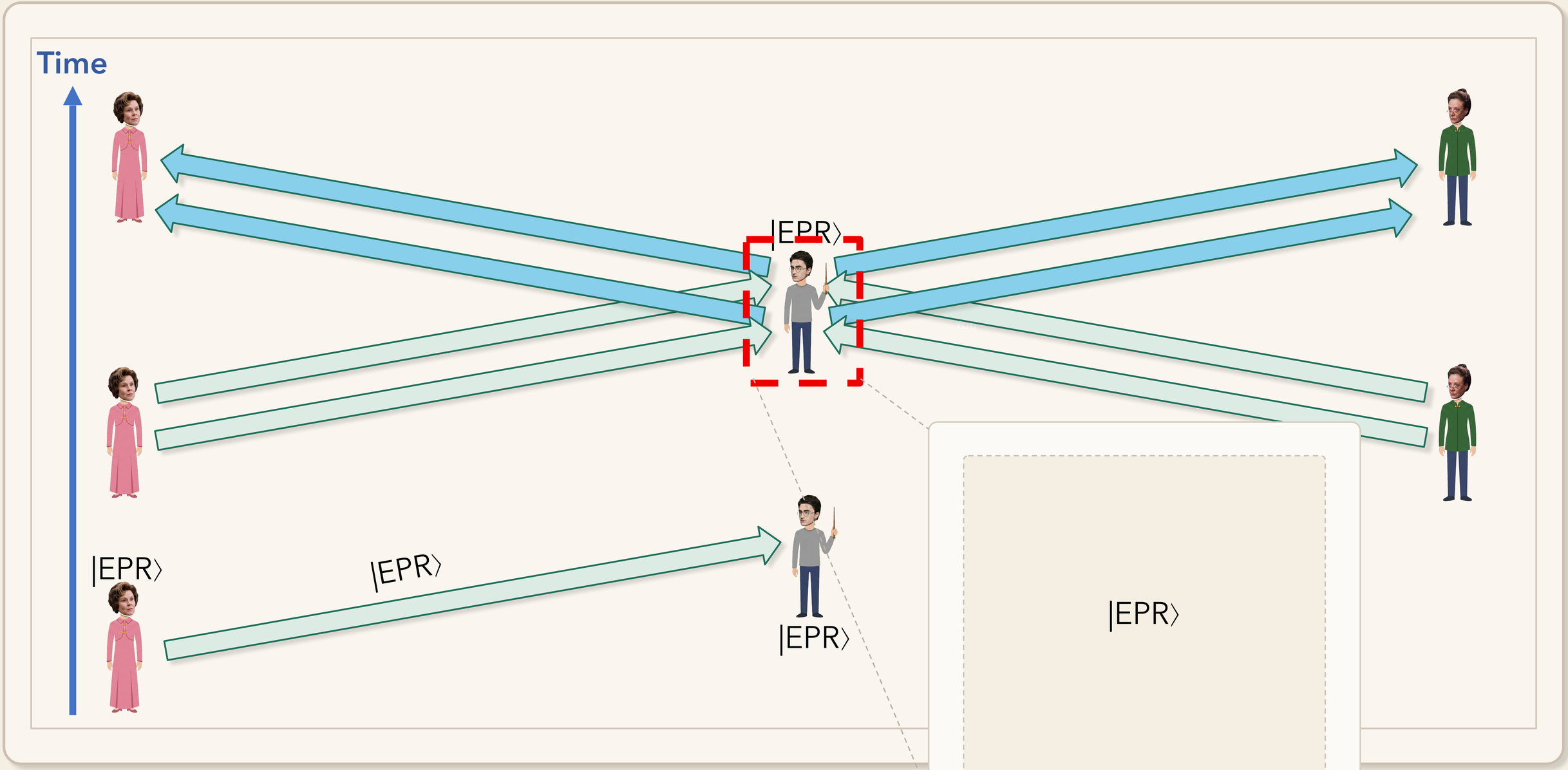

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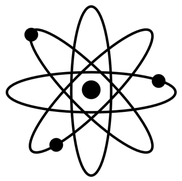
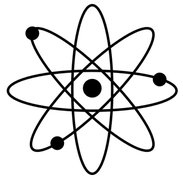
EPR Extraction:

 : “I am an EPR half”



Task: Entanglement Localization

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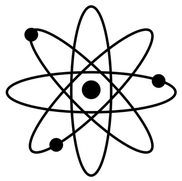
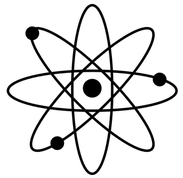
 : “I am an EPR half”

“Prove it”



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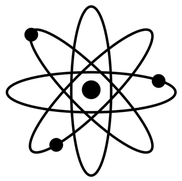
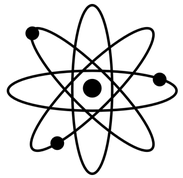


“Measure in the standard basis”



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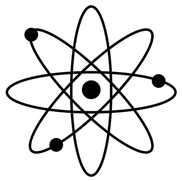 : “I got $|0\rangle$”

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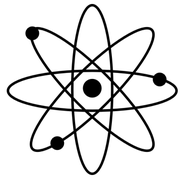

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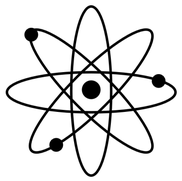
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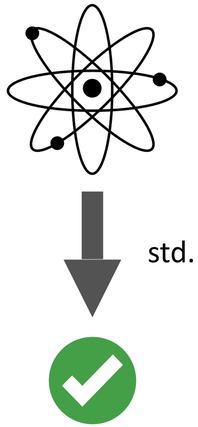

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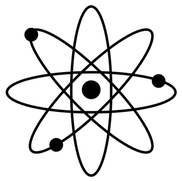
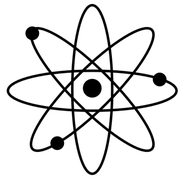


OR



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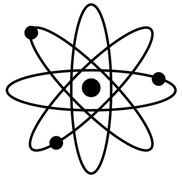
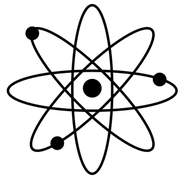


OR

“Measure in the Hadamard basis”



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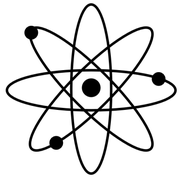 : “I got $|+\rangle$”

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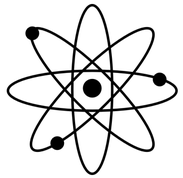

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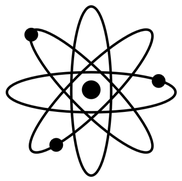
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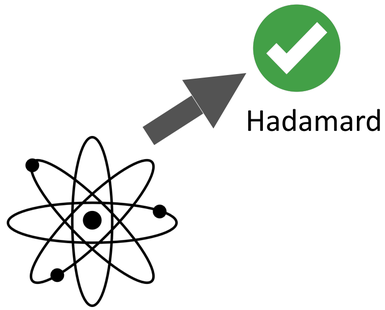

Task: Entanglement Localization

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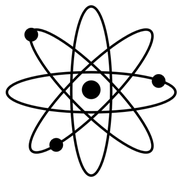
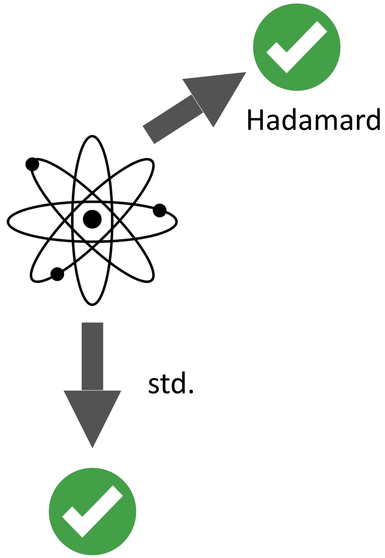


: “I got $|+\rangle$”

w.p. 1/2 **“Measure in the standard basis”**

OR

w.p. 1/2 **“Measure in the Hadamard basis”**



Task: Entanglement Localization

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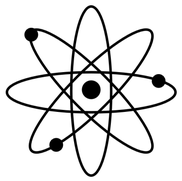

Assume
$\Pr[\text{win}] = 1 - \varepsilon$

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OR

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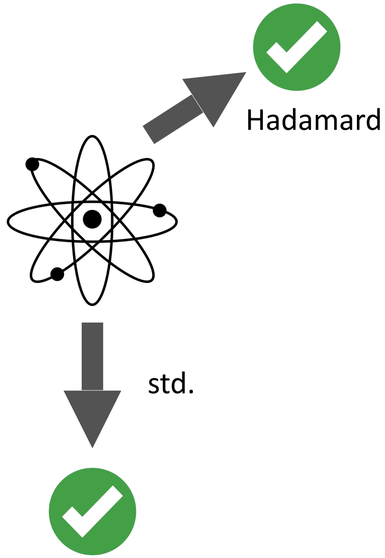

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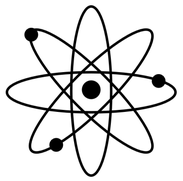


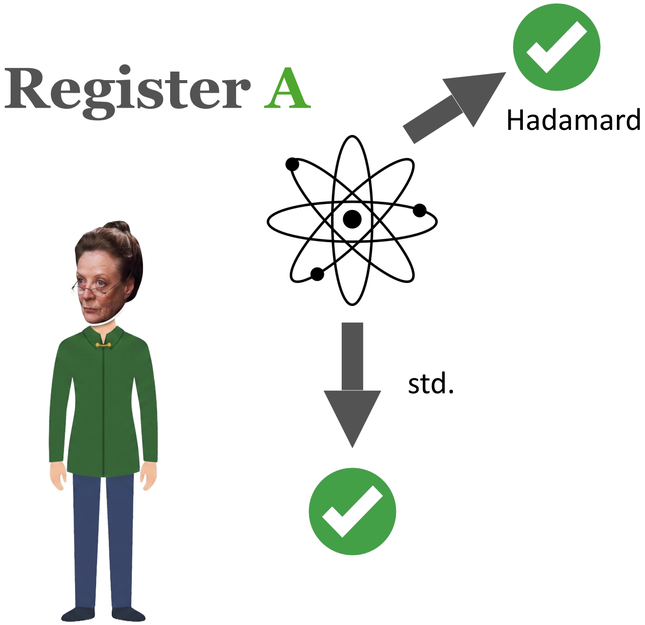
Register **B**

Assume
$\Pr[\text{win}] = 1 - \varepsilon$

w.p. 1/2 **“Measure in the standard basis”**

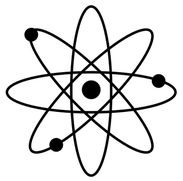
OR

w.p. 1/2 **“Measure in the Hadamard basis”**



Task: Entanglement Localization

EPR Extraction:



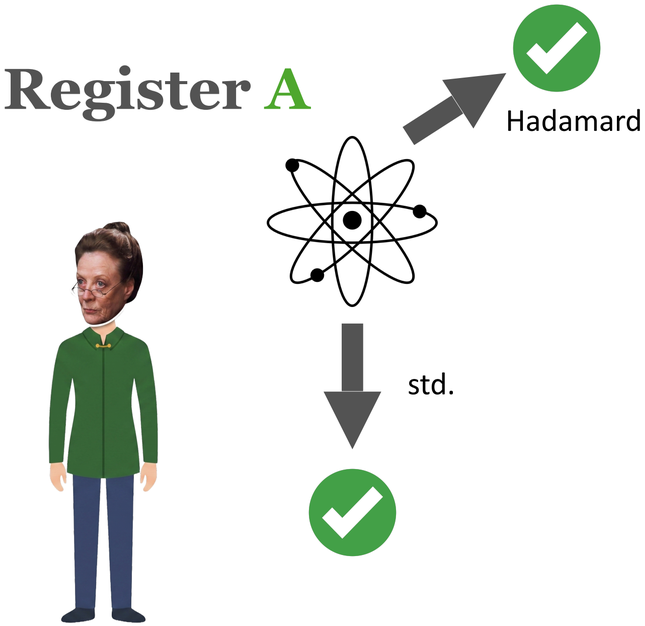
Register B

Assume
Pr[win] = 1 - ε

w.p. 1/2 **“Measure in the standard basis”**

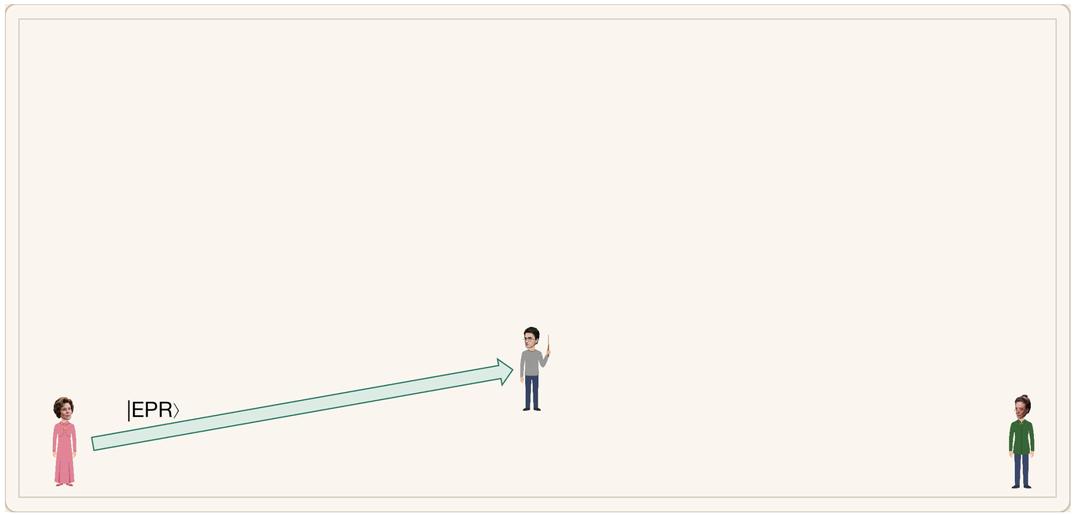
OR

w.p. 1/2 **“Measure in the Hadamard basis”**



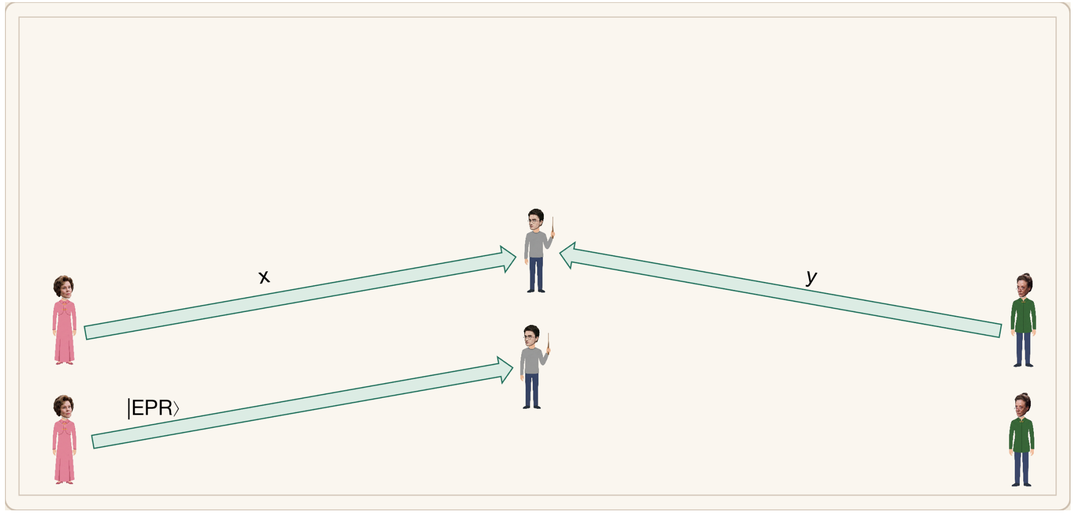
[VZ’21]: There exists an isometry E(B) -> CX such that the state on AC is ε-close to |EPR⟩

Task: Entanglement Localization



Doesn’t f-BB84 already include this measurement game?

Task: Entanglement Localization



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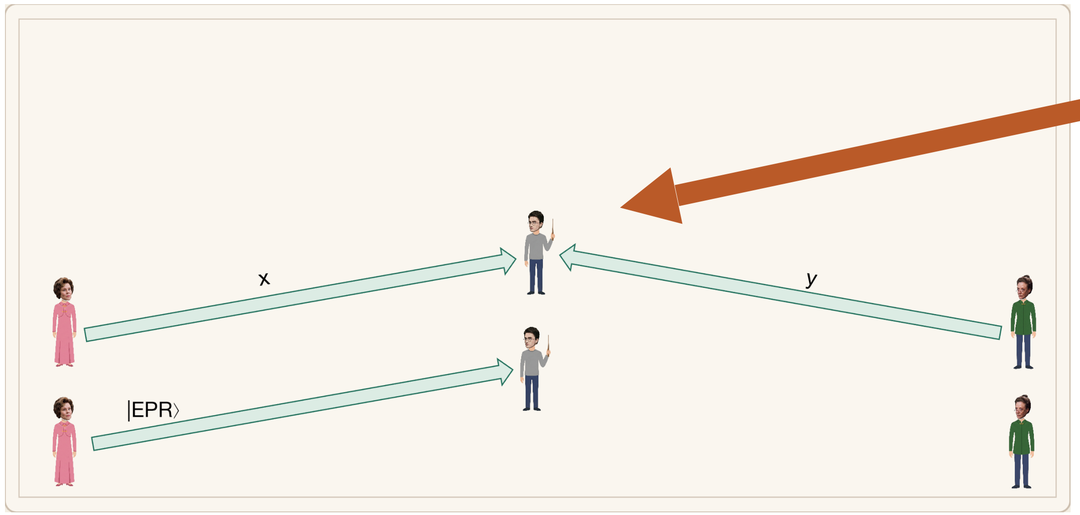
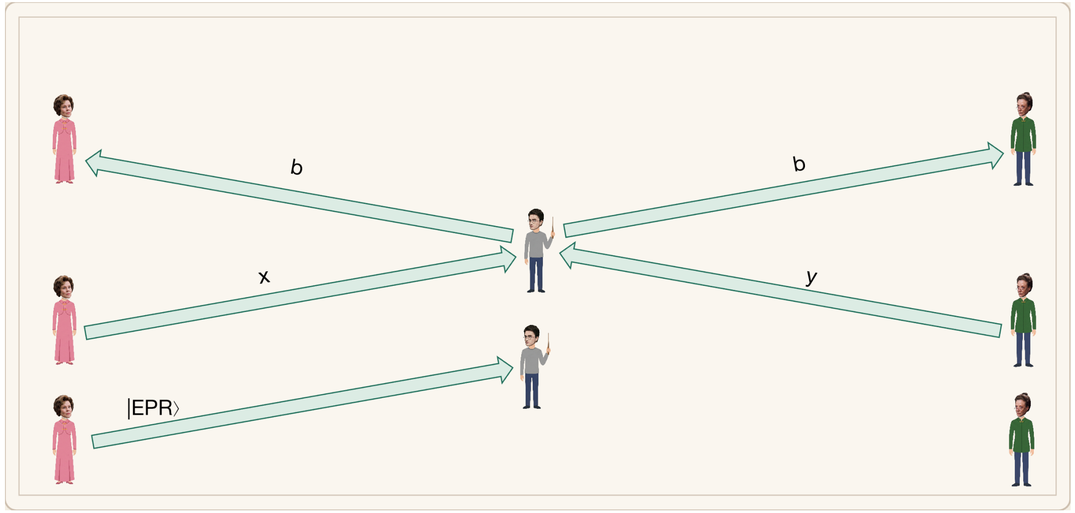


Can be either standard or Hadamard

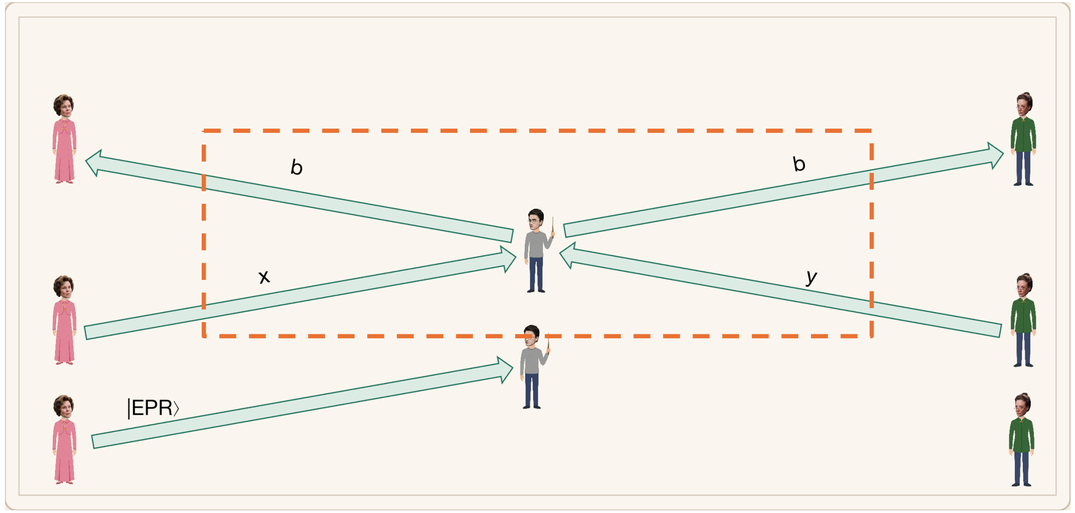
Doesn’t f-BB84 already include this measurement game?

Task: Entanglement Localization



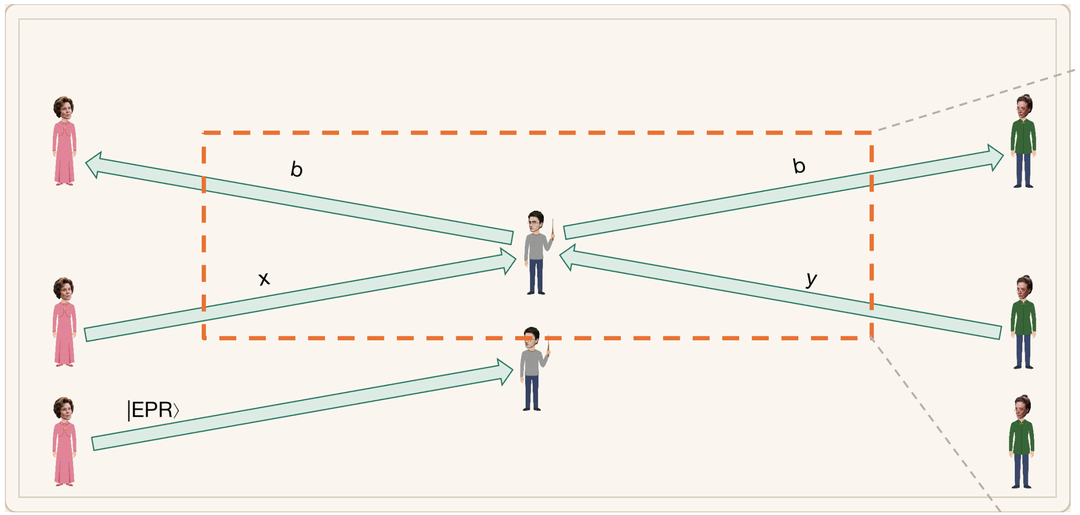
Doesn’t f-BB84 already include this measurement game?

Task: Entanglement Localization



This box passes the [VZ'21] game with high probability, therefore ∃ isometry => ≈|EPR⟩

Task: Entanglement Localization



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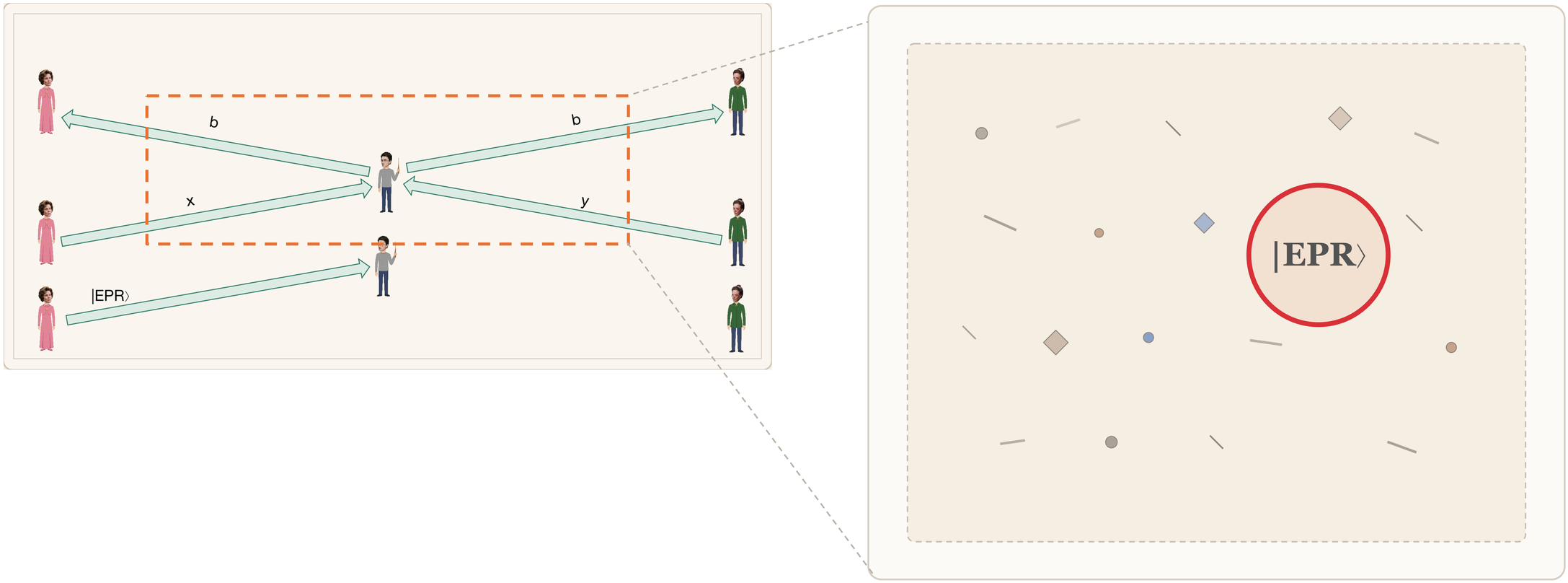
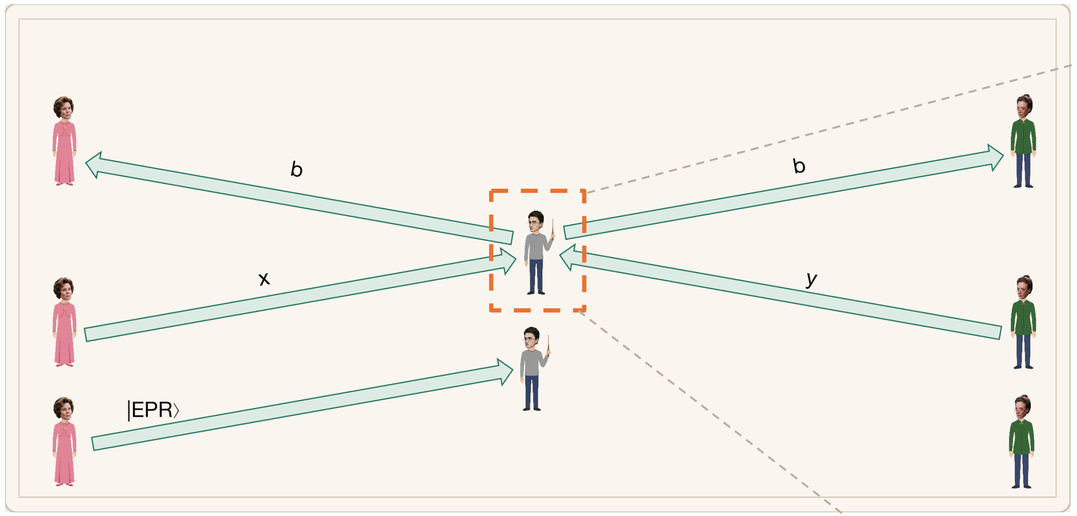


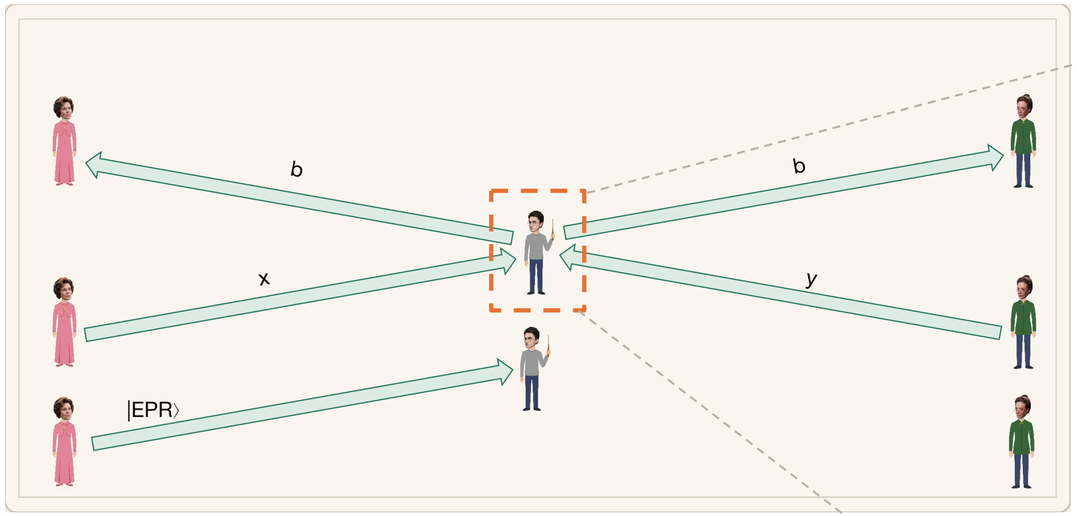
Sure, but… that box is huge. Want a claim only about the middle!

Task: Entanglement Localization



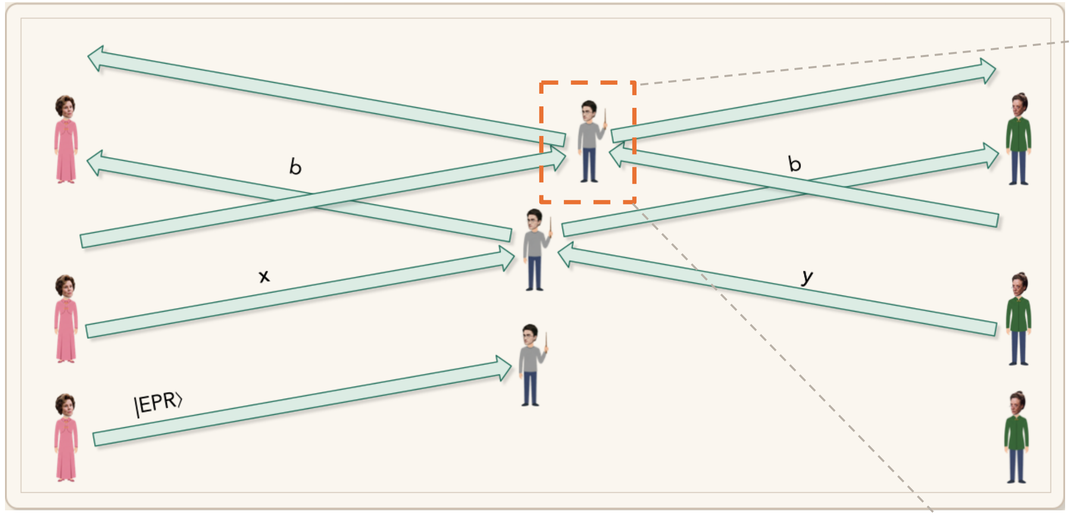
?

Task: Entanglement Localization



Also, what if this didn't destroy the EPR pair…

Task: Entanglement Localization



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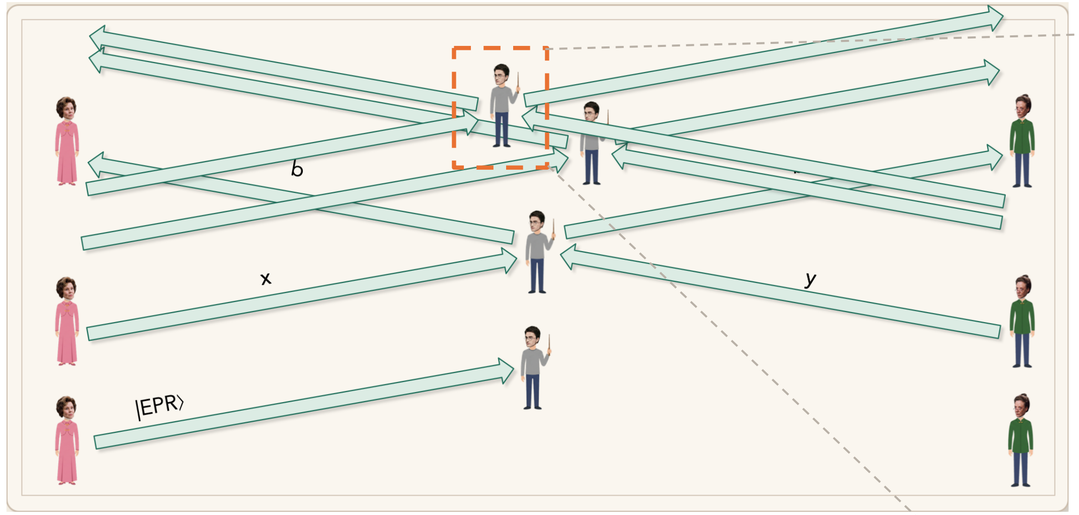
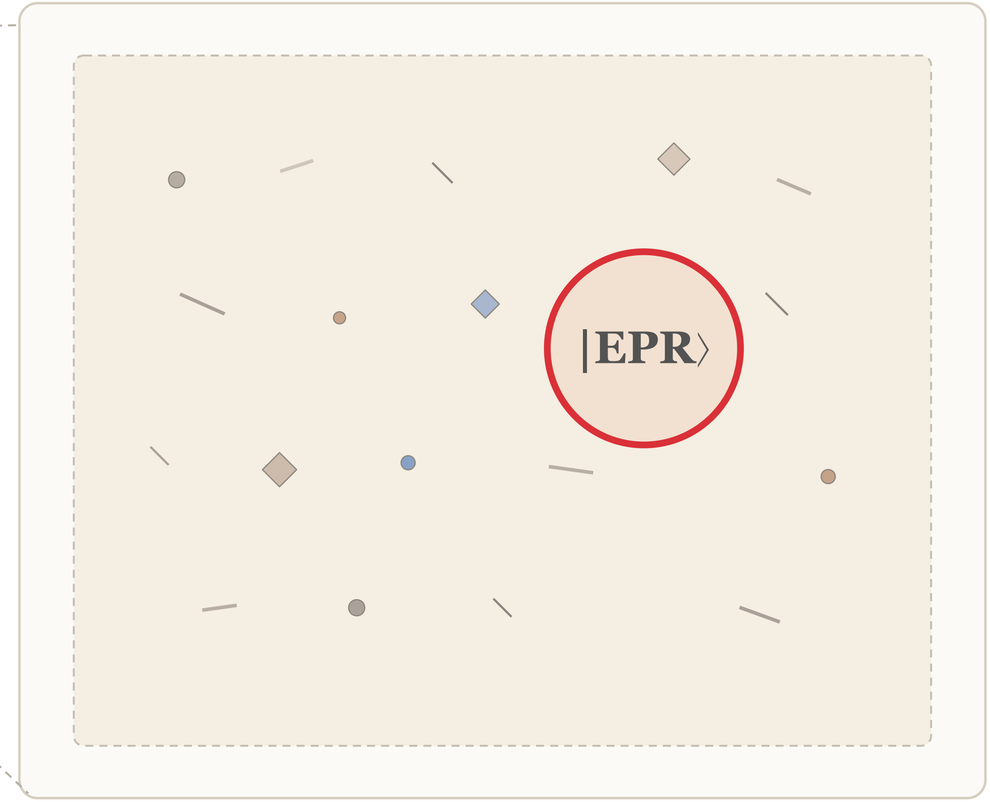
Task: Entanglement Localization





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Task: Entanglement Localization

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Main Questions:

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- *Can we localize entanglement to a small region?*

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1. **Not without new protocols:** f-BB84 admits distributed and *non-localizable* prover strategies.

Task: Entanglement Localization

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Task: Entanglement Localization

Main Questions:

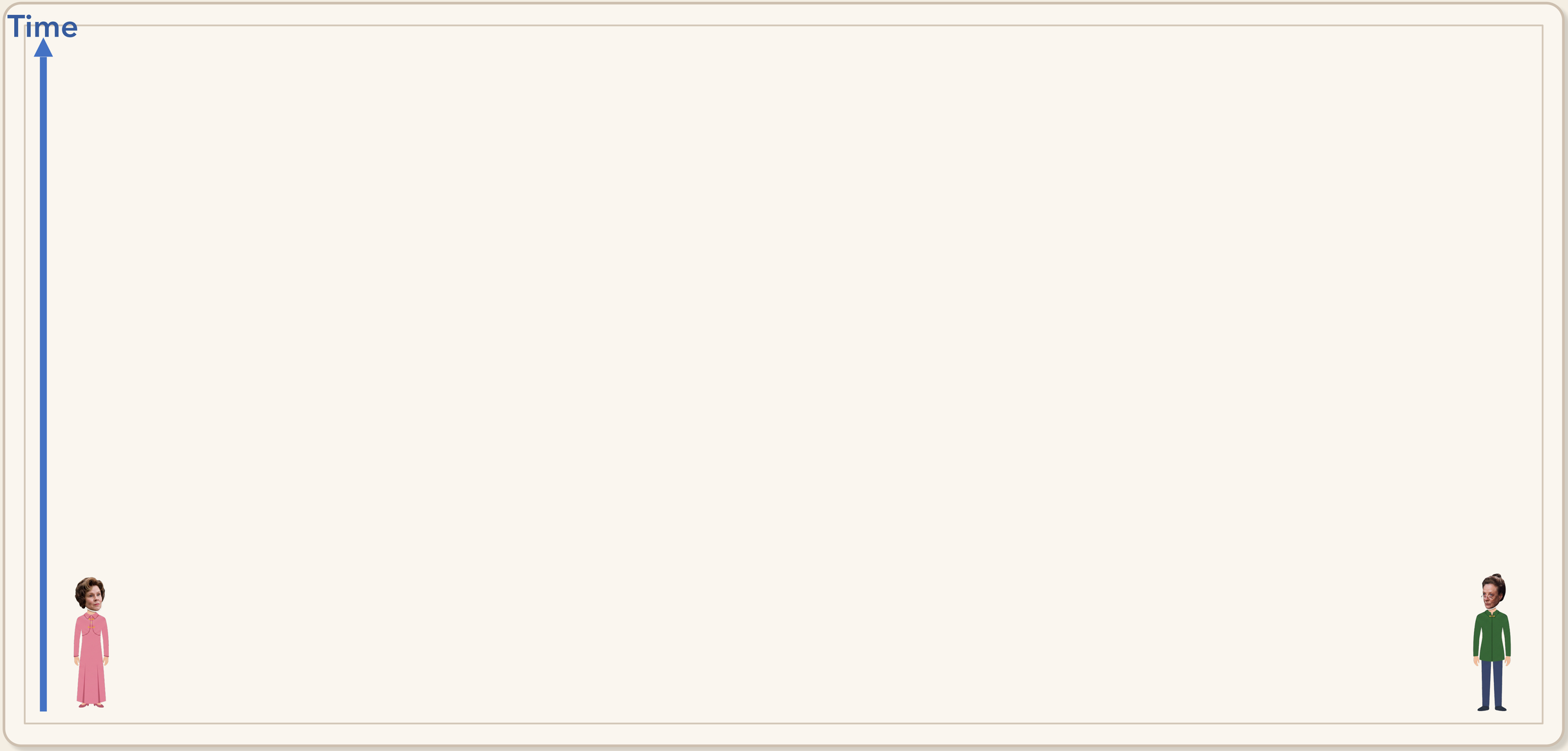
- *Can we localize entanglement to a small region?*
- *Can we do so non-destructively?*

This work:

1. **Not without new protocols:** f-BB84 admits distributed and *non-localizable* prover strategies.

2. **With oracles, it's possible:** We construct non-destructive entanglement protocols in the *classical oracle model*. The state is extracted from an arbitrarily small region around the target.

Precise Localization via Non-Destructive Tests



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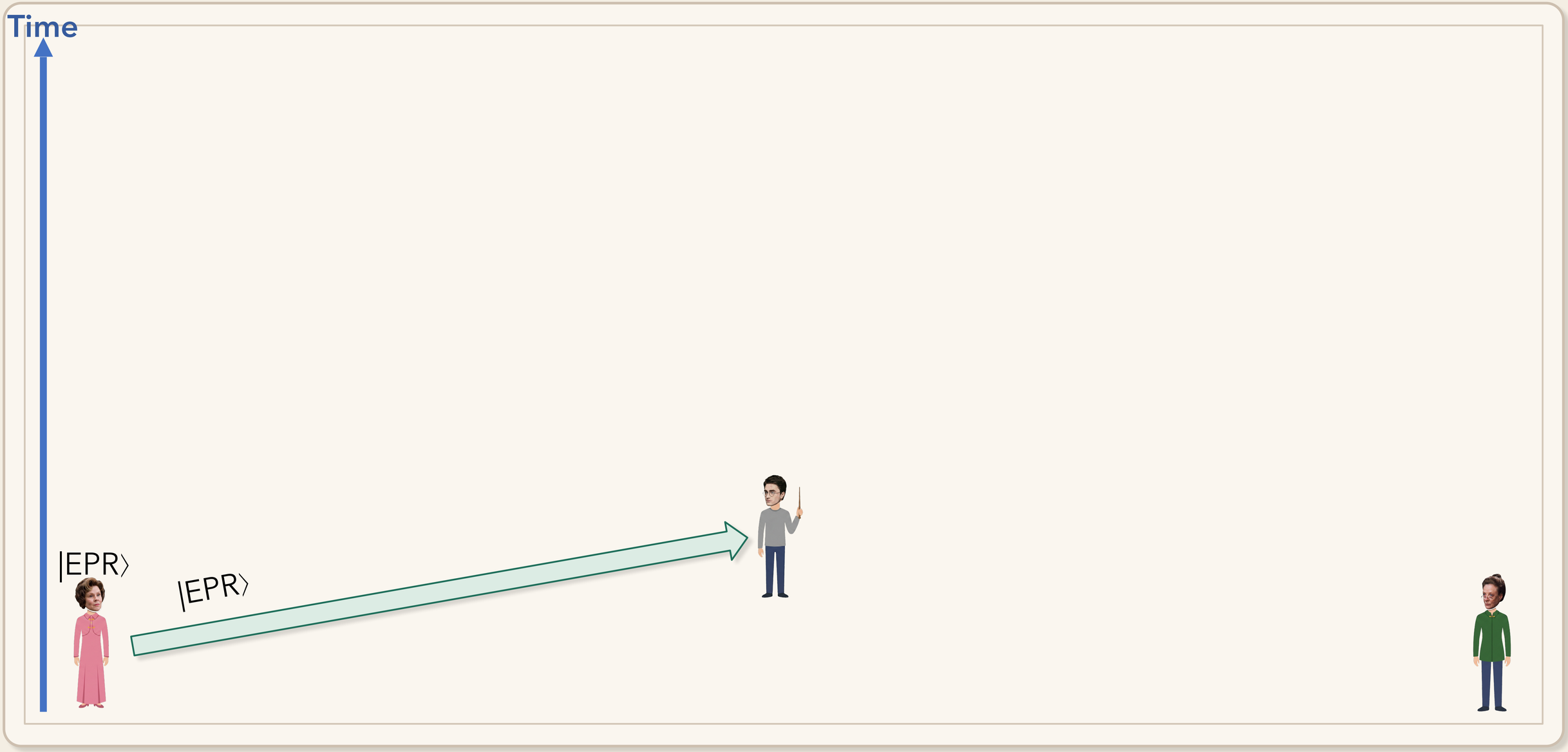

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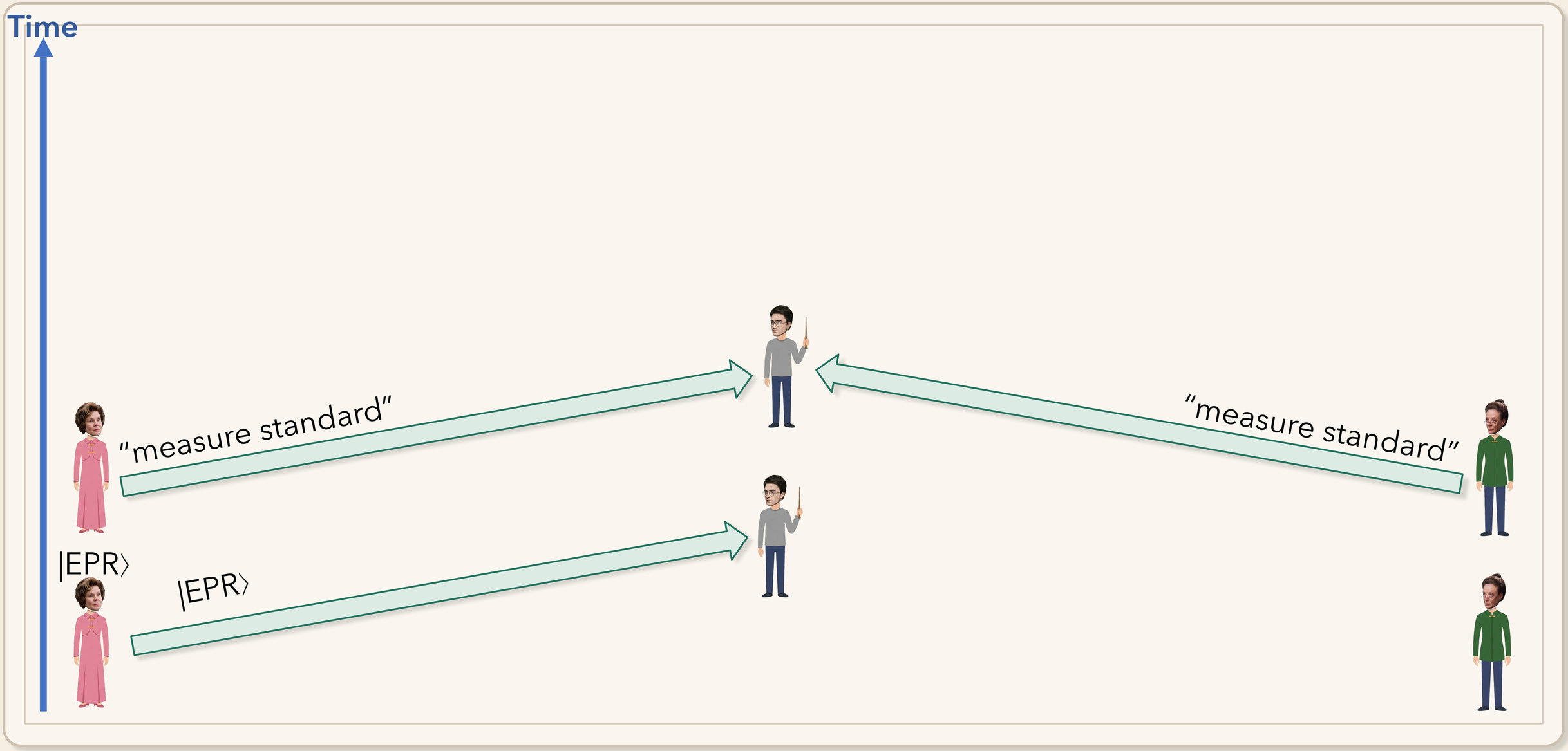

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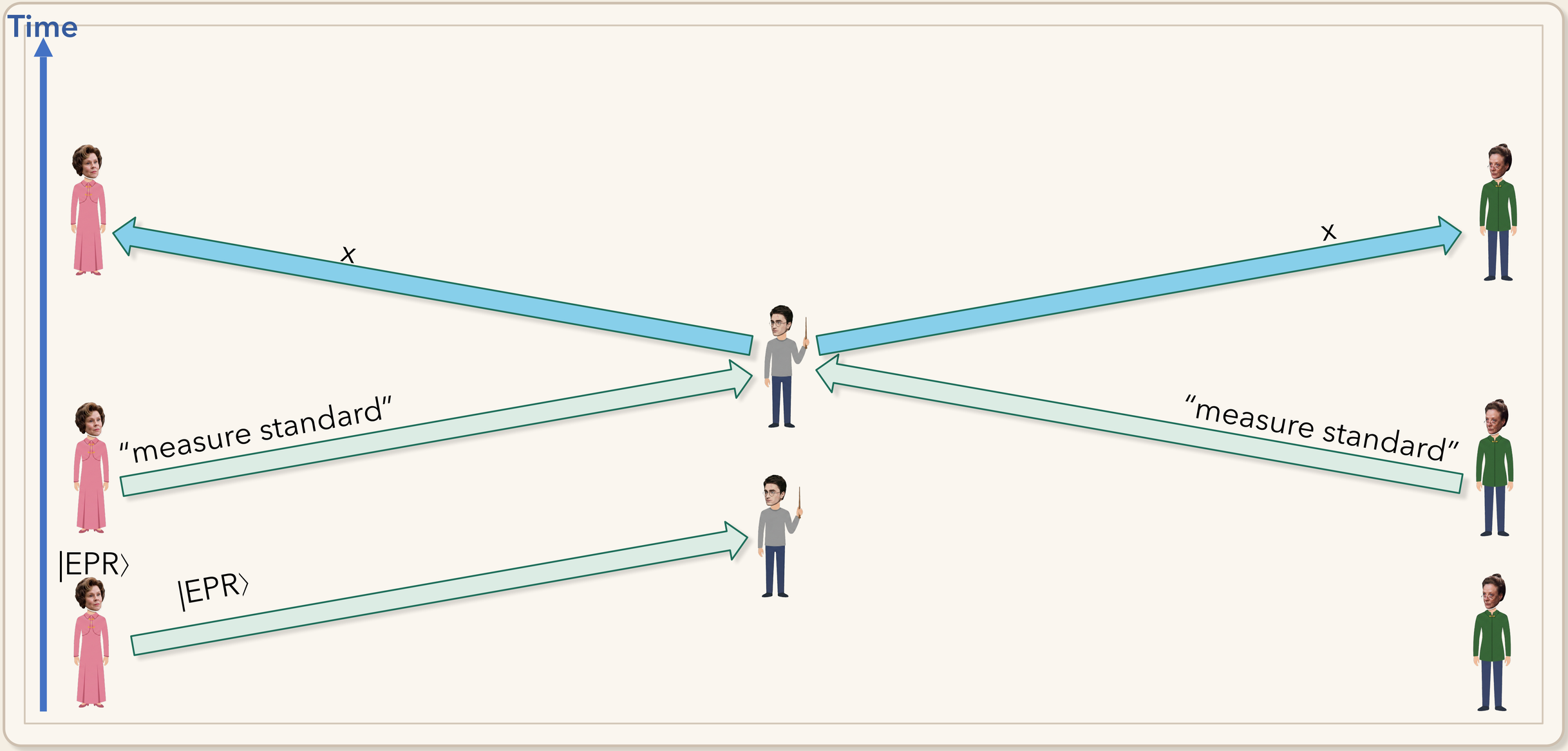

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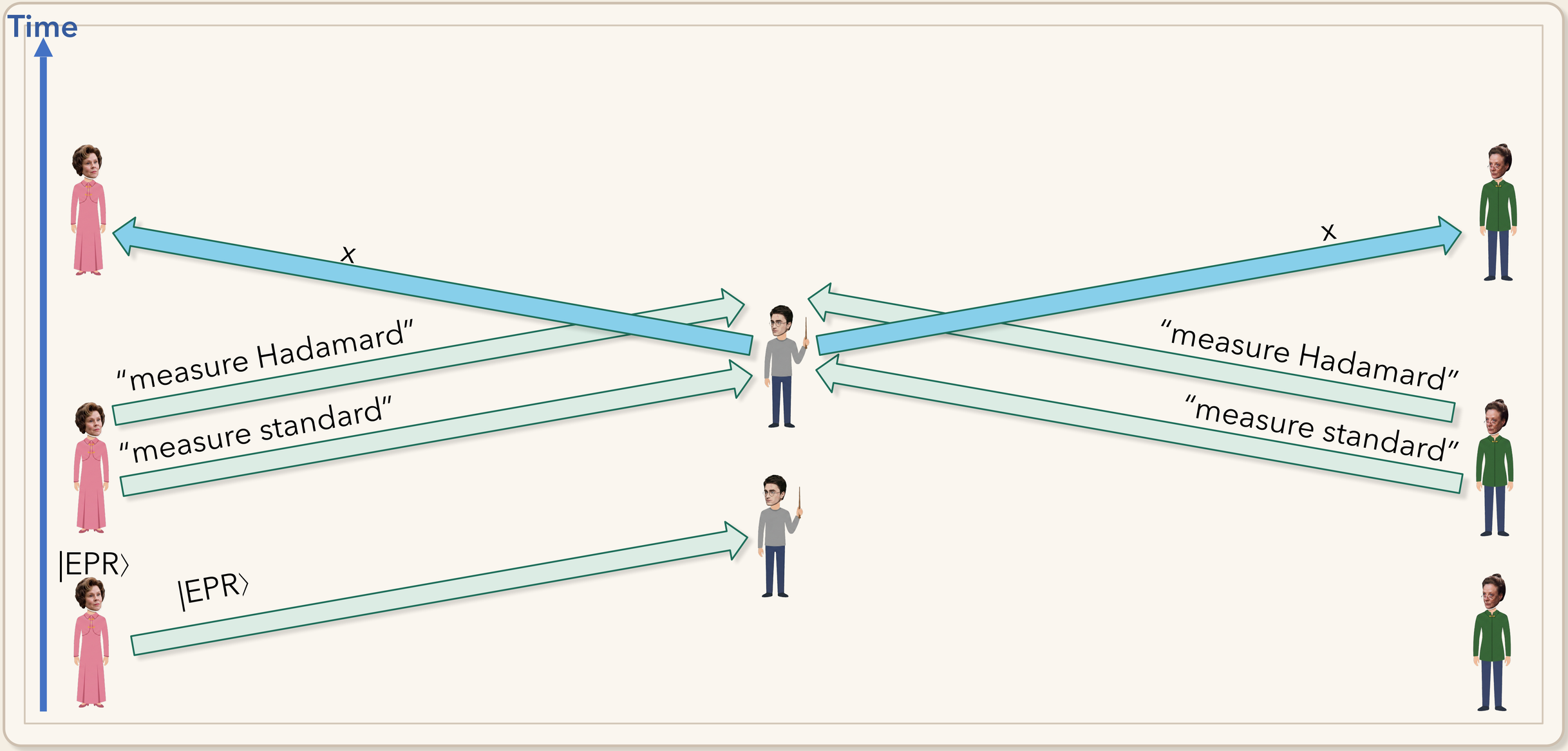

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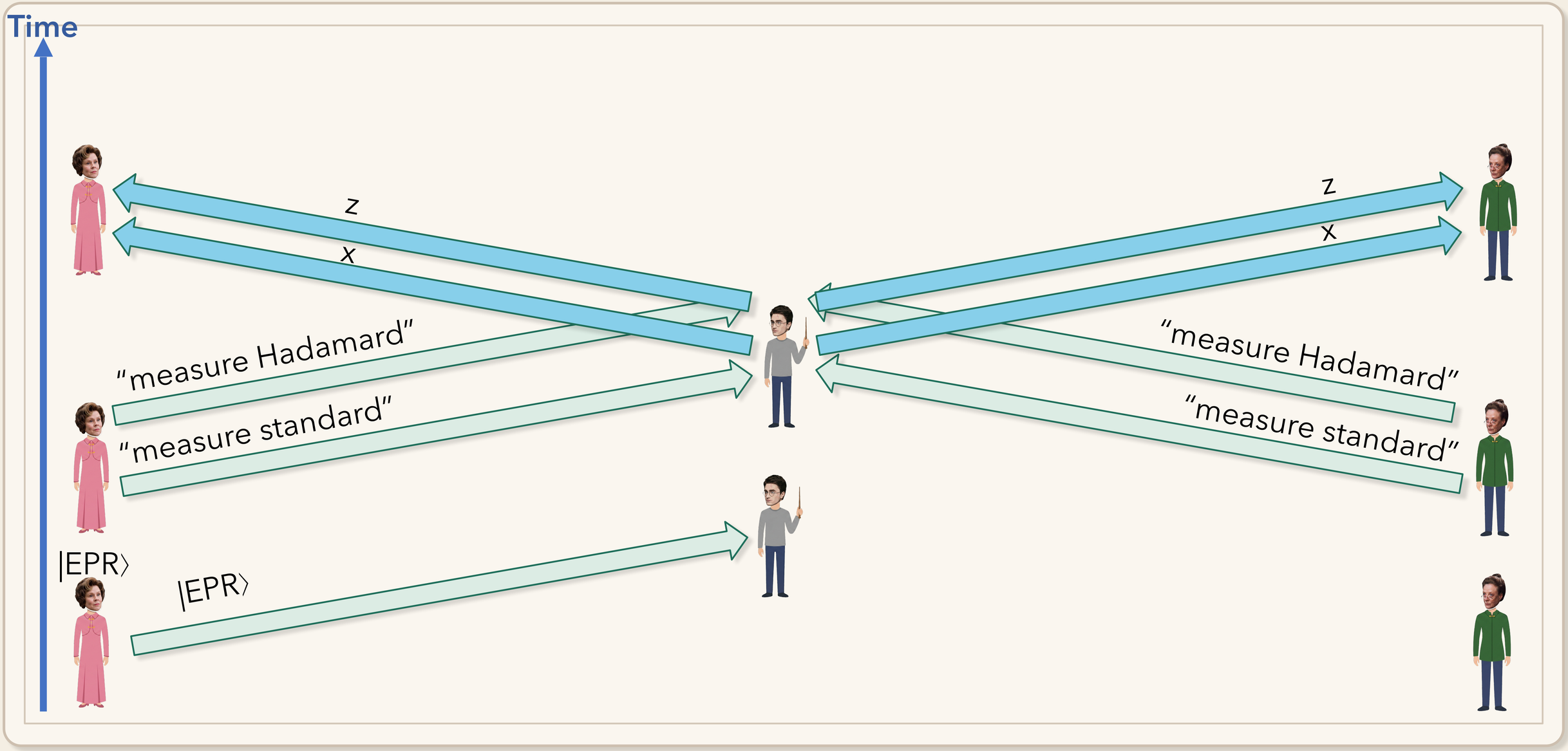

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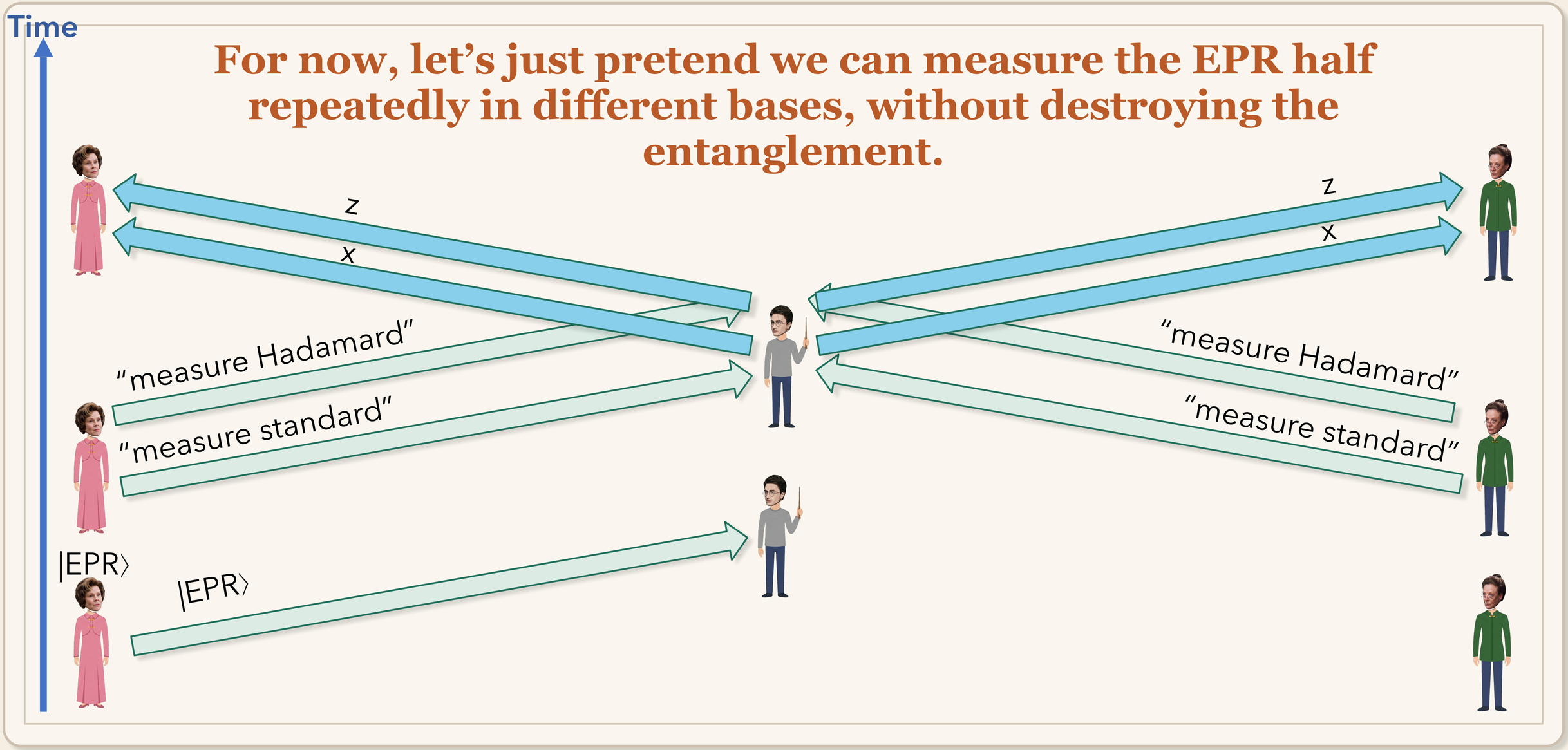

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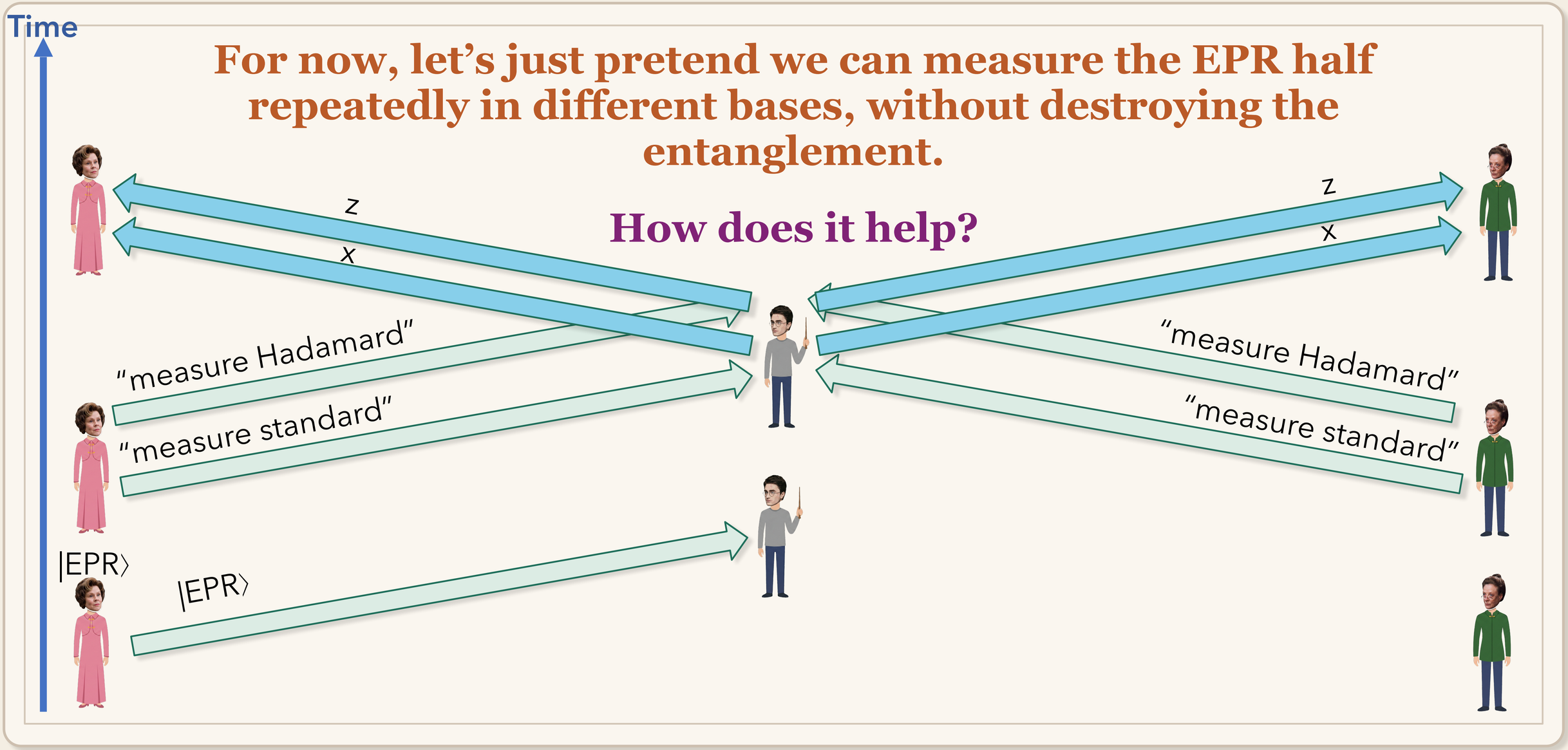
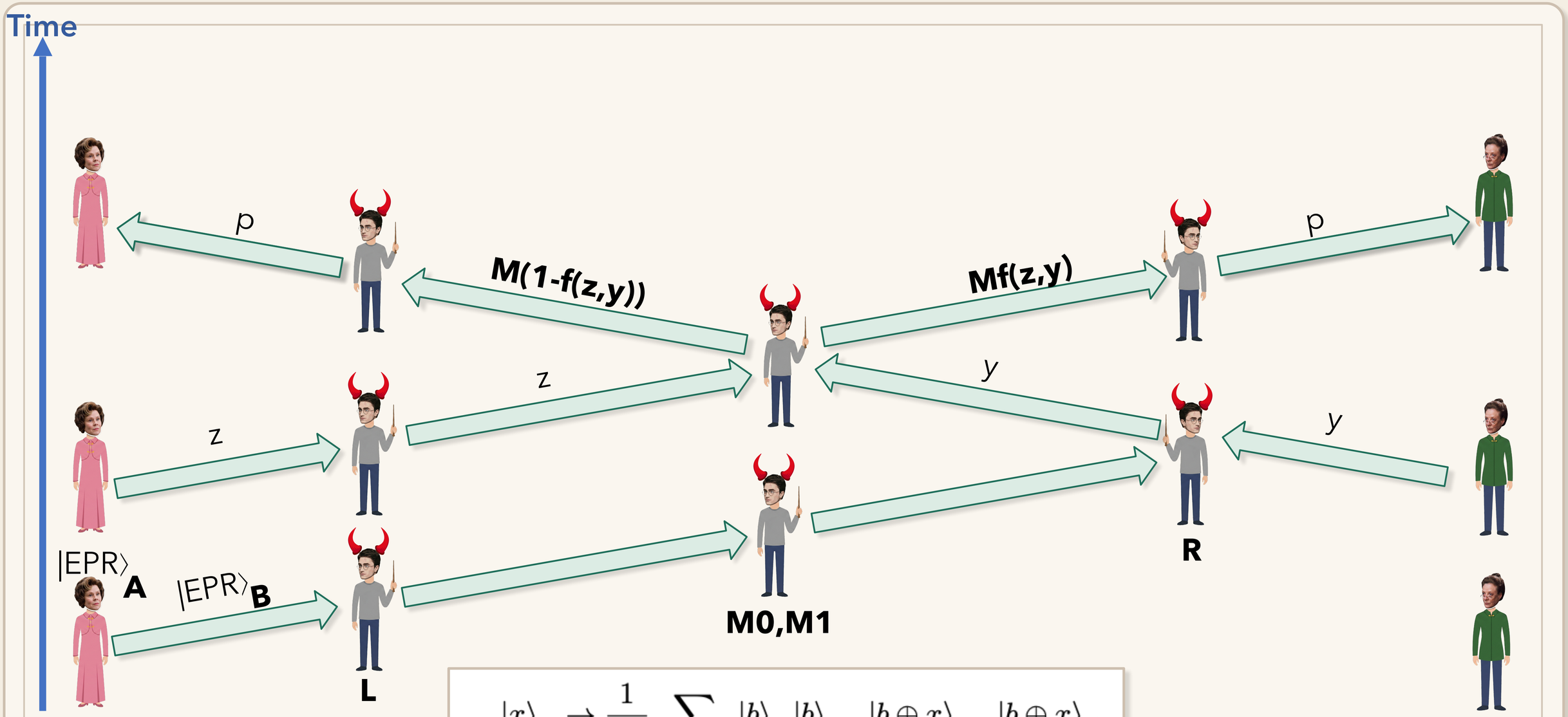


(FLASHBACK) Non-Localizability of f-BB84



$$|x\rangle_{\mathbf{B}} \to \frac{1}{\sqrt{2}} \sum_{b\in\{0,1\}} |b\rangle_{\mathbf{L}} |b\rangle_{\mathbf{M}_0} |b\oplus x\rangle_{\mathbf{M}_1} |b\oplus x\rangle_{\mathbf{R}}$$

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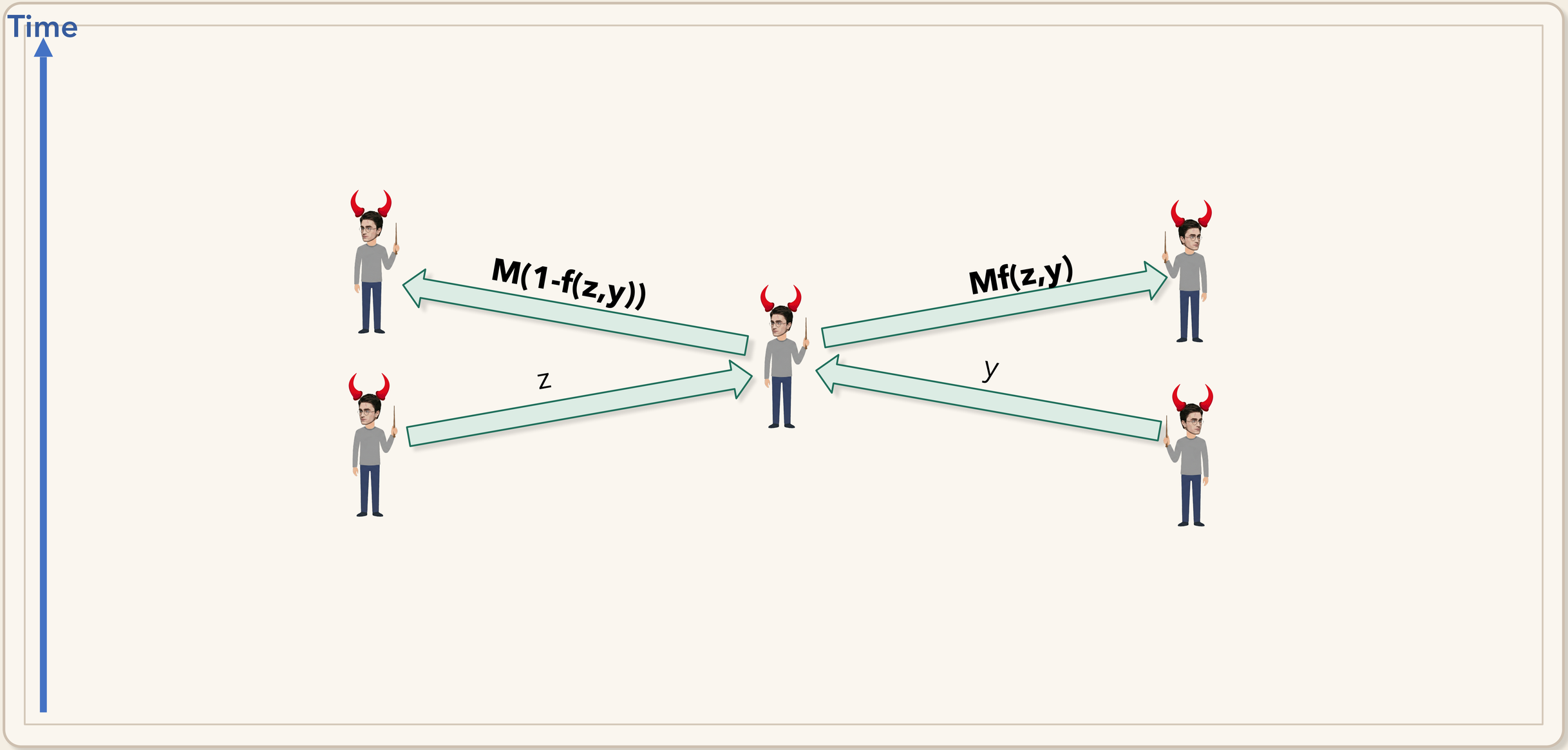
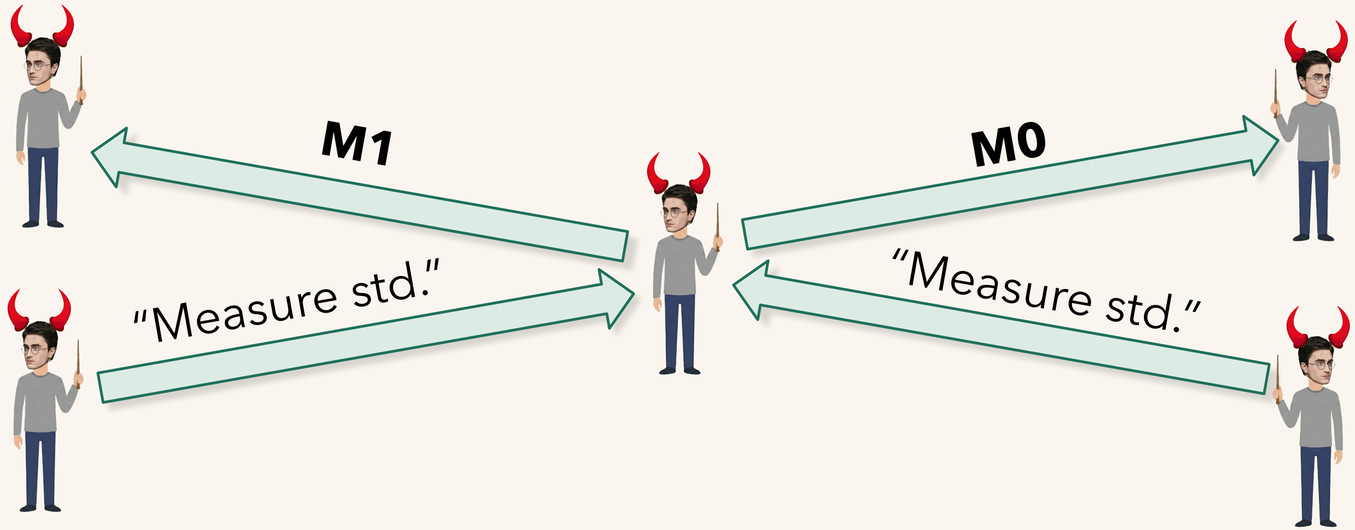


(FLASHBACK) Non-Localizability of f-BB84

Time

(When f(z,y)=0)



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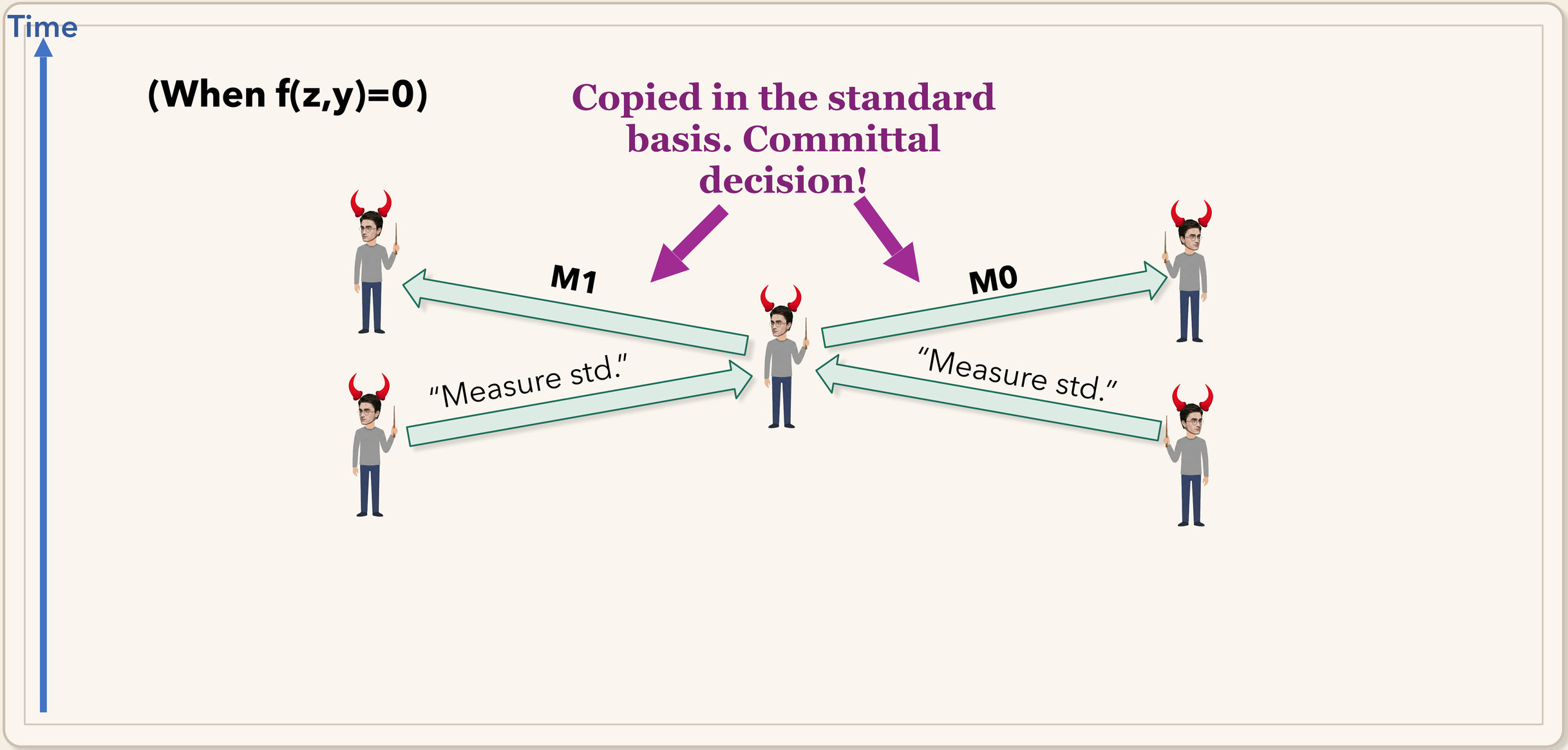

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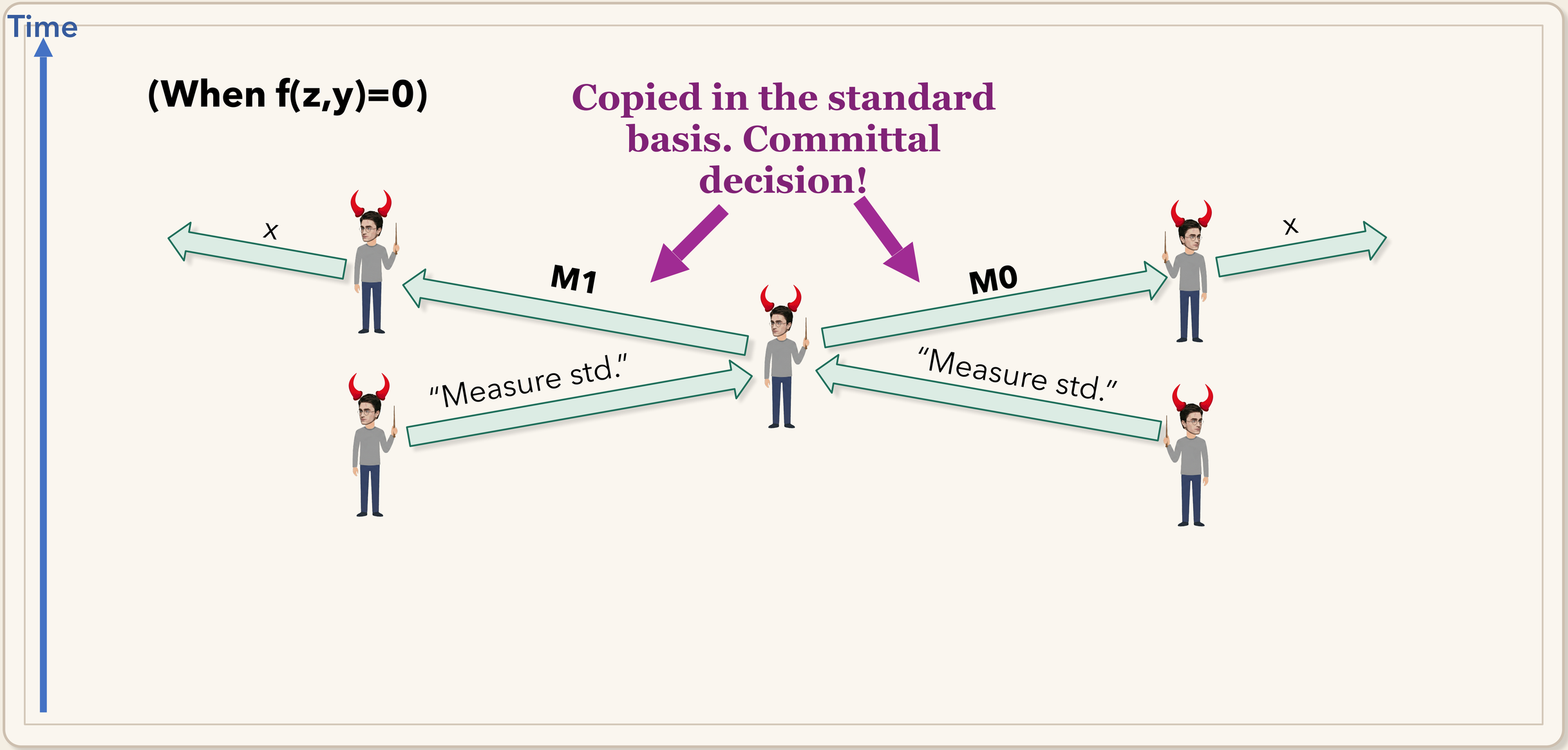

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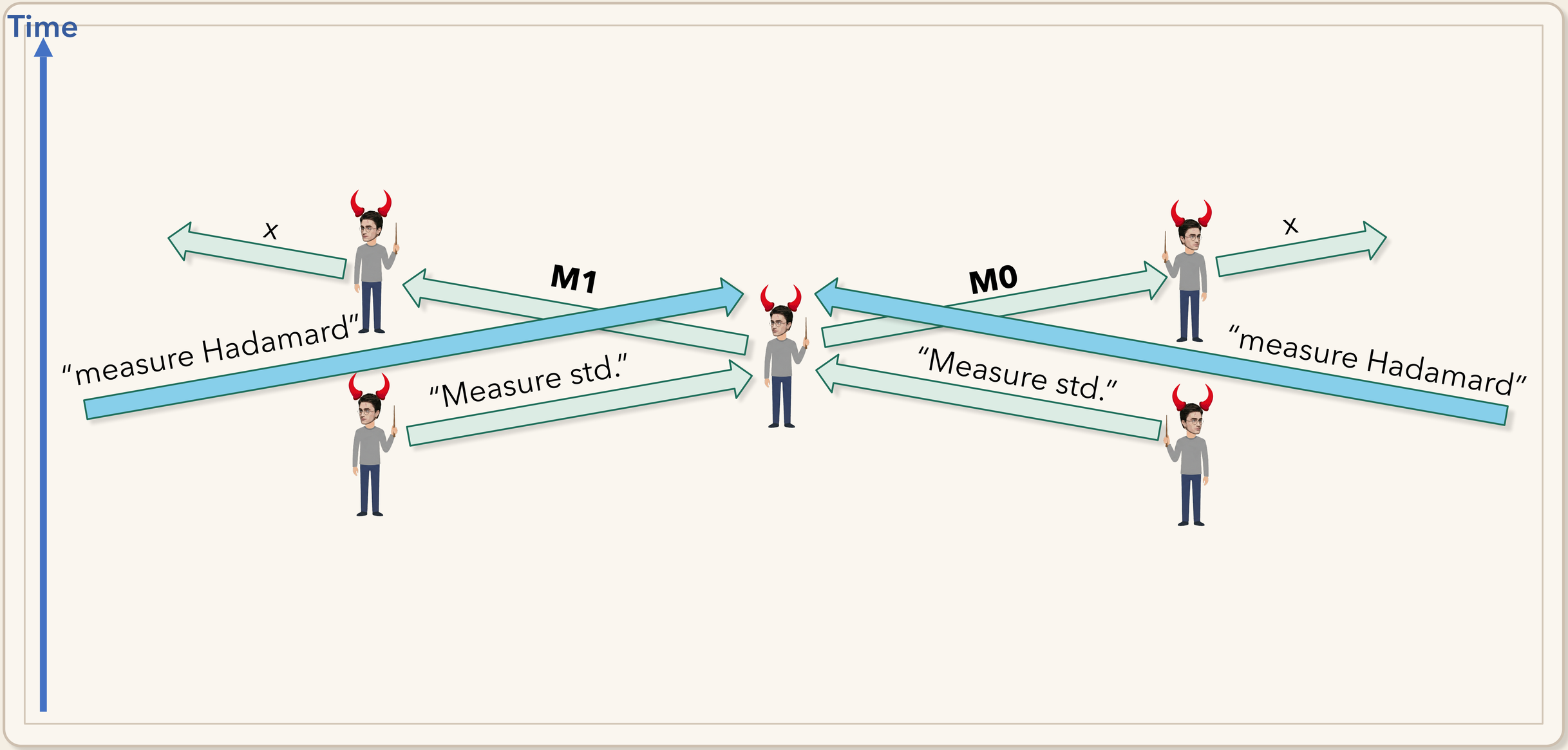

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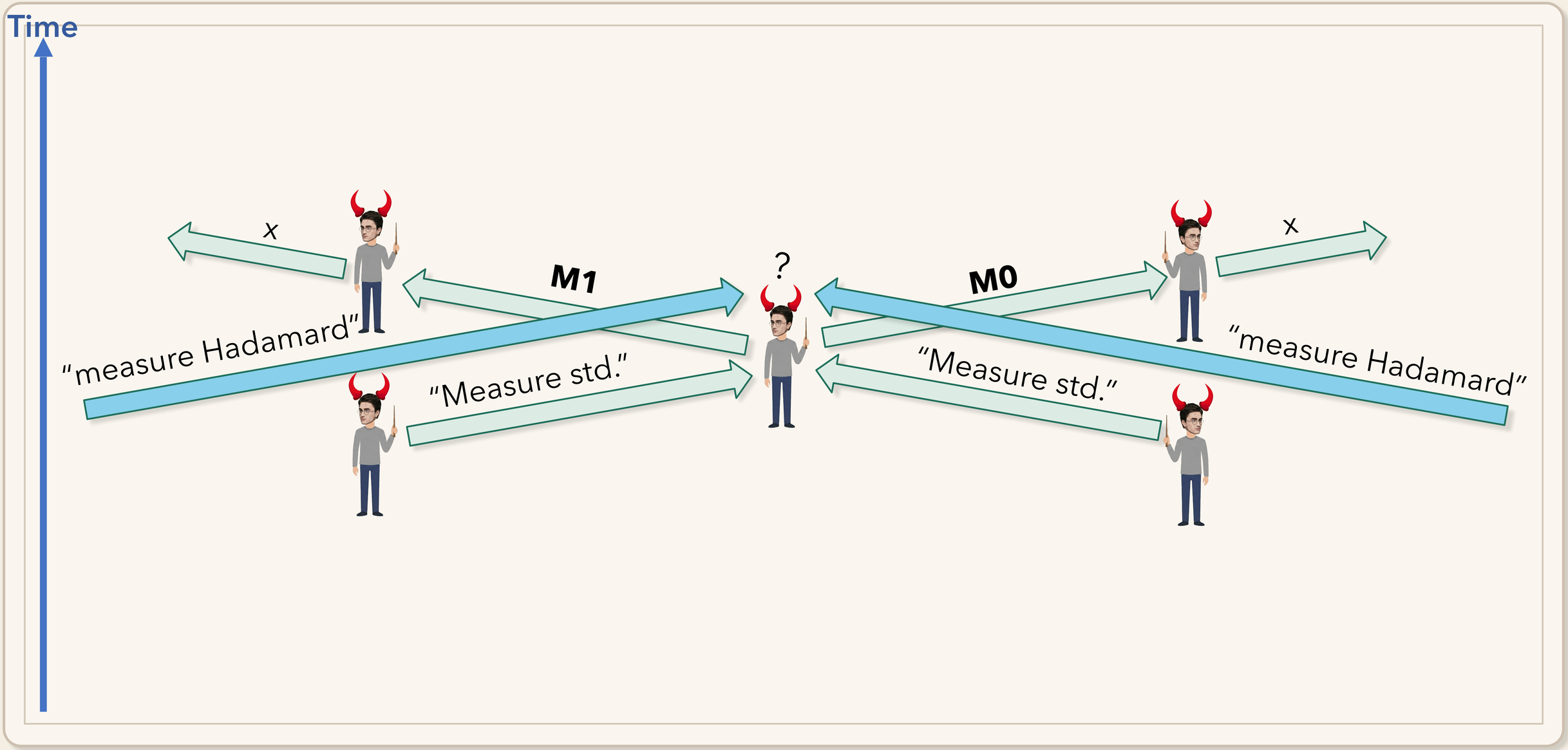

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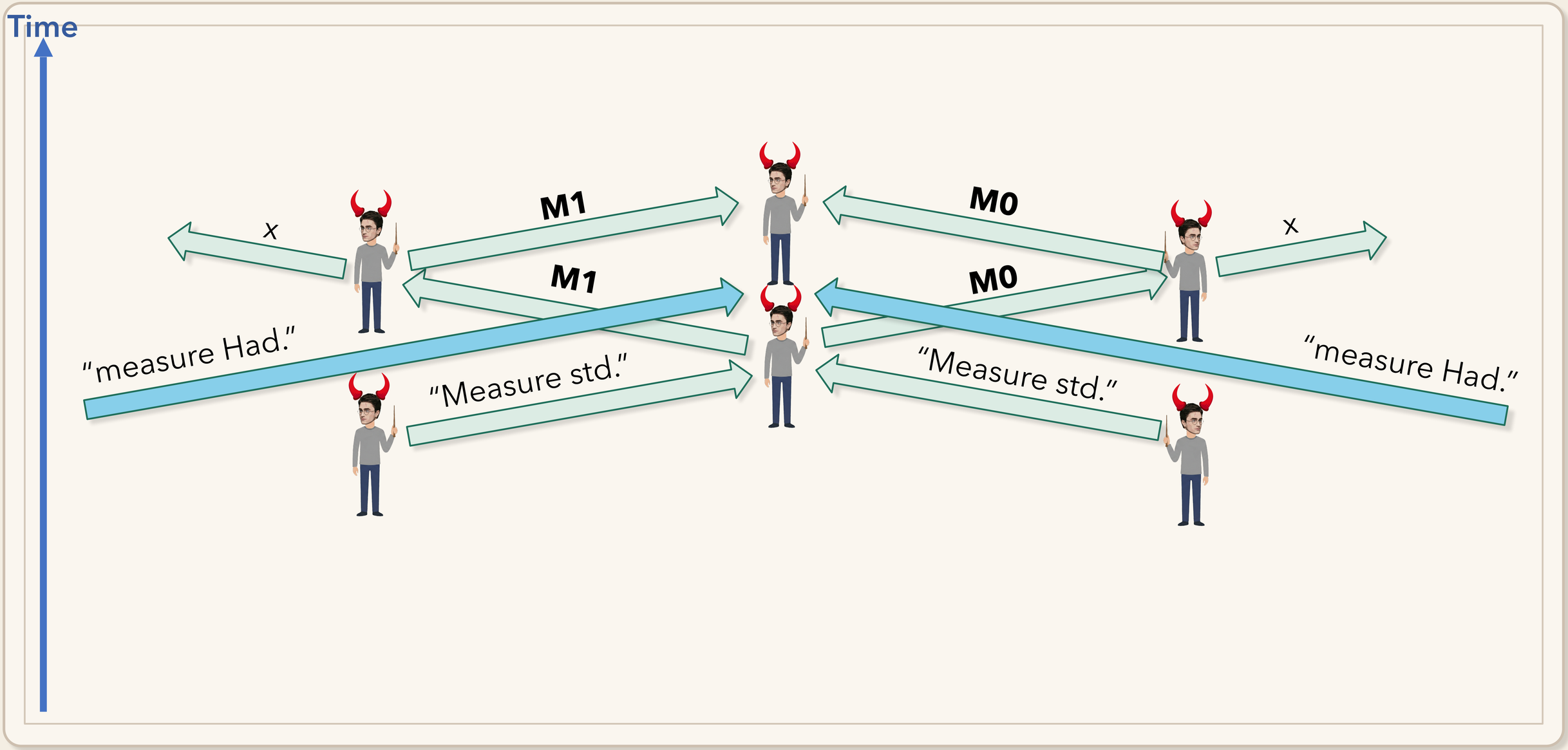

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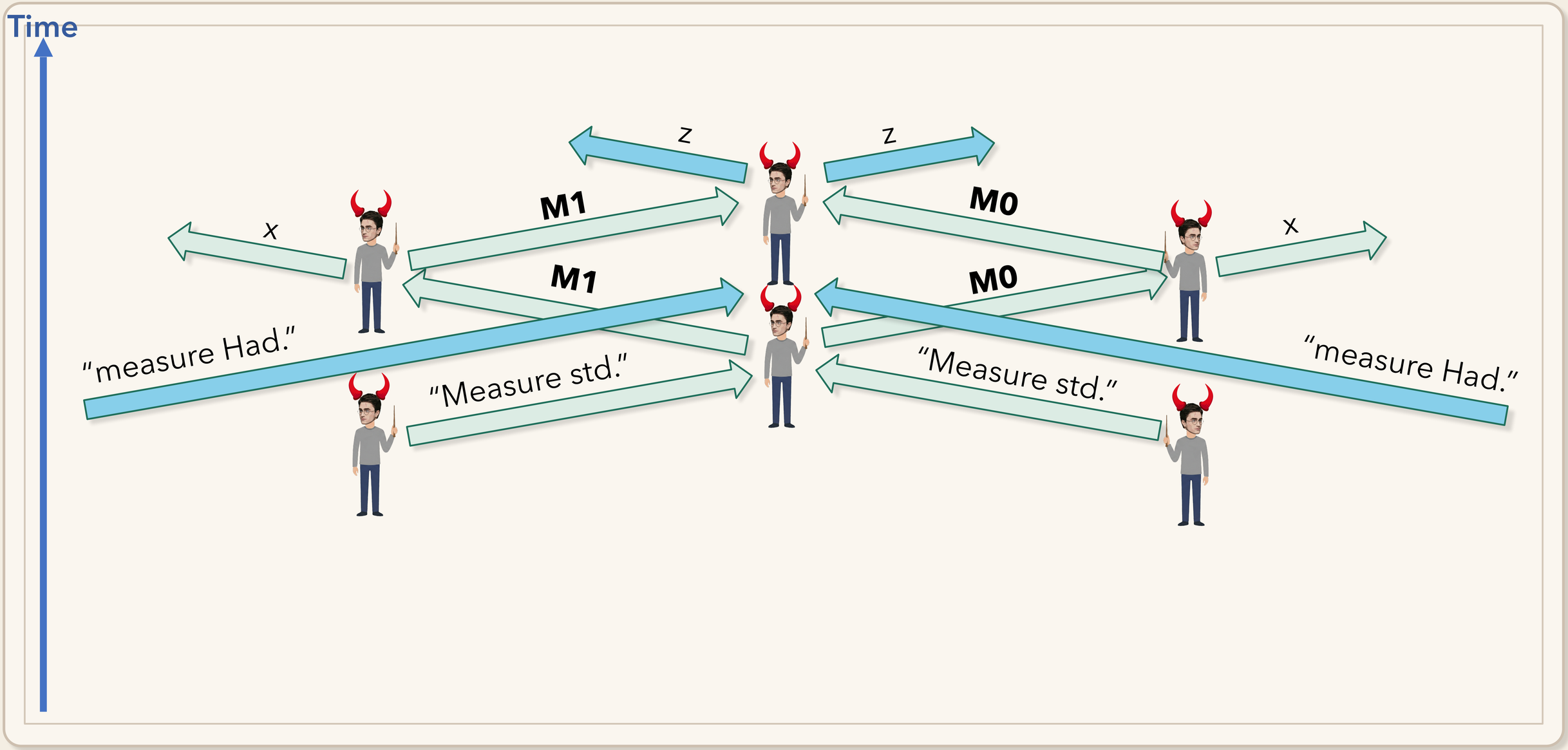

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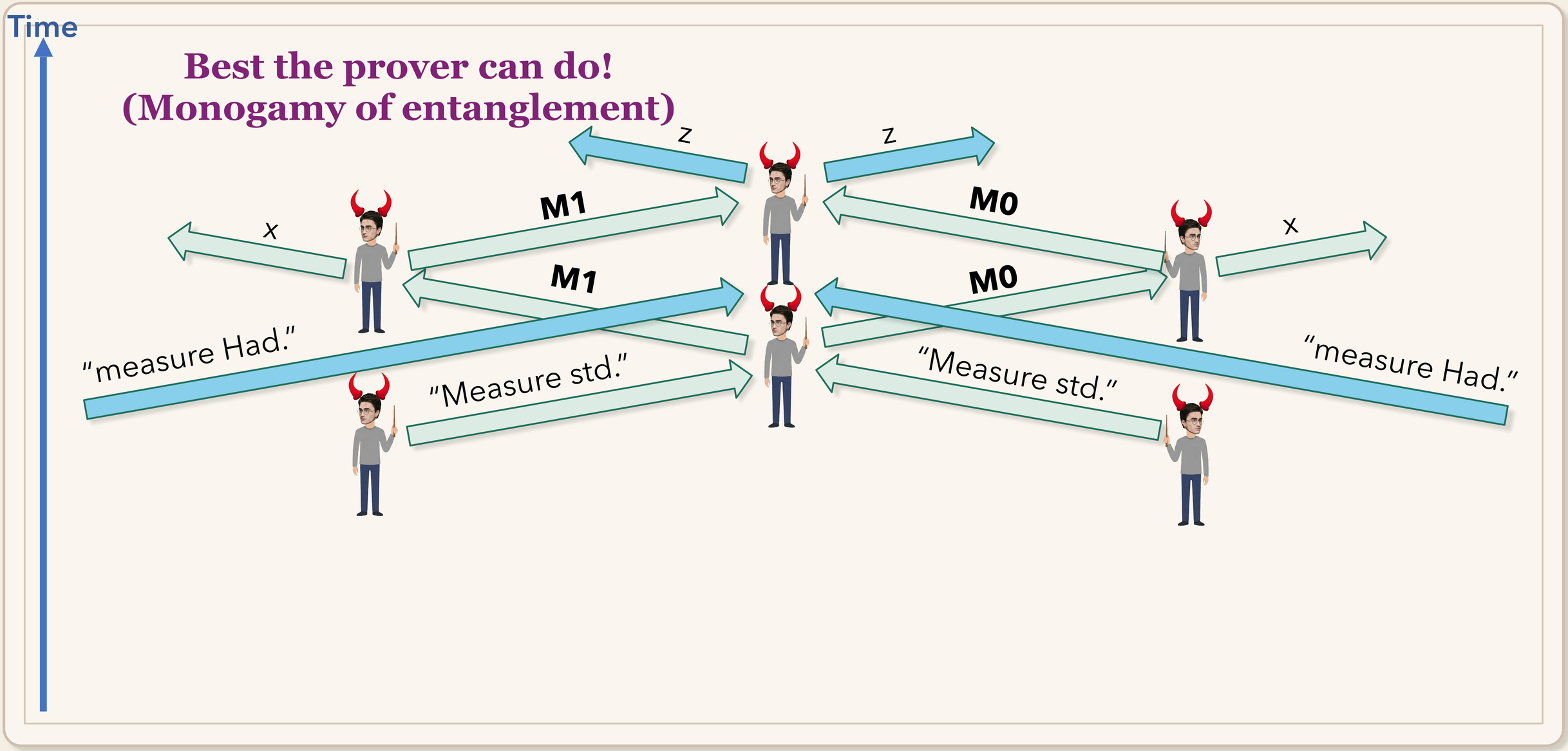

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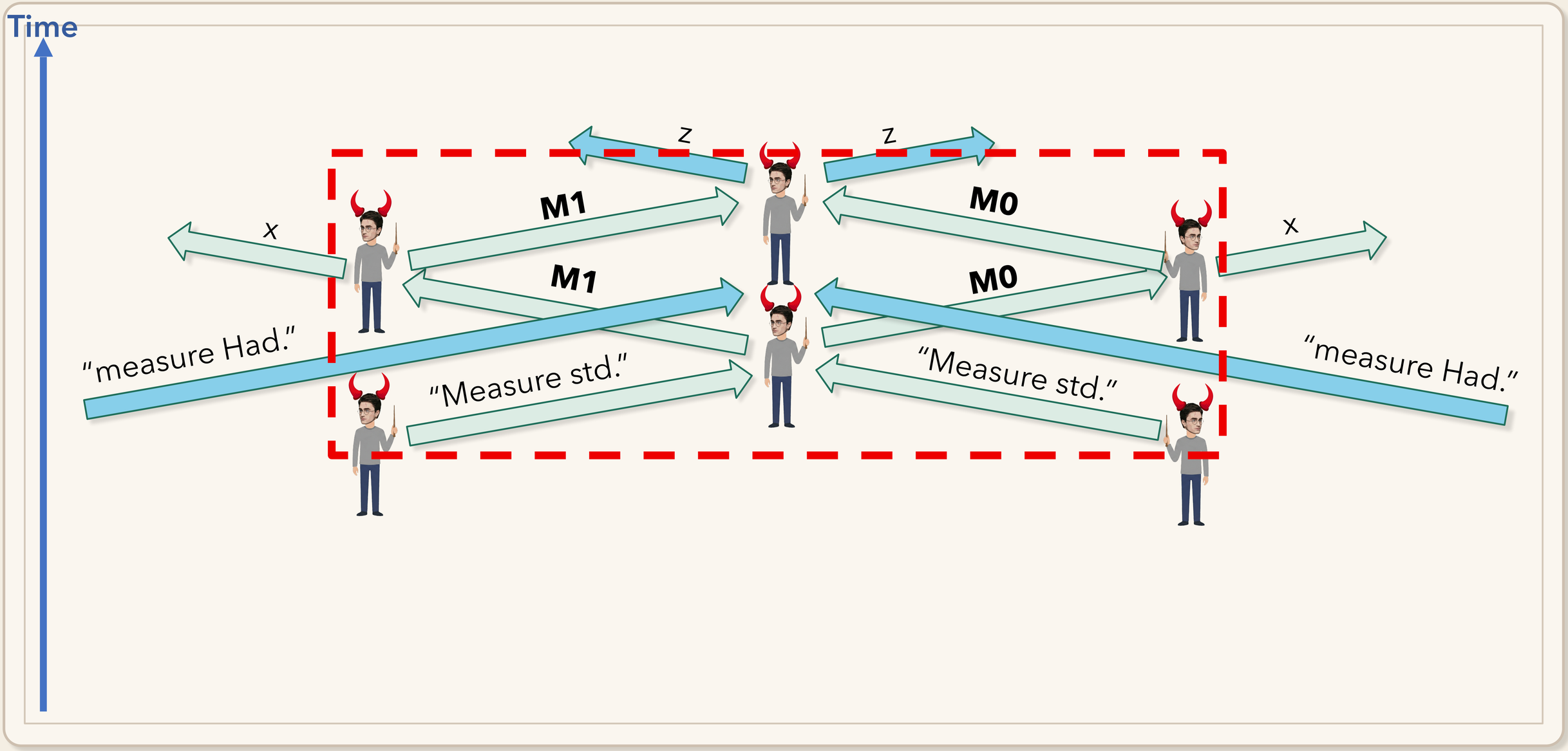

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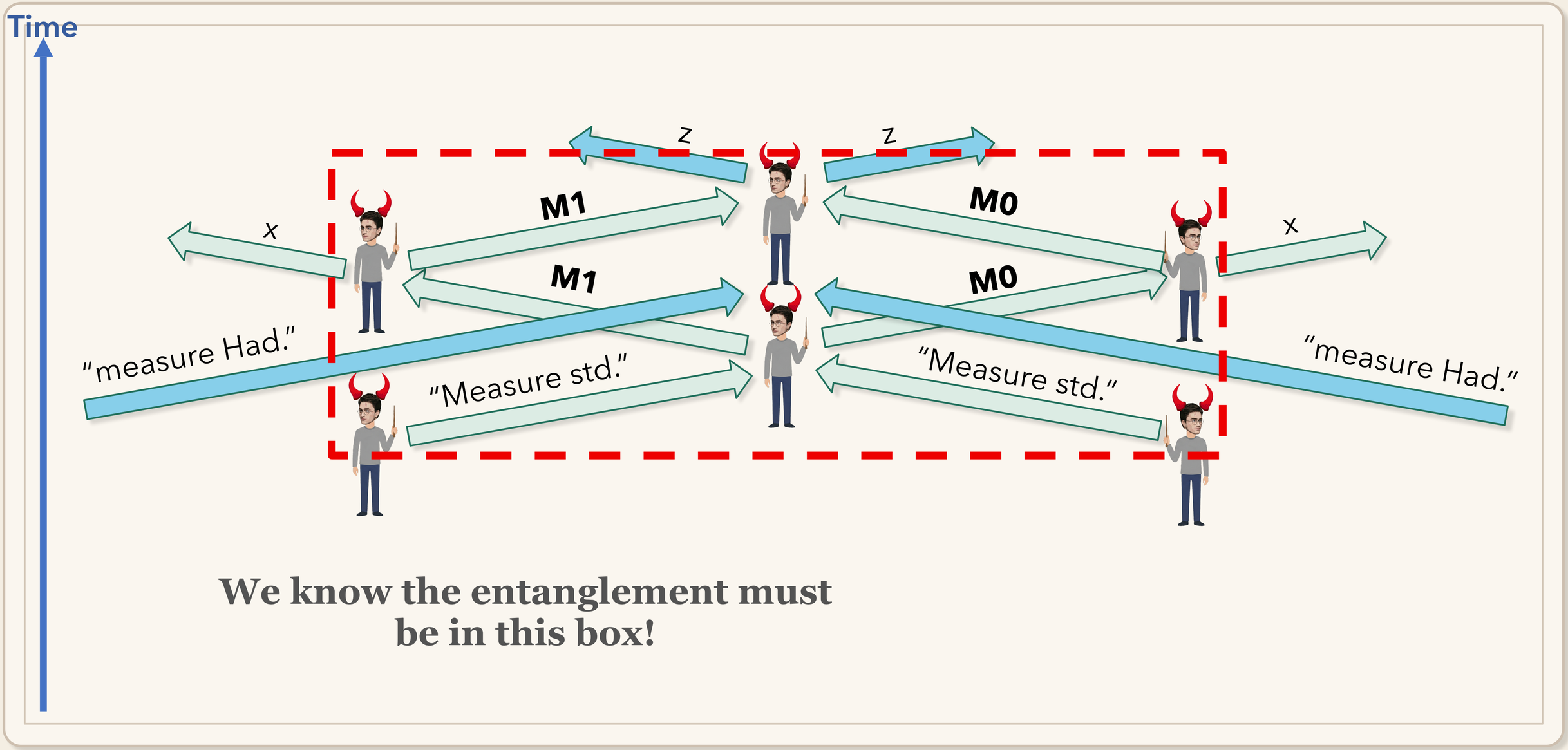

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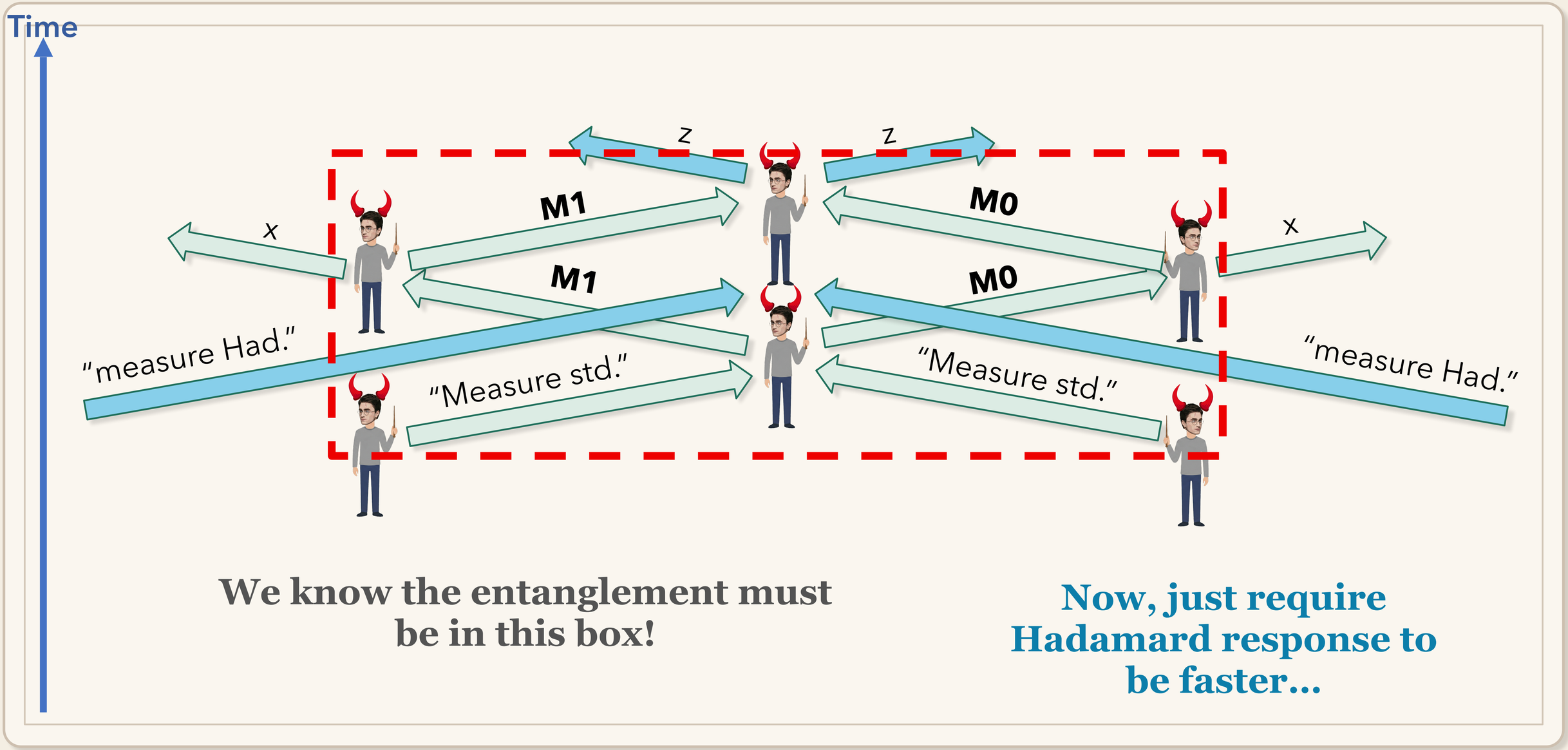

We know the entanglement must be in this box!

Now, just require Hadamard response to be faster…

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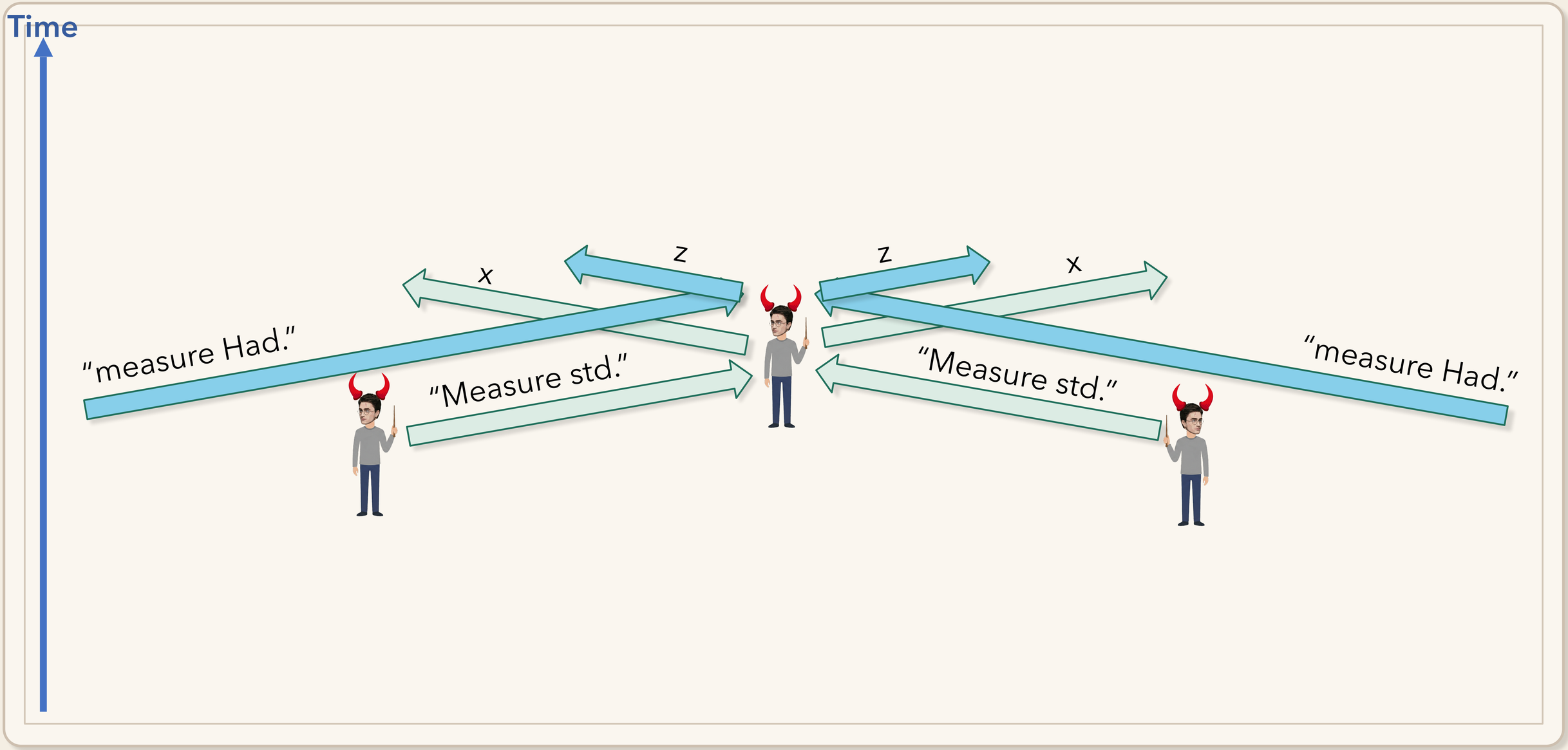

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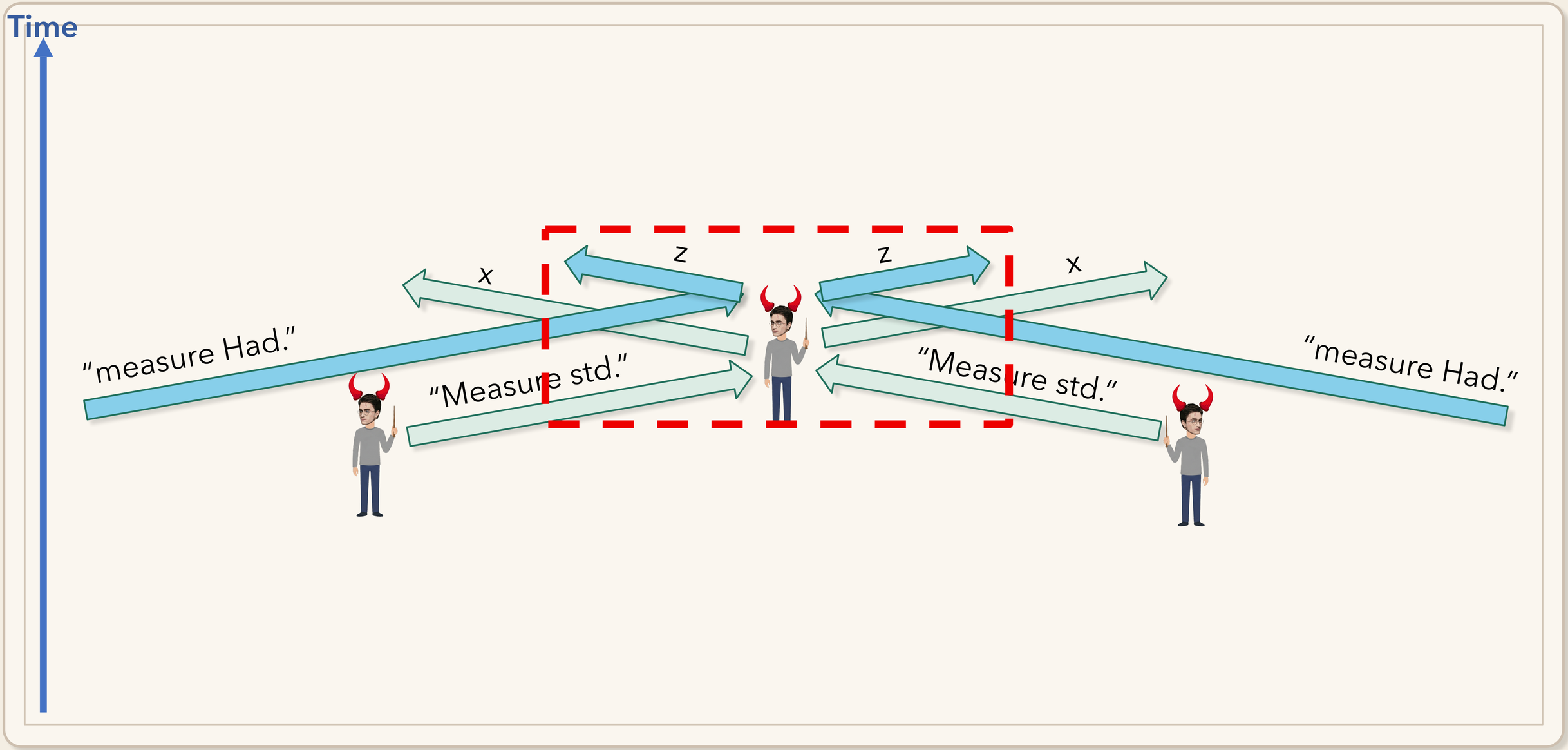

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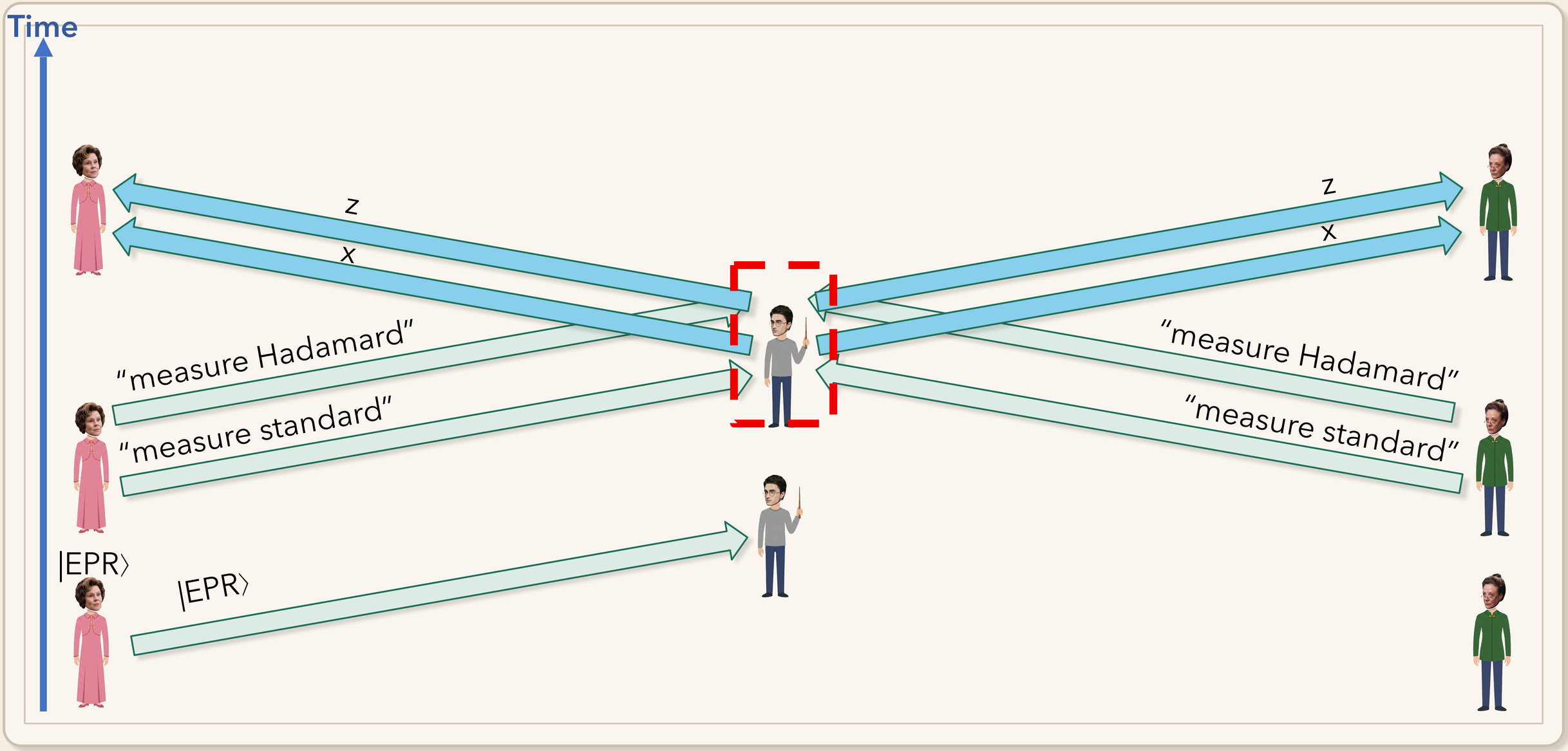

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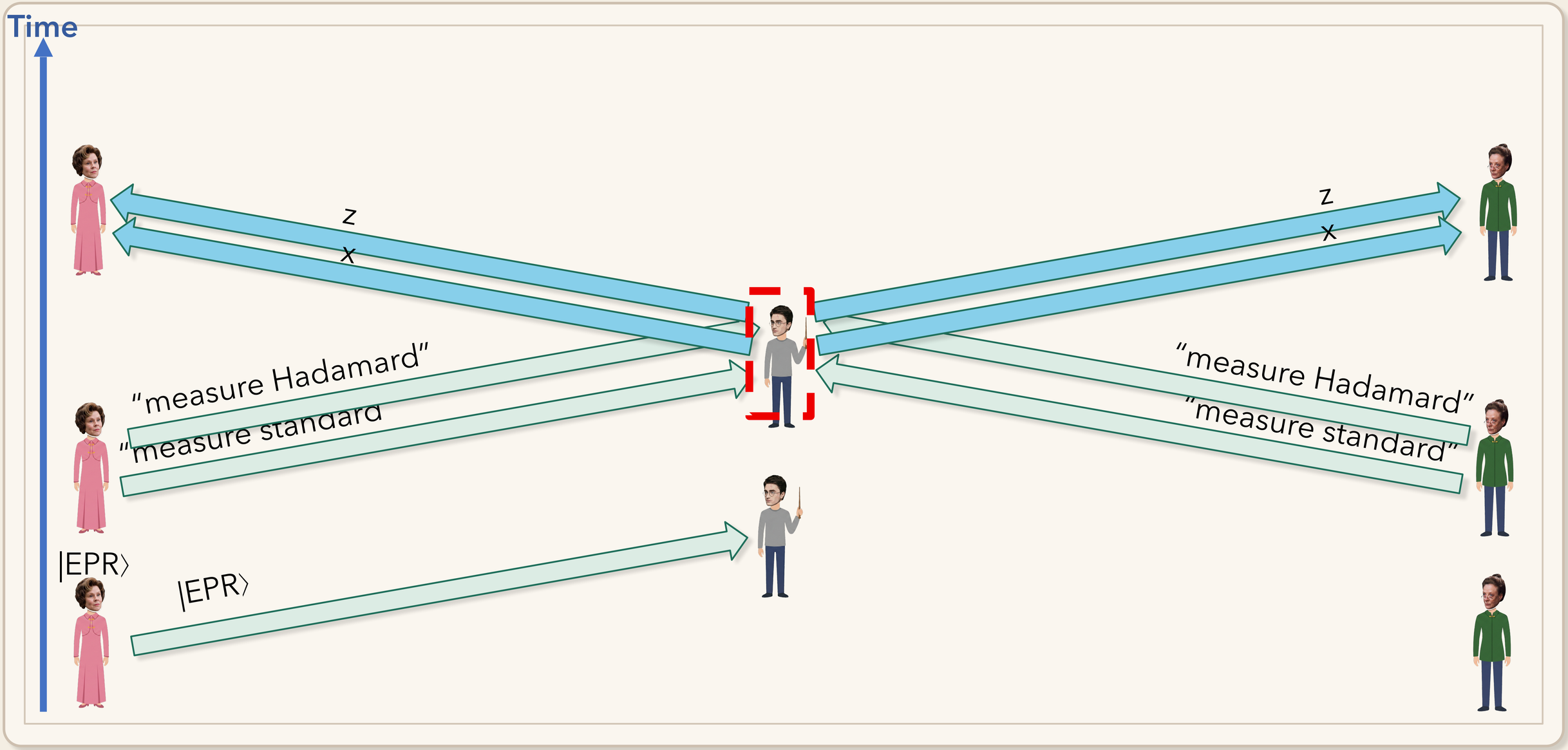

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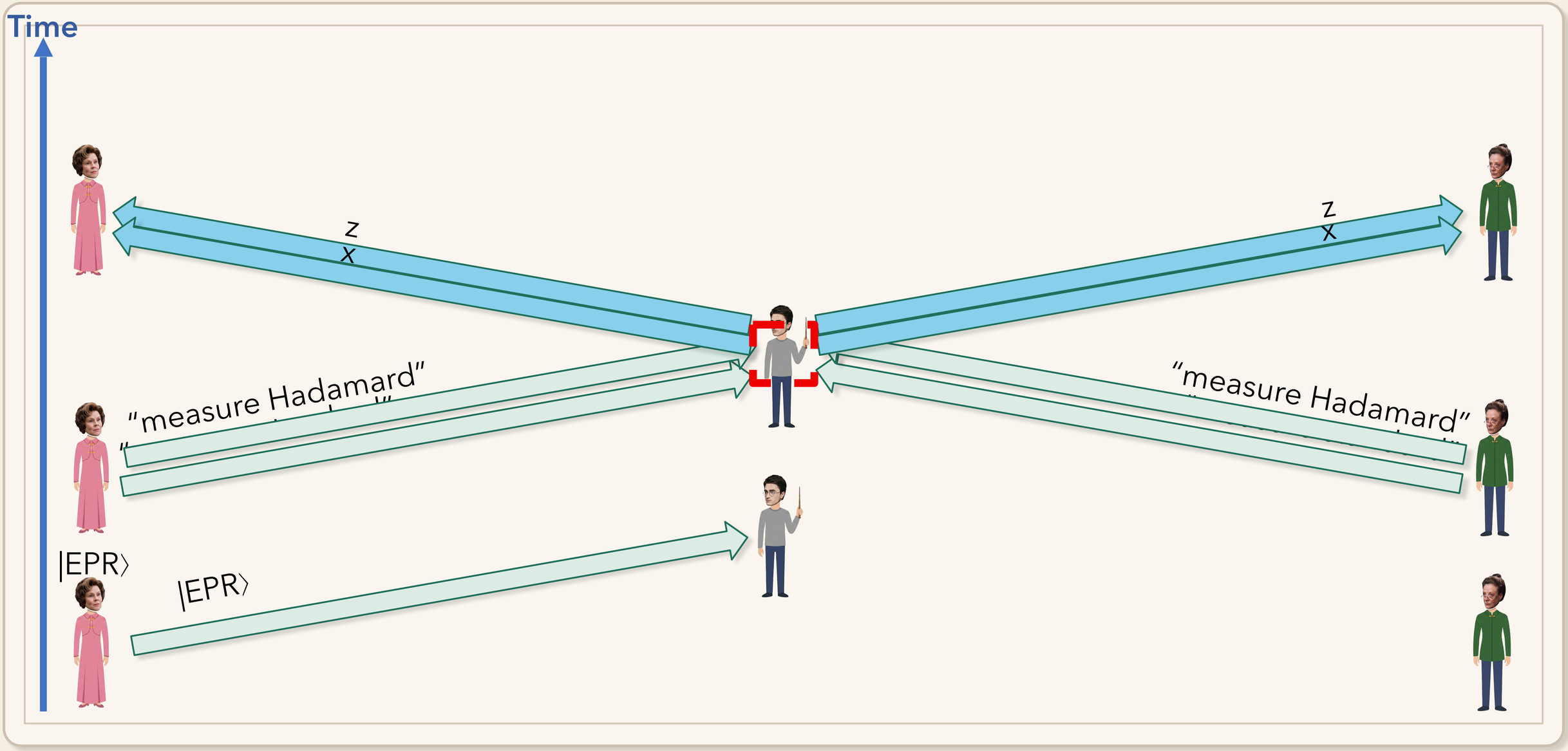

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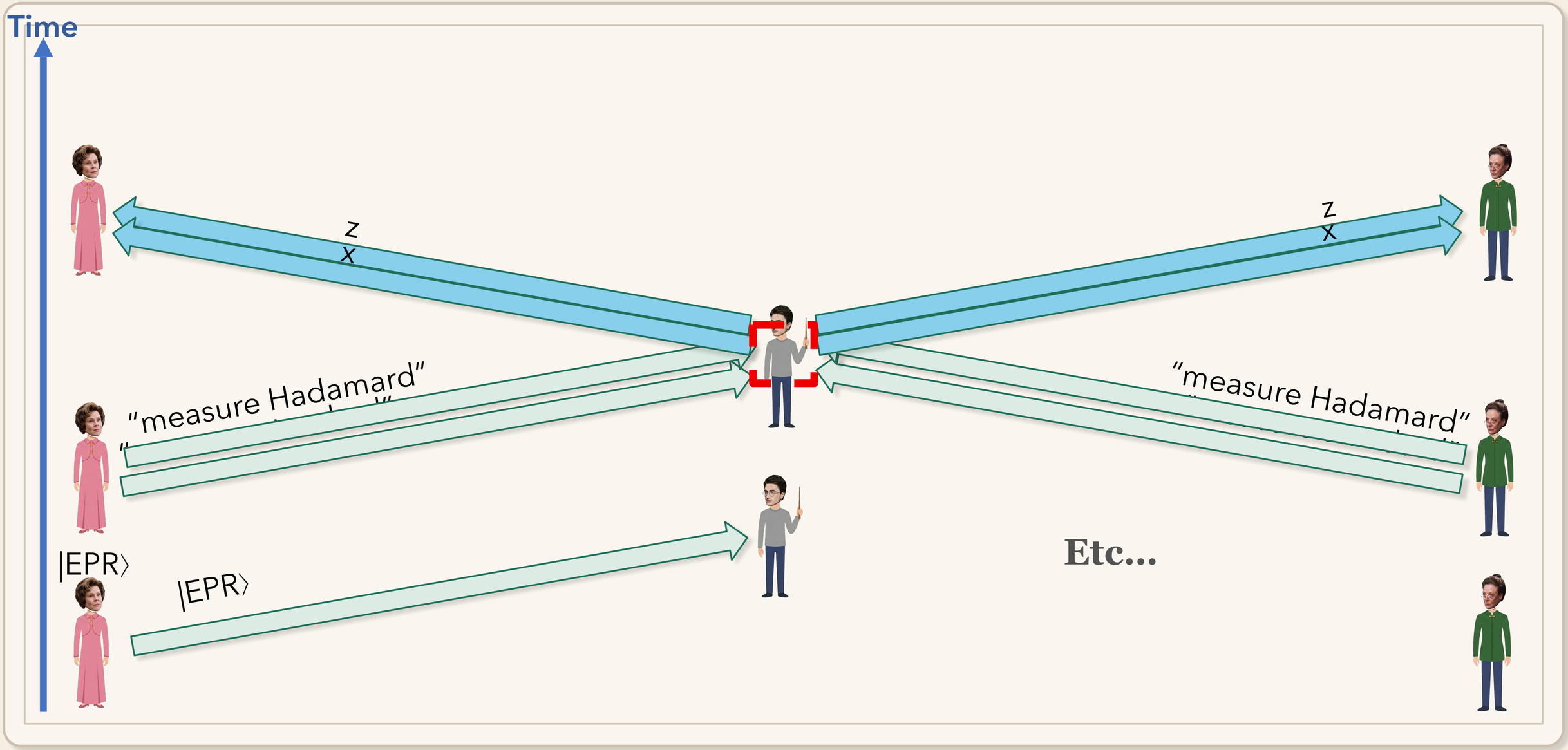
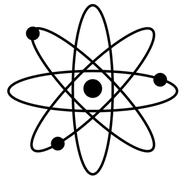


Non-Destructive Entanglement Testing

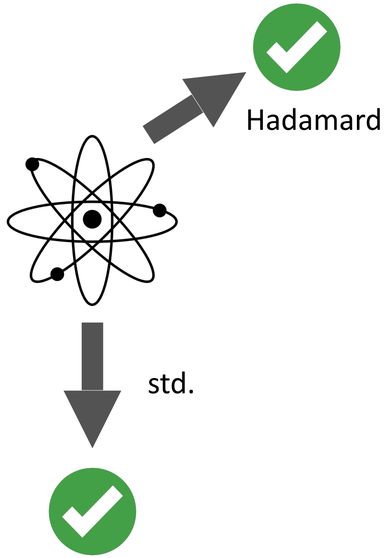


: “I got $|+\rangle$/I got $|0\rangle$”

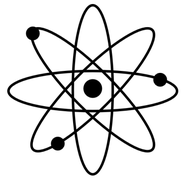
w.p. 1/2 **“Measure in the standard basis”**

OR

w.p. 1/2 **“Measure in the Hadamard basis”**



Non-Destructive Entanglement Testing



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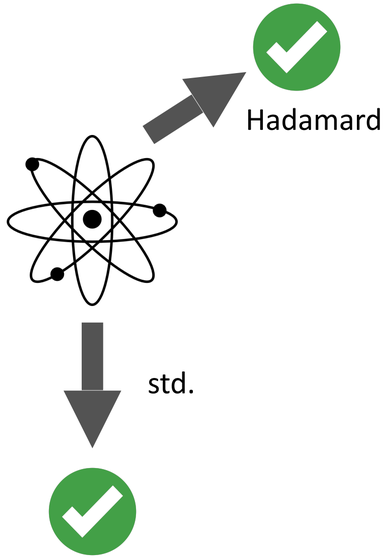
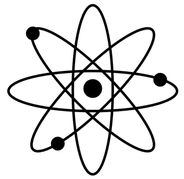


This is destructive! But need it be?

Non-Destructive Entanglement Testing

Adding Decoy Qubits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OR

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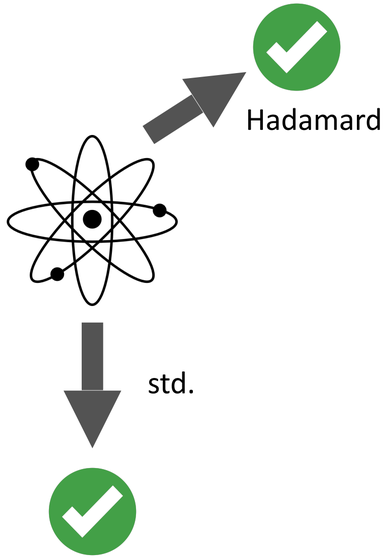
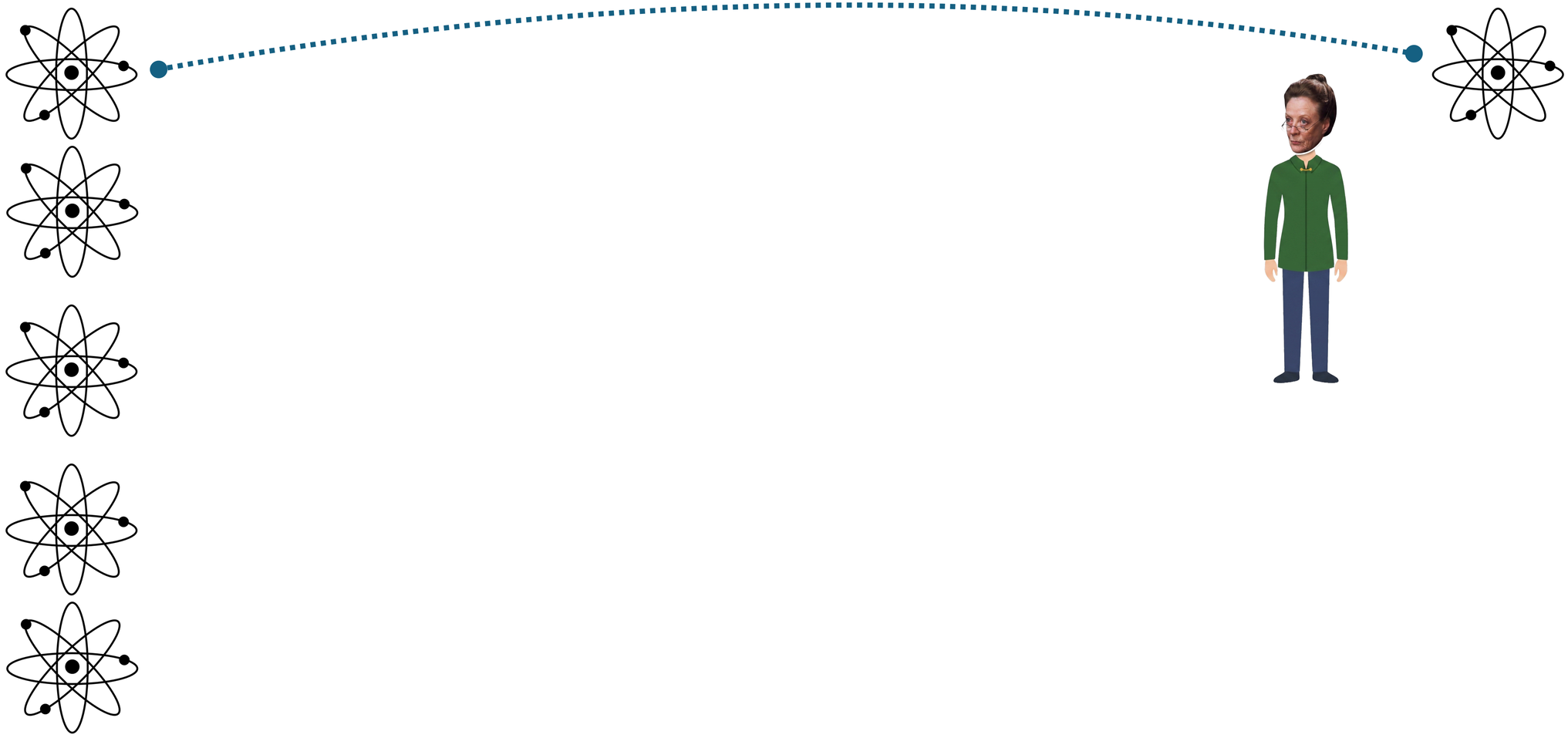


Non-Destructive Entanglement Testing

Adding Decoy Qubits

Non-Destructive Entanglement Testing

Adding Decoy Qubits

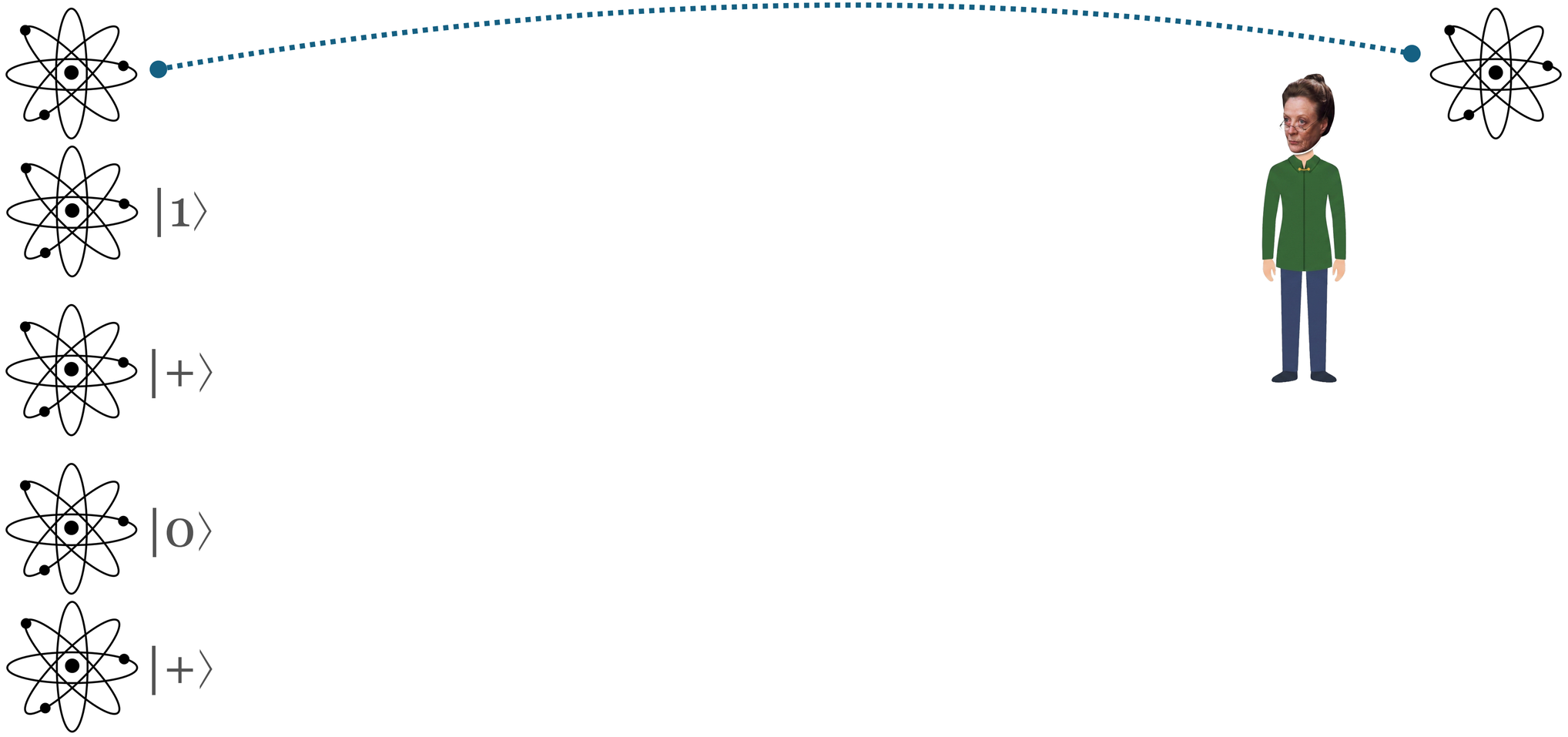


Non-Destructive Entanglement Testing

Adding Decoy Qubits

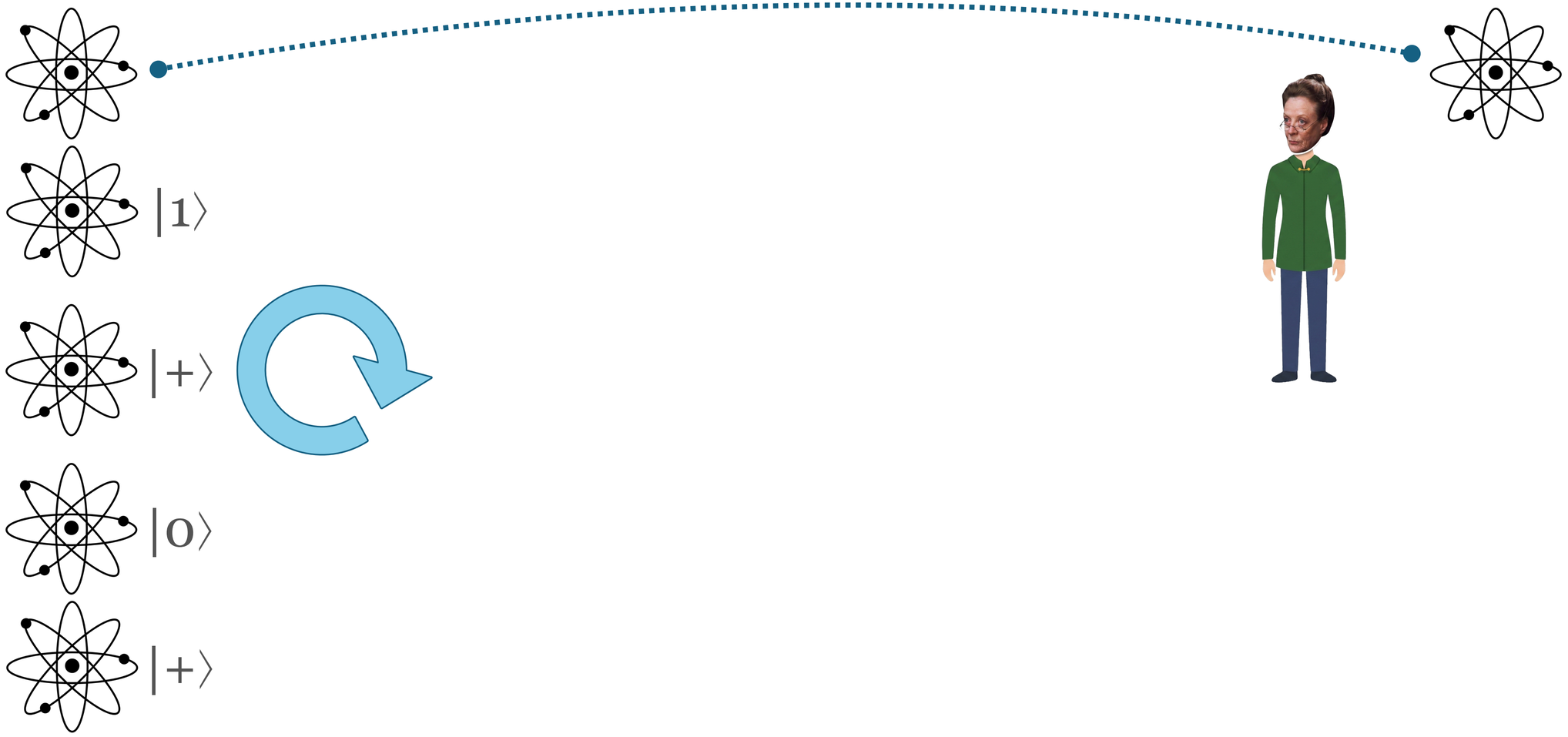
Non-Destructive Entanglement Testing

Adding Decoy Qubits



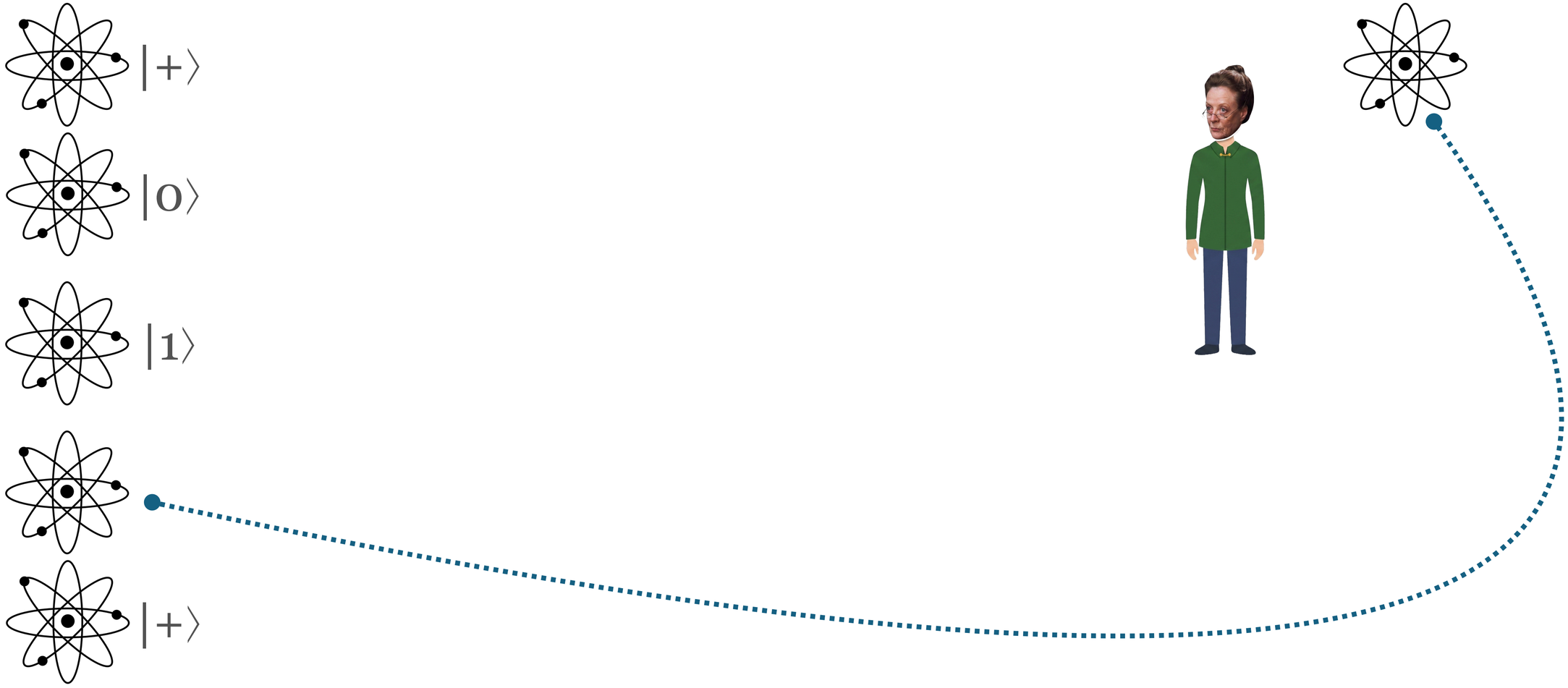
Non-Destructive Entanglement Testing

Adding Decoy Qubits



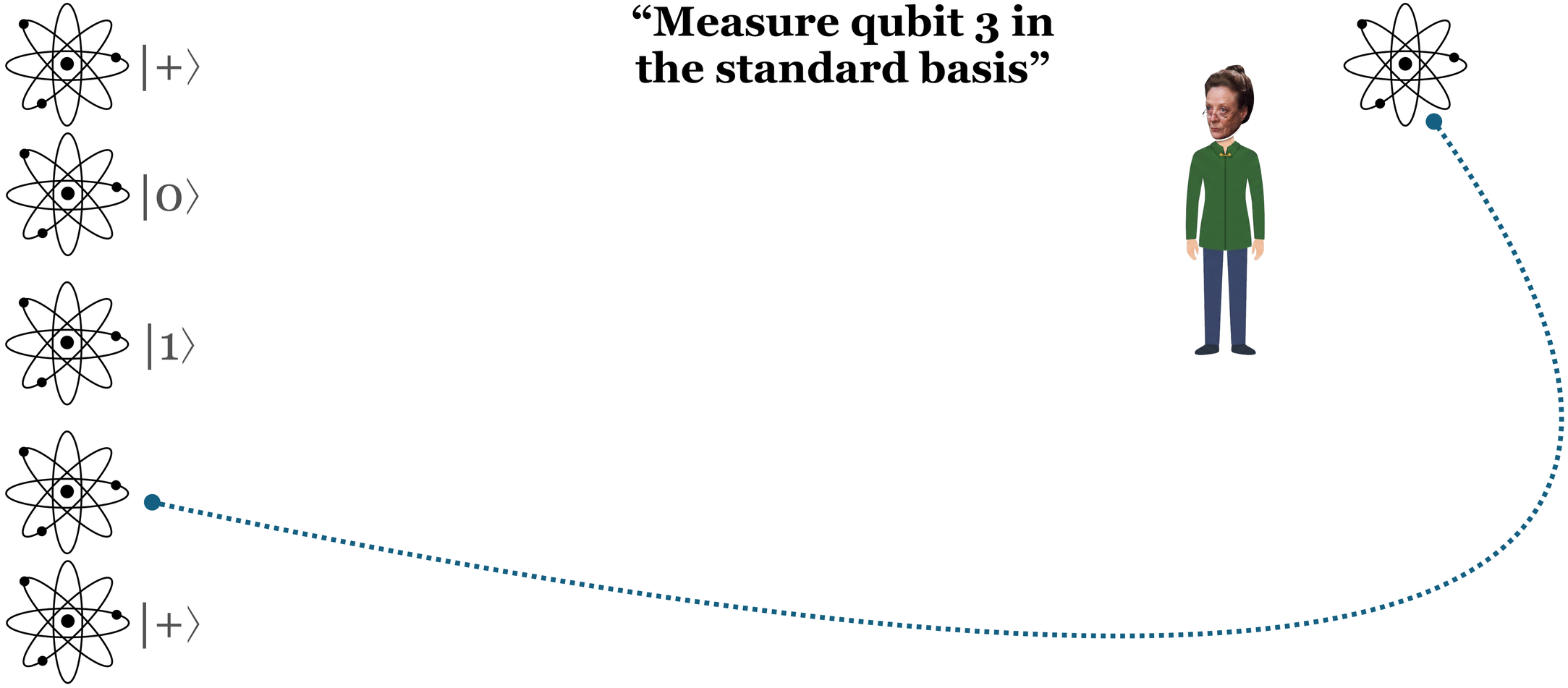
Non-Destructive Entanglement Testing

Adding Decoy Qubits



Non-Destructive Entanglement Testing

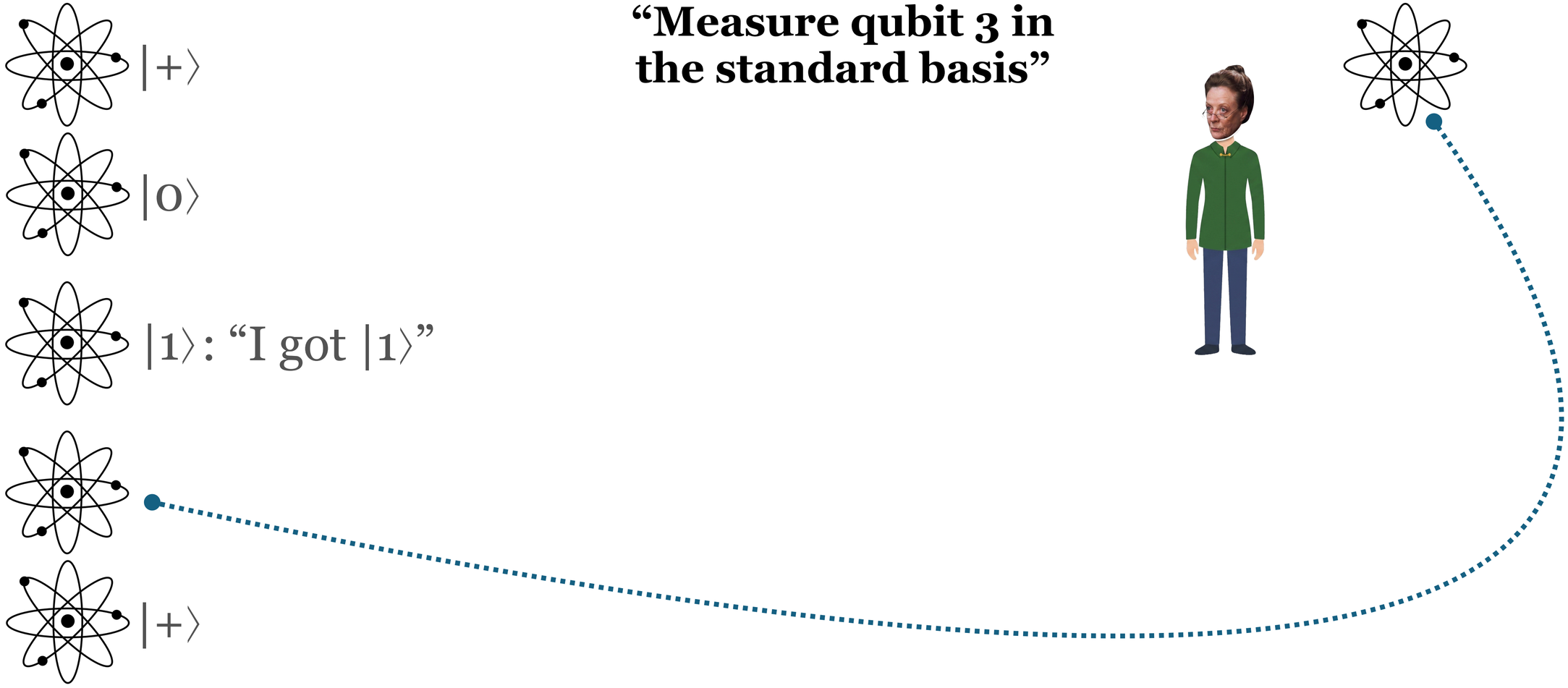
Adding Decoy Qubits



“Measure qubit 3 in the standard basis”

Non-Destructive Entanglement Testing

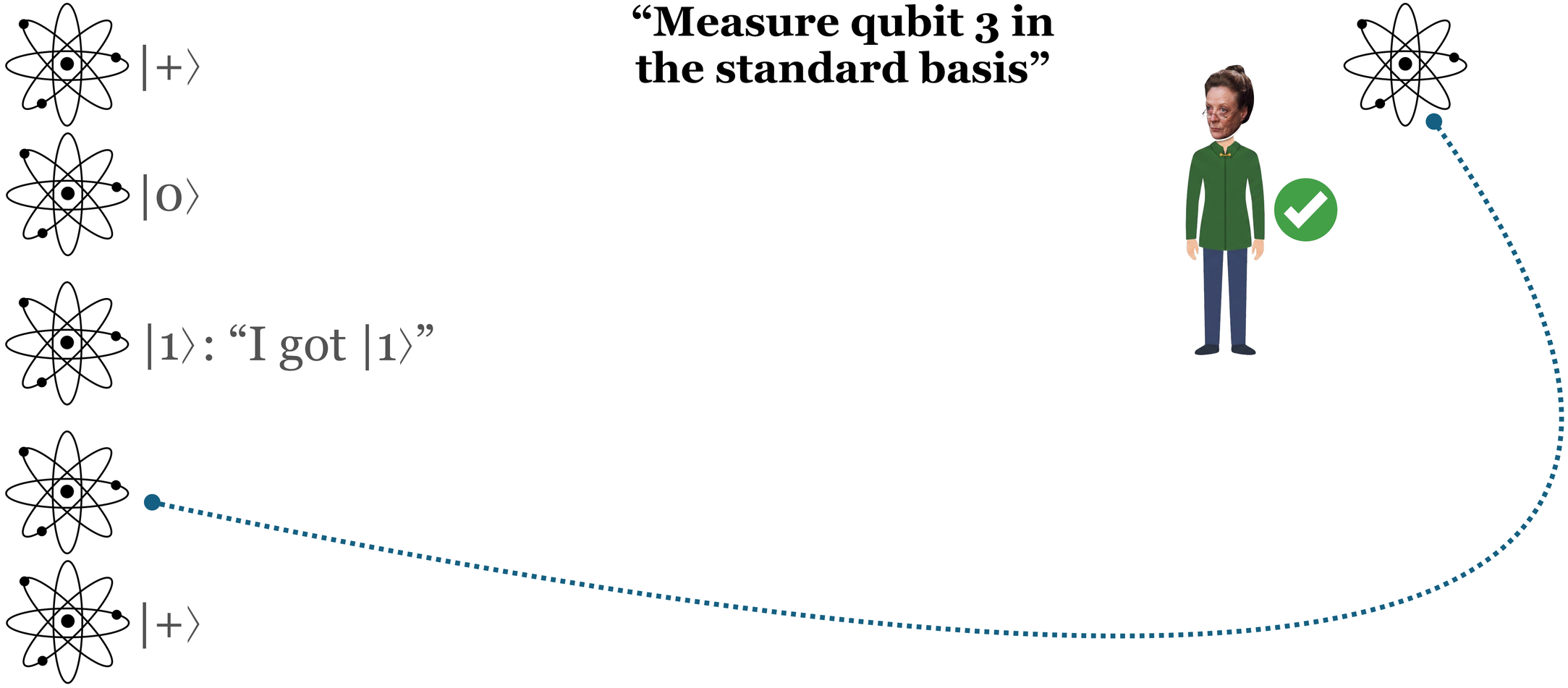
Adding Decoy Qubits



“Measure qubit 3 in the standard basis”

Non-Destructive Entanglement Testing

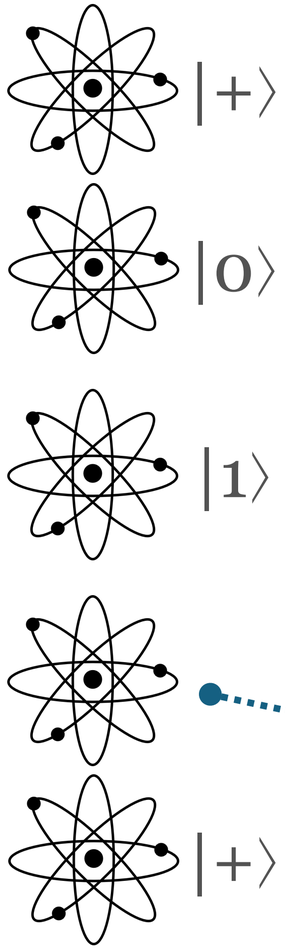
Adding Decoy Qubits



“Measure qubit 3 in the standard basis”

Non-Destructive Entanglement Testing

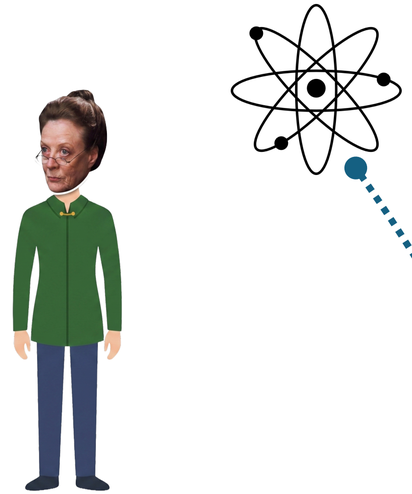
Adding Decoy Qubits



“Measure qubit 3 in the standard basis”

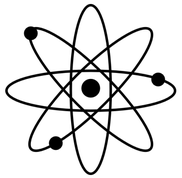
OR

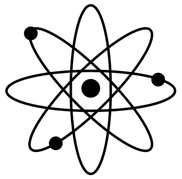
“Measure qubit 1 in the Hadamard basis”

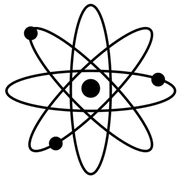


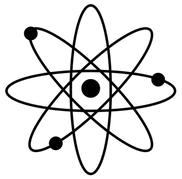
Non-Destructive Entanglement Testing

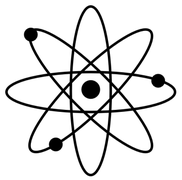
Adding Decoy Qubits


$|+\rangle$: “I got $|+\rangle$”


$|0\rangle$


$|1\rangle$

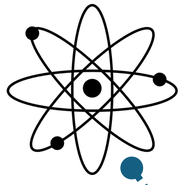



$|+\rangle$

“Measure qubit 3 in the standard basis”

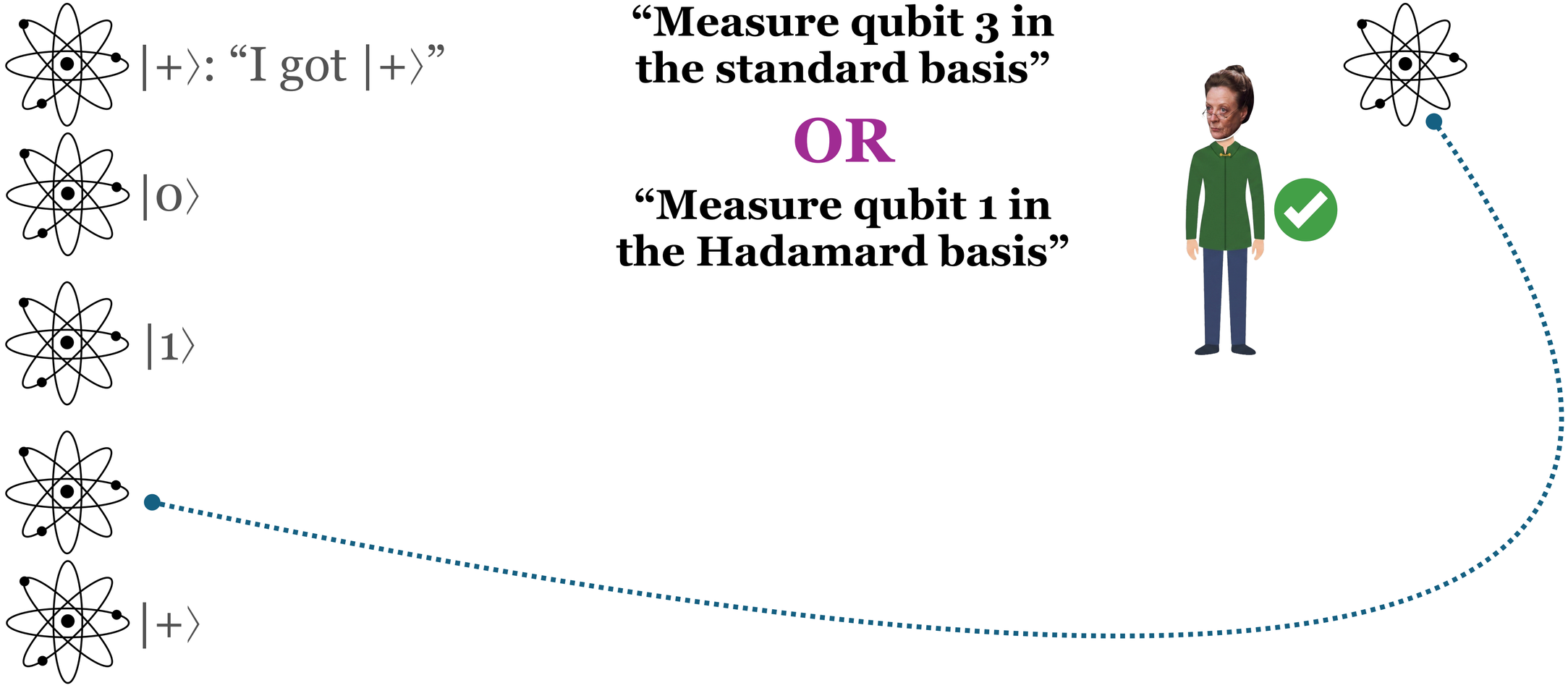
OR

“Measure qubit 1 in the Hadamard basis”



Non-Destructive Entanglement Testing

Adding Decoy Qubits



“Measure qubit 3 in the standard basis”

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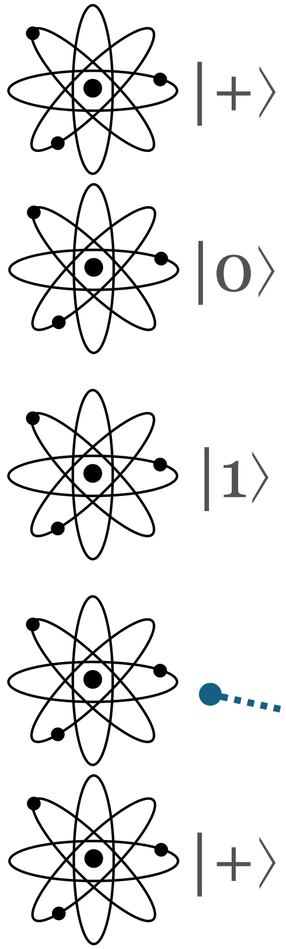
Non-Destructive Entanglement Testing

Adding Decoy Qubits



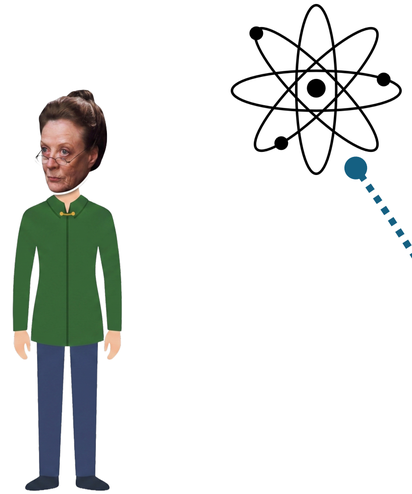
“Measure qubit 3 in the standard basis”

OR

“Measure qubit 1 in the Hadamard basis”

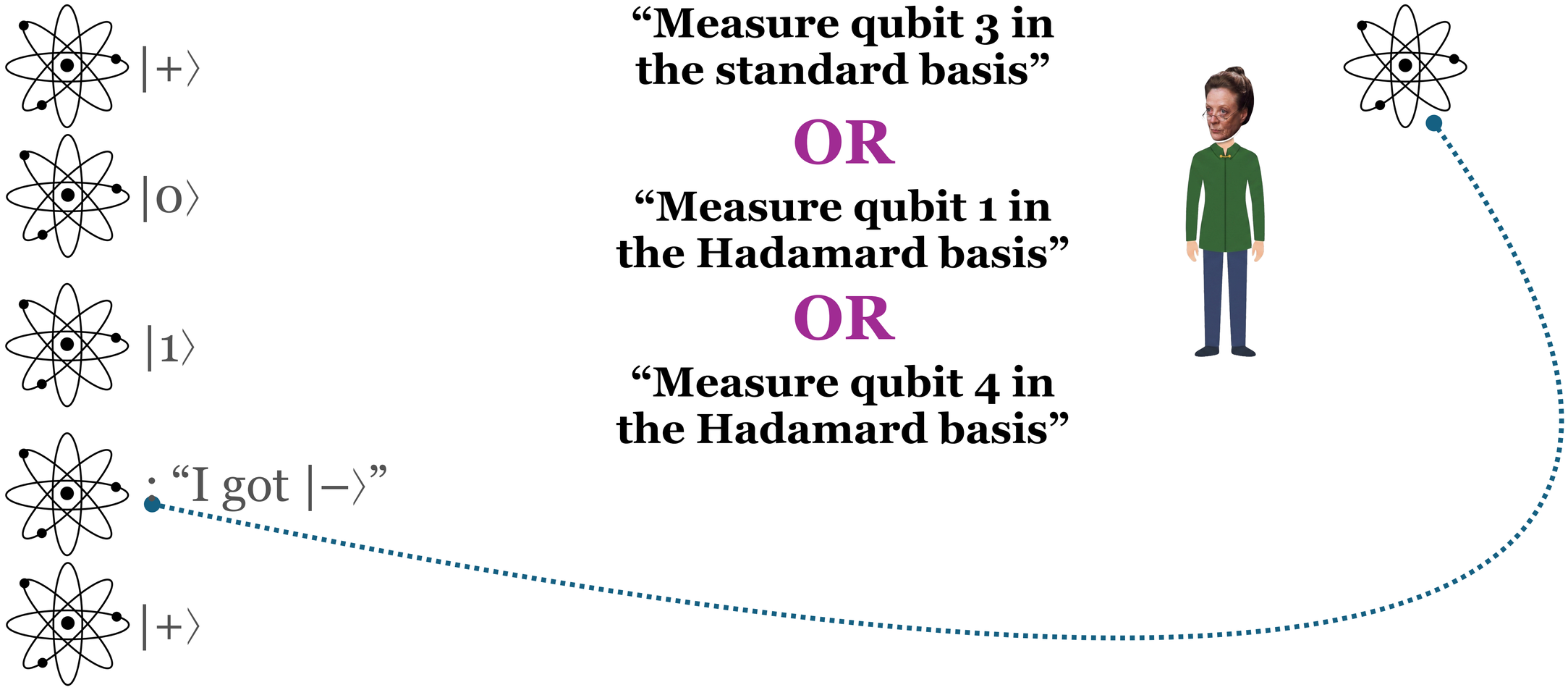
OR

“Measure qubit 4 in the Hadamard basis”



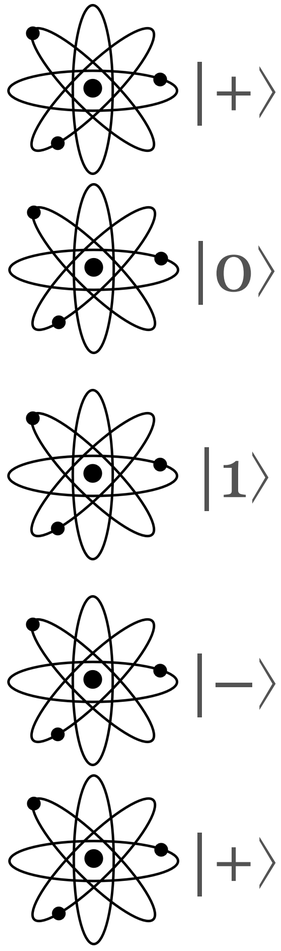
Non-Destructive Entanglement Testing

Adding Decoy Qubits



Non-Destructive Entanglement Testing

Adding Decoy Qubits



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“Measure qubit 1 in the Hadamard basis”

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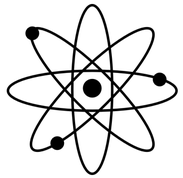
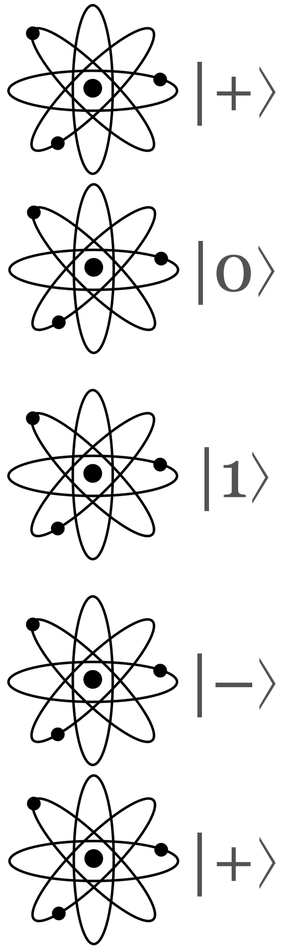


Non-Destructive Entanglement Testing

Adding Decoy Qubits



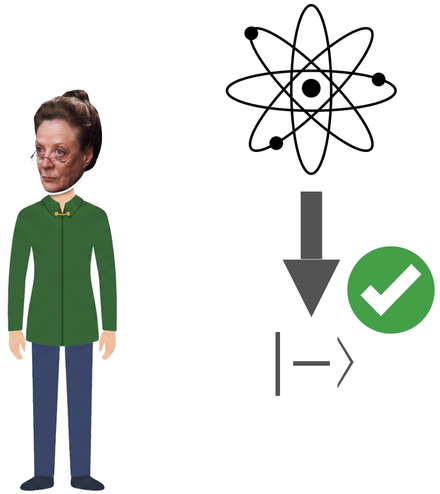
“Measure qubit 3 in the standard basis”

OR

“Measure qubit 1 in the Hadamard basis”

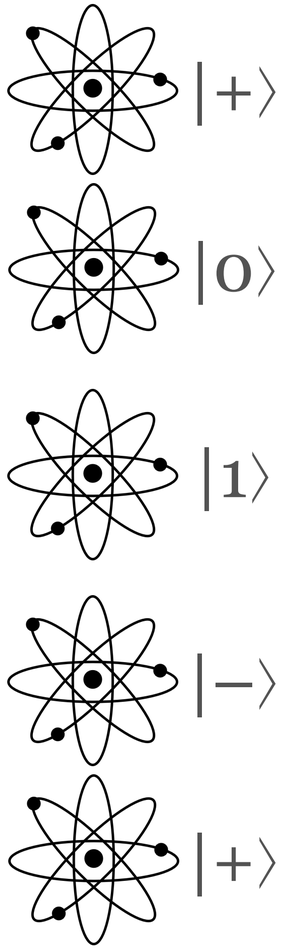
OR

“Measure qubit 4 in the Hadamard basis”



Non-Destructive Entanglement Testing

Adding Decoy Qubits



Non-destructive

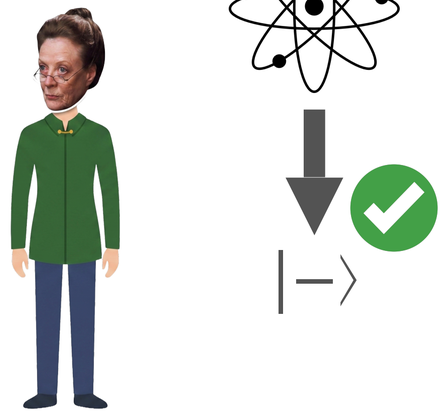
“Measure qubit 3 in the standard basis”

OR

“Measure qubit 1 in the Hadamard basis”

OR

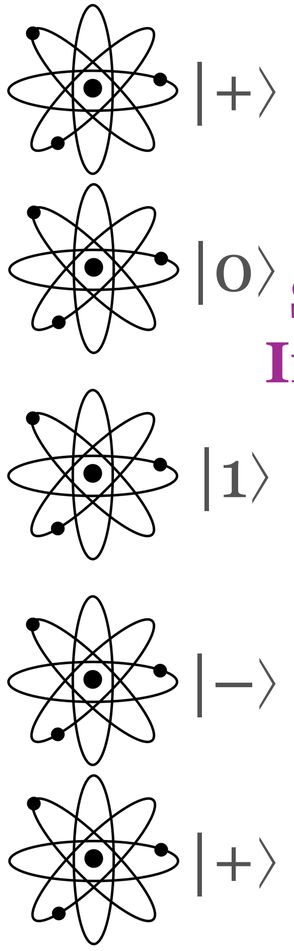
“Measure qubit 4 in the Hadamard basis”



Destructive!

Non-Destructive Entanglement Testing

Adding Decoy Qubits



Statistically Indistinguishable

“Measure qubit 3 in the standard basis”

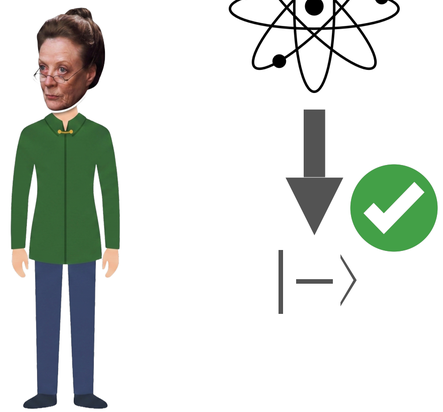
OR

“Measure qubit 1 in the Hadamard basis”

OR

“Measure qubit 4 in the Hadamard basis”

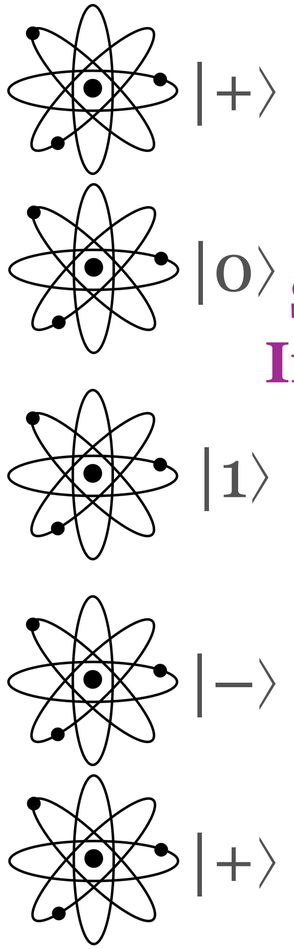
Non-destructive



Destructive!

Non-Destructive Entanglement Testing

Adding Decoy Qubits



Statistically Indistinguishable

"Measure qubit 3 in the standard basis"

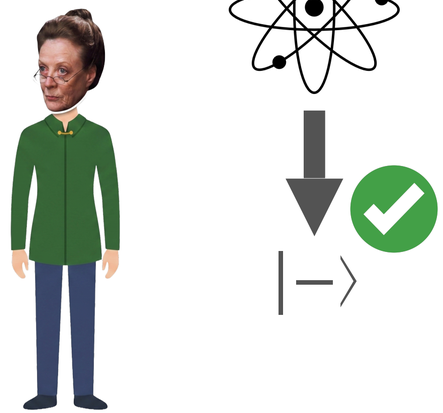
OR

"Measure qubit 1 in the Hadamard basis"

OR

"Measure qubit 4 in the Hadamard basis"

Non-destructive

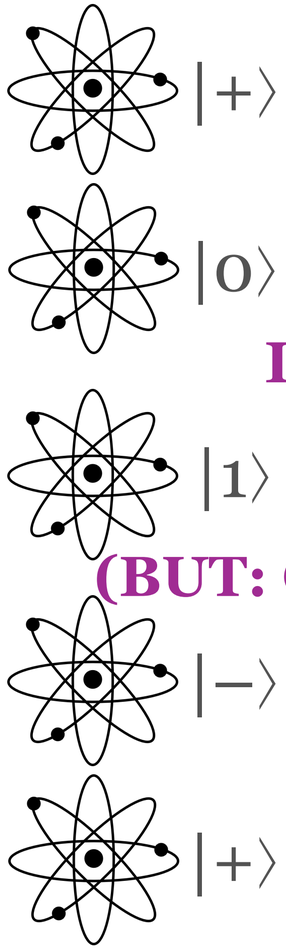


Destructive!

Moral: Real experiment performs *non-destructive* test, then analysis argues that it *could* have been the destructive test, thus certifying entanglement

Non-Destructive Entanglement Testing

Adding Decoy Qubits



Statistically Indistinguishable

(BUT: Only single-use)

Non-destructive

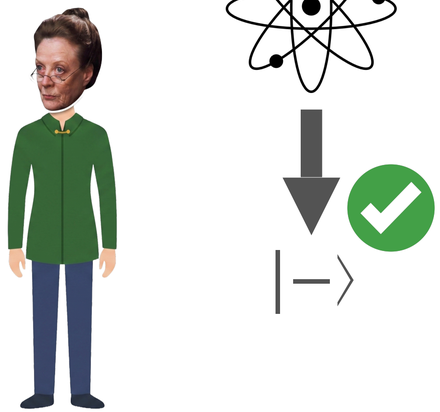
"Measure qubit 3 in the standard basis"

OR

"Measure qubit 1 in the Hadamard basis"

OR

"Measure qubit 4 in the Hadamard basis"

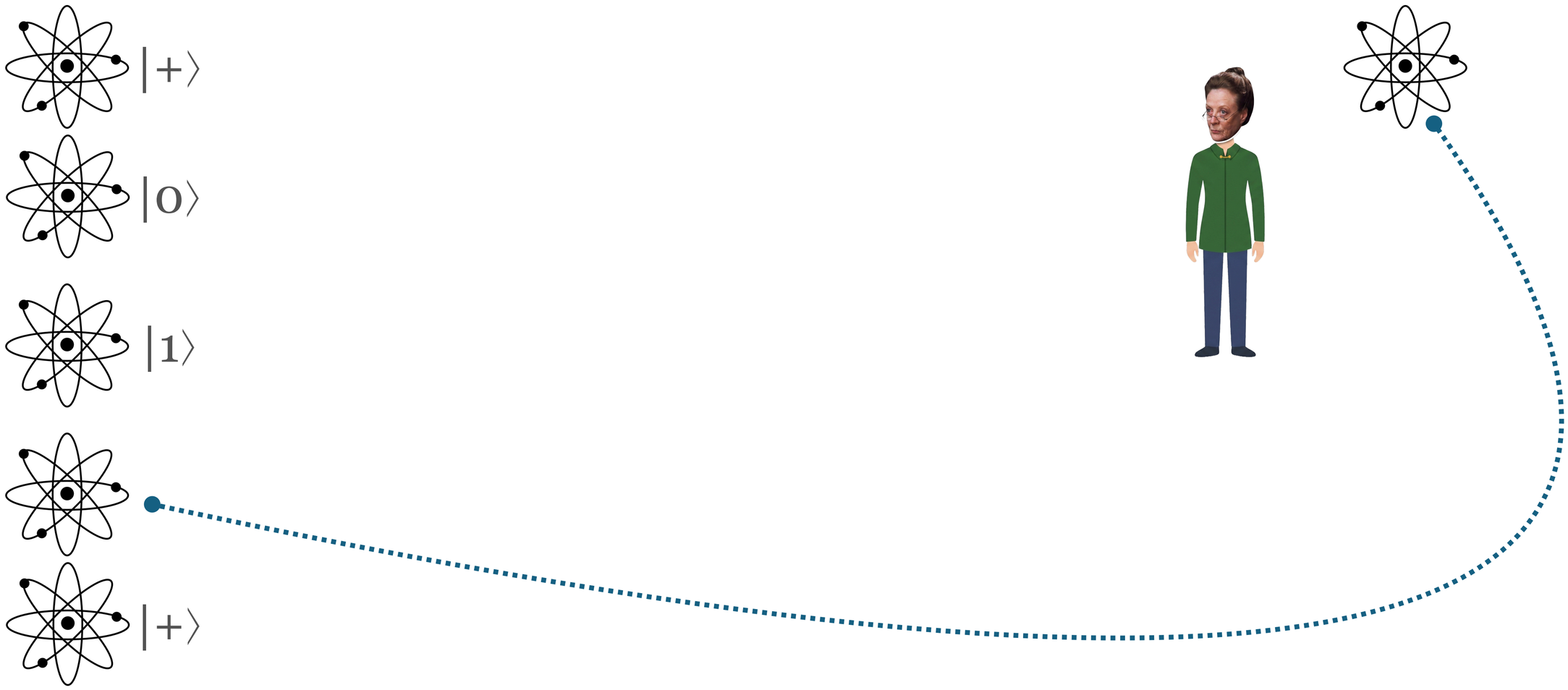


Destructive!

Moral: Real experiment performs *non-destructive* test, then analysis argues that it *could* have been the destructive test, thus certifying entanglement

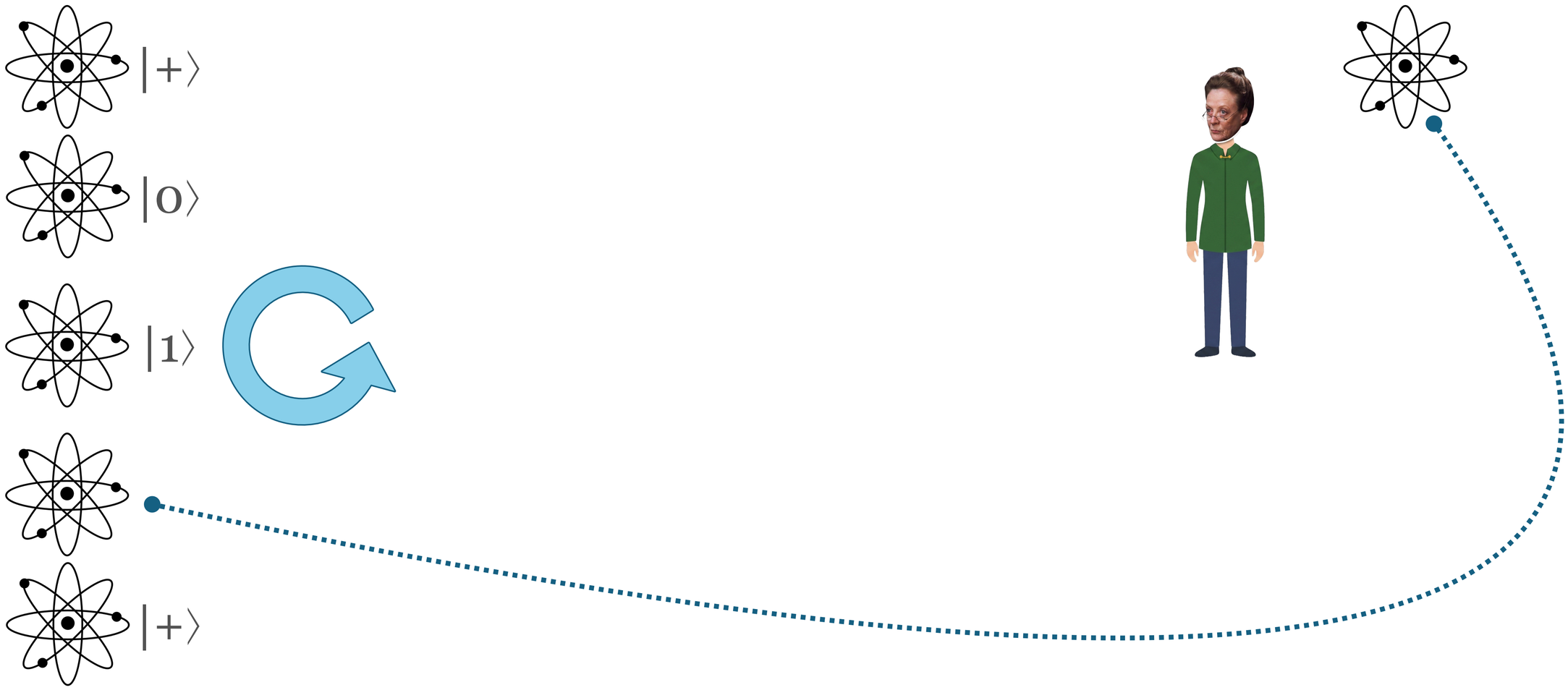
Non-Destructive Entanglement Testing

Hiding Entanglement



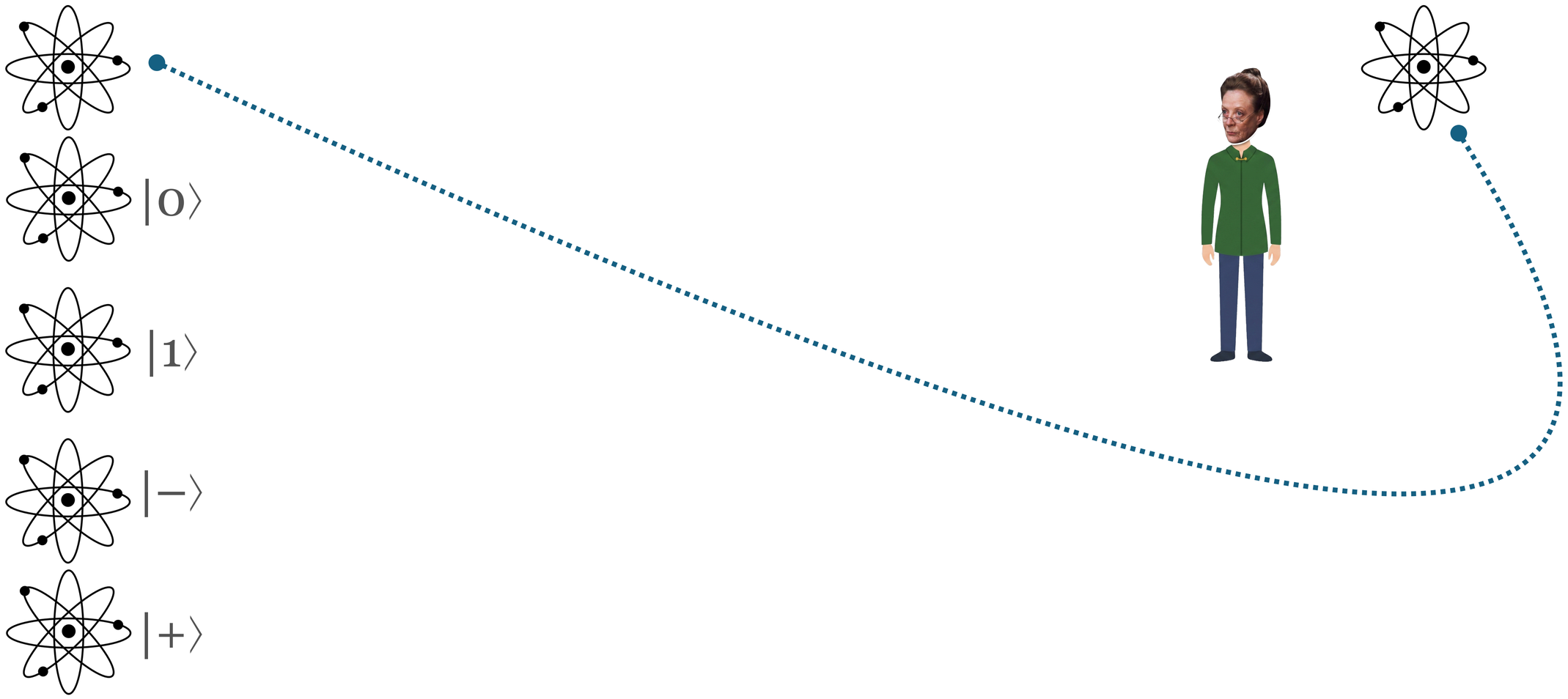
Non-Destructive Entanglement Testing

Hiding Entanglement

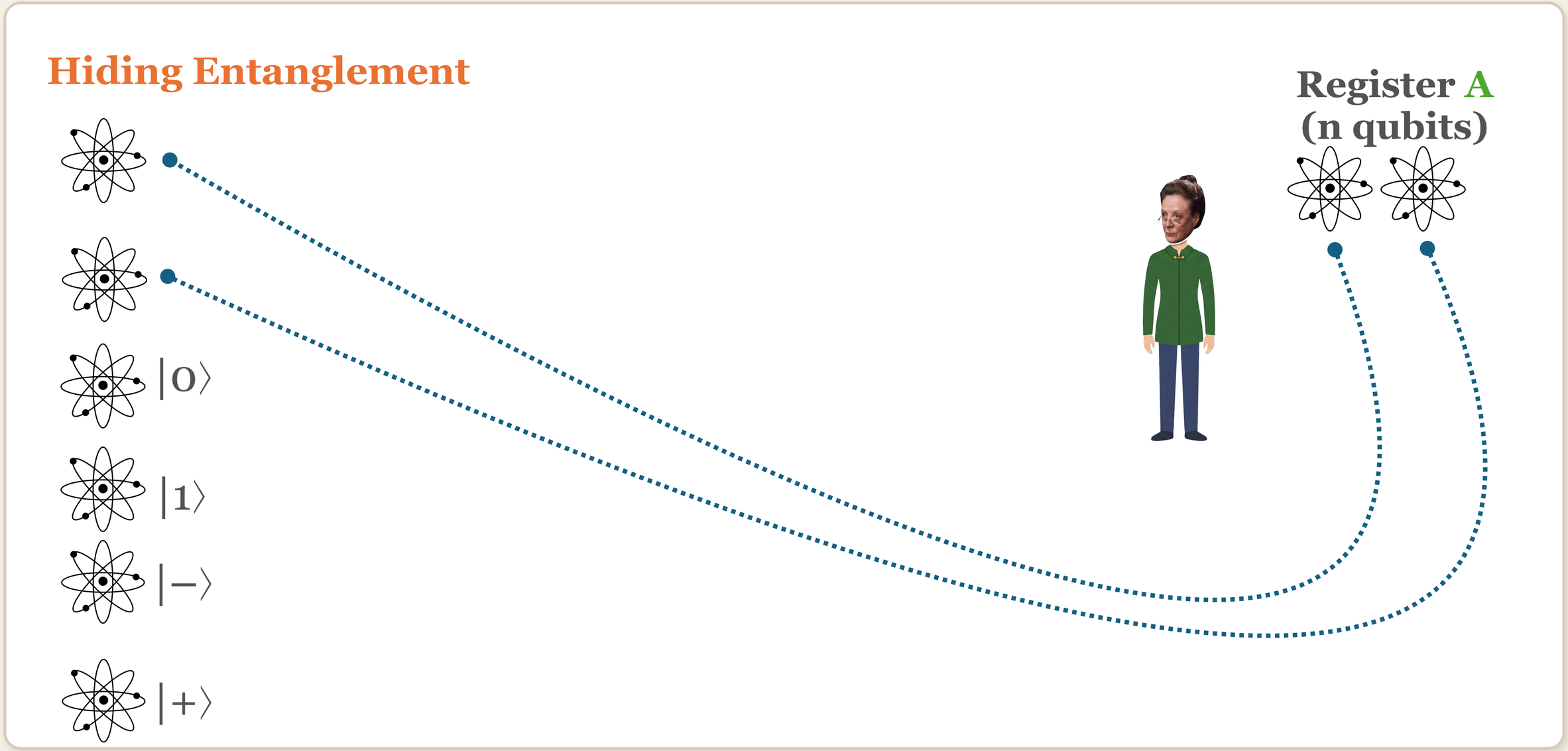


Non-Destructive Entanglement Testing

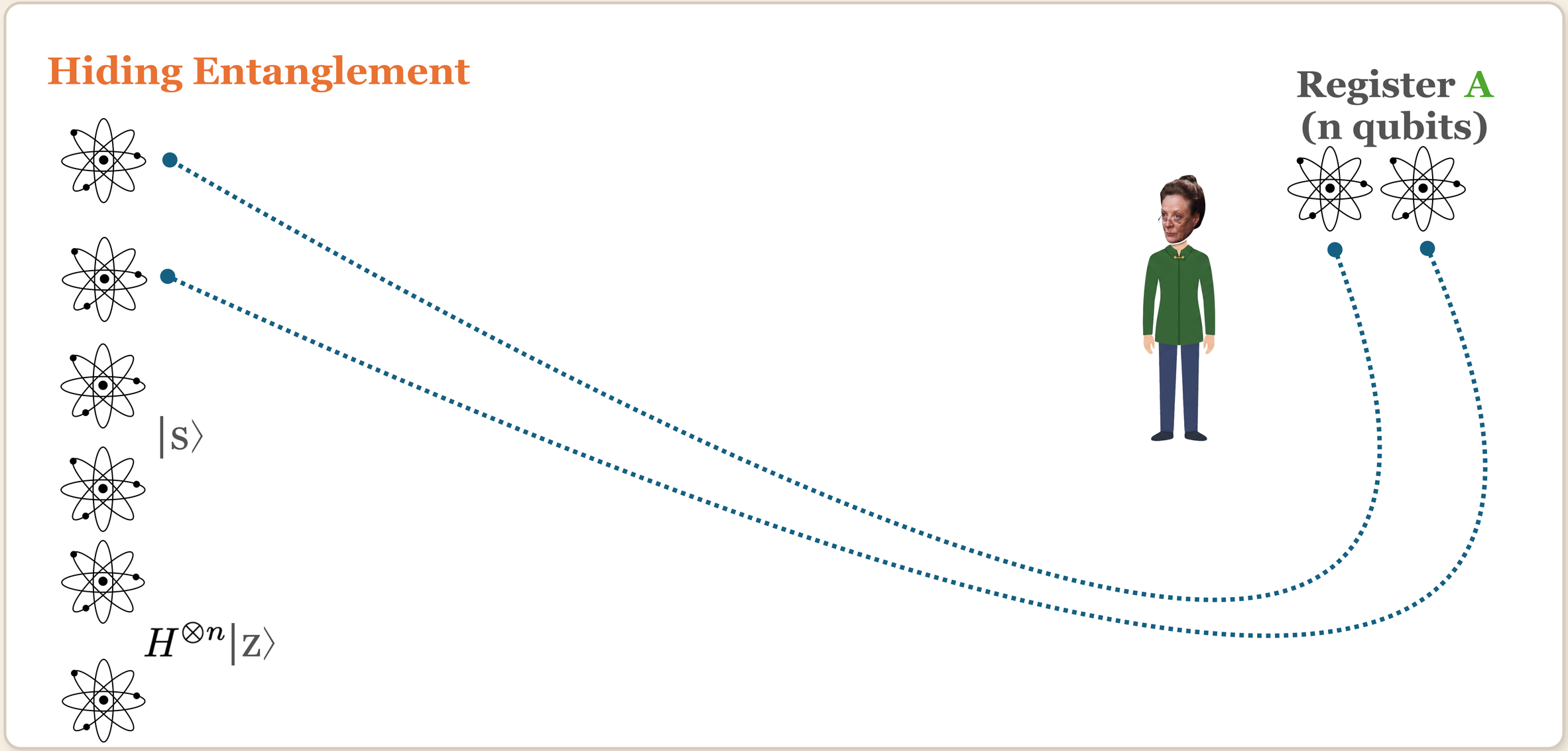
Hiding Entanglement



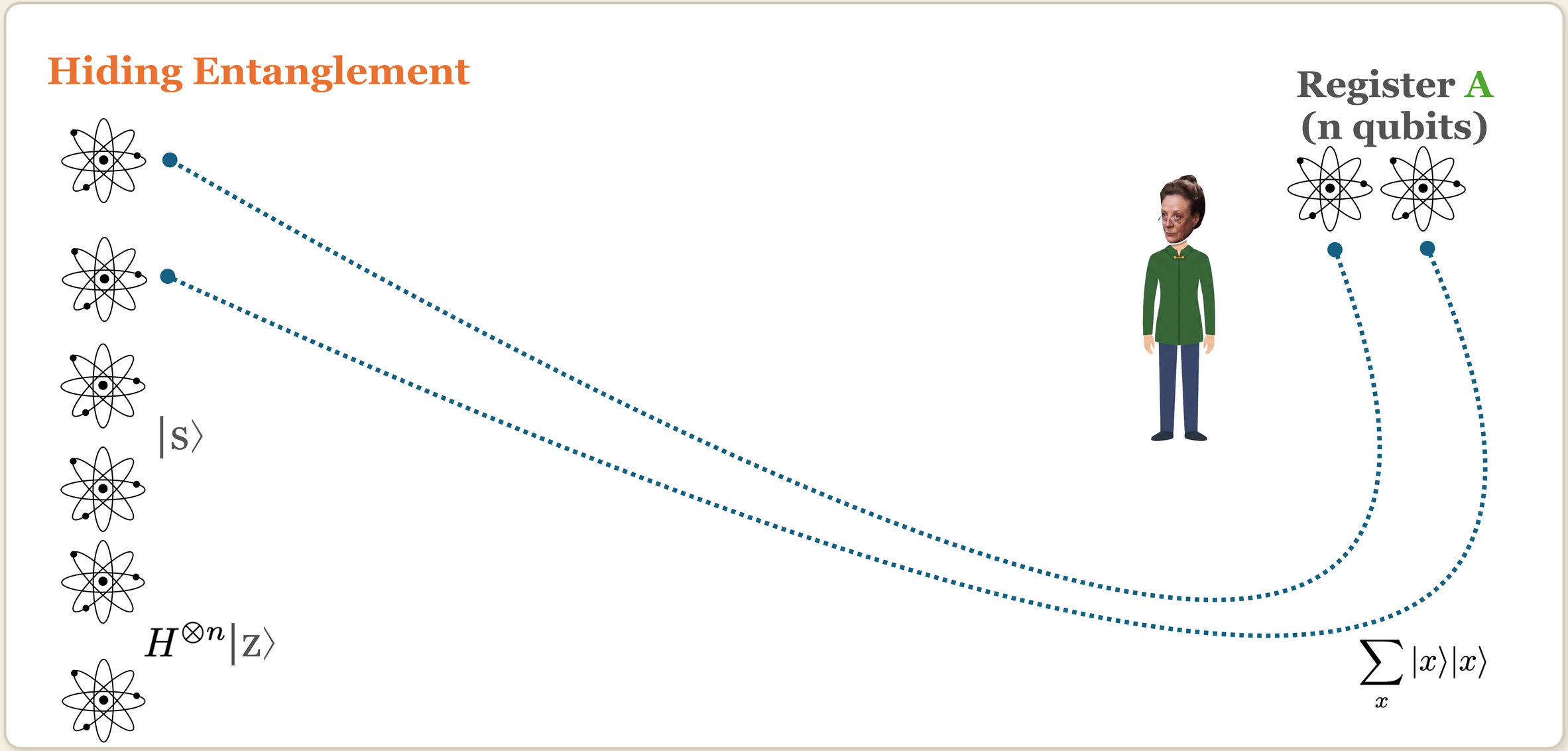
Non-Destructive Entanglement Testing



Non-Destructive Entanglement Testing

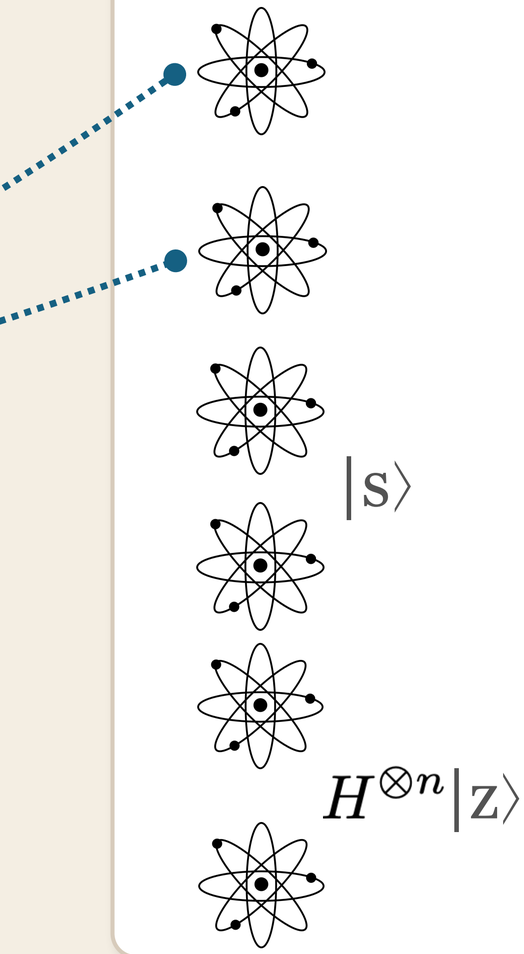


Non-Destructive Entanglement Testing

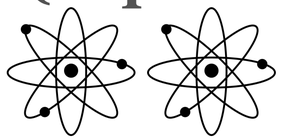


Non-Destructive Entanglement Testing

Hiding Entanglement



Register **A** (n qubits)

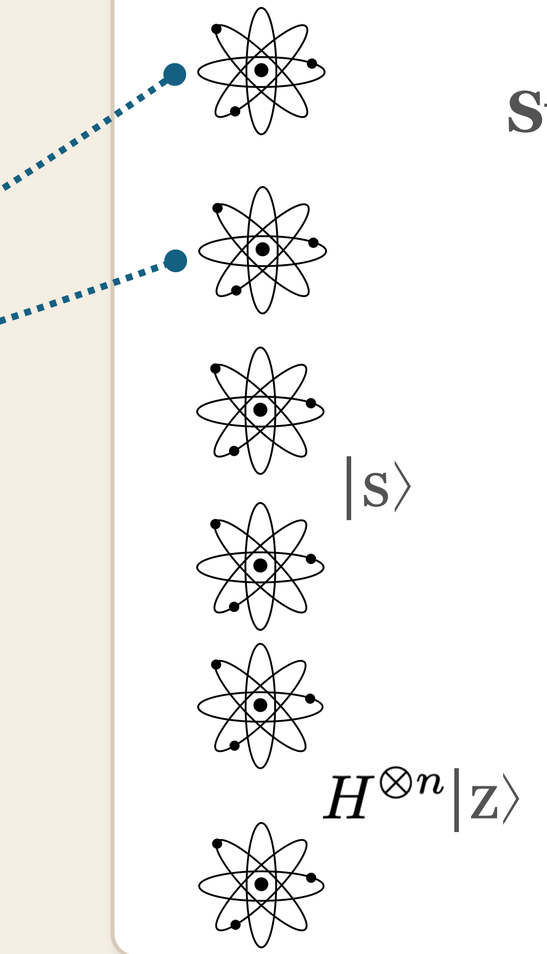


$$\sum_x |x\rangle|x\rangle$$

Register **B** (3n qubits)

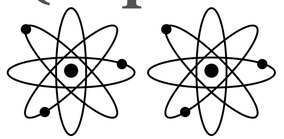
Non-Destructive Entanglement Testing

Hiding Entanglement



State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z\rangle}|a\rangle\right) \otimes |s\rangle$

Register A (n qubits)



$\sum_x |x\rangle|x\rangle$

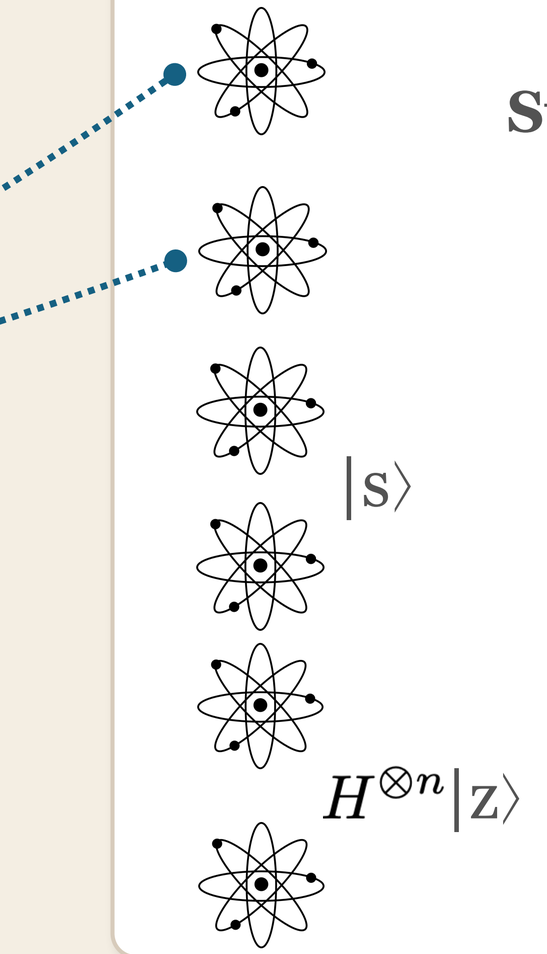
Register B (3n qubits)

Non-Destructive Entanglement Testing

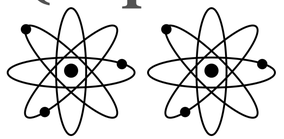
Hiding Entanglement

Randomly sample a change-of-basis matrix $U_{\text{shift}} \in \mathbb{F}_2^{3n \times 3n}$

State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z\rangle} |a\rangle\right) \otimes |s\rangle$



Register **A** (n qubits)



$\sum_x |x\rangle|x\rangle$

Register **B** (3n qubits)

Non-Destructive Entanglement Testing

Hiding Entanglement

Randomly sample a change-of-basis matrix

$$U_{\text{shift}} \in \mathbb{F}_2^{3n \times 3n}$$

State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z\rangle} |a\rangle\right) \otimes |s\rangle$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Register A (n qubits)

$$\sum_x |x\rangle|x\rangle$$

$|s\rangle$

$H^{\otimes n}|z\rangle$

Register B (3n qubits)

Non-Destructive Entanglement Testing

Hiding Entanglement

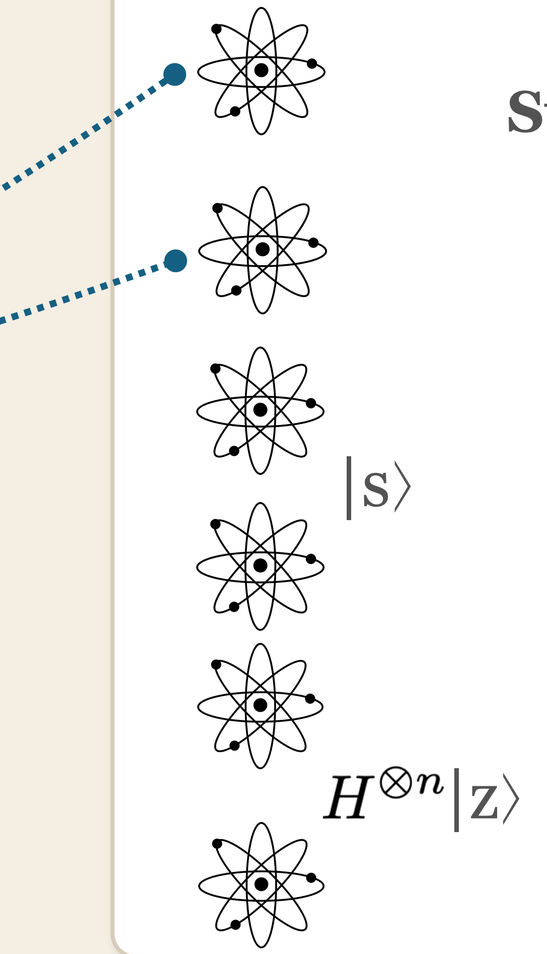
Randomly sample a change-of-basis matrix

$$U_{\text{shift}} \in \mathbb{F}_2^{3n \times 3n}$$

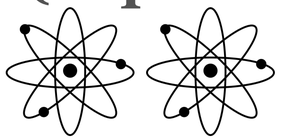
$$T = \overbrace{\underbrace{S}_{}\quad \underbrace{S^{\perp} \cap T}_{}}^{T} \quad \underbrace{T^{\perp}}_{}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z \rangle} |a\rangle\right) \otimes |s\rangle$



Register A (n qubits)

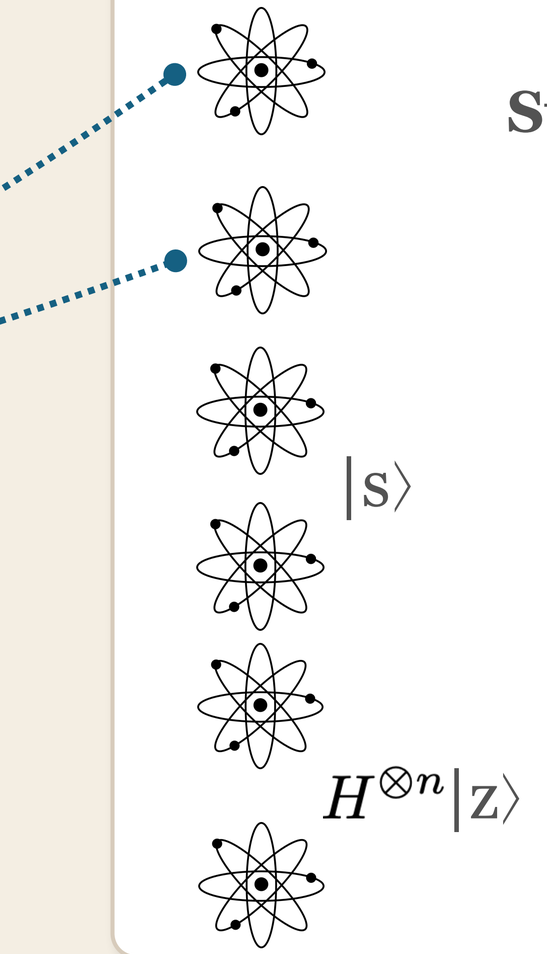


$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

Non-Destructive Entanglement Testing

Hiding Entanglement



Randomly sample a change-of-basis matrix

$$U_{\text{shift}} \in \mathbb{F}_2^{3n \times 3n}$$

$$\overbrace{\begin{matrix} \underbrace{}_{S} & \underbrace{}_{S^\perp \cap T} & \underbrace{}_{T^\perp} \end{matrix}}^{T}$$

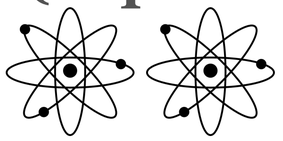
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z\rangle} |a\rangle\right) \otimes |s\rangle$

$$\mathcal{U}_{\text{shift}}|x\rangle := |U_{\text{shift}} x\rangle$$

$$\sum_{x \in \mathbb{F}_2^n} |x\rangle_{\mathbf{A}} \sum_{a \in S} (-1)^{\langle a,u\rangle} |a + U_{\text{shift}}(0^n \times x \times 0^n) + v\rangle_{\mathbf{B}}$$

Register A (n qubits)



$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

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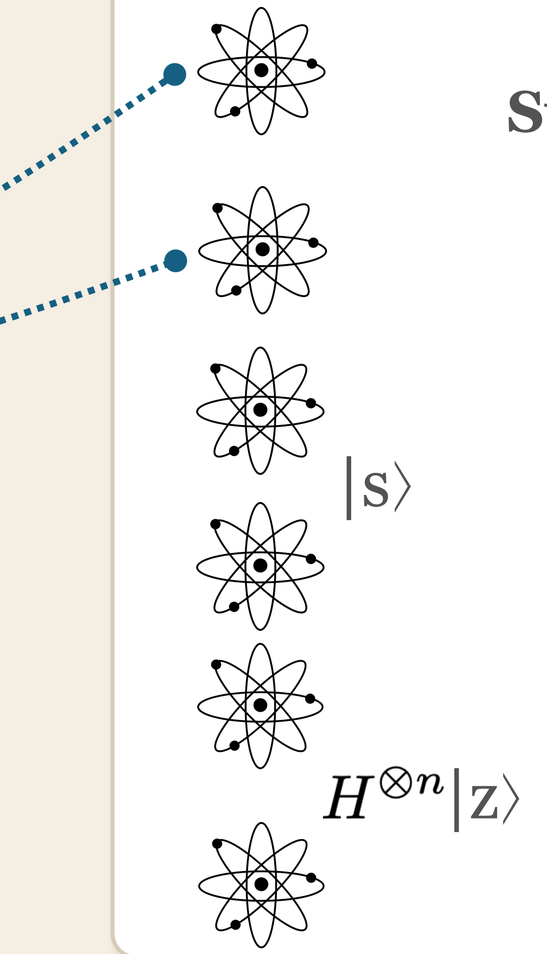

$|s\rangle$

$H^{\otimes n}|z\rangle$

Randomly sample a change-of-basis matrix

$$U_{\text{shift}} \in \mathbb{F}_2^{3n\times 3n}$$

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$$\overbrace{\phantom{\begin{pmatrix} 1 & 0 \end{pmatrix}}}^{S}\ \overbrace{\phantom{\begin{pmatrix} 0 & 0 \end{pmatrix}}}^{S^\perp\cap T}\ \overbrace{\phantom{\begin{pmatrix} 0 & 0 \end{pmatrix}}}^{T^\perp}$$

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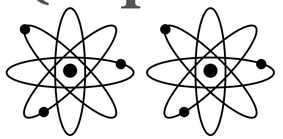
S=span(first n columns of U_{shift})

Register A (n qubits)



$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

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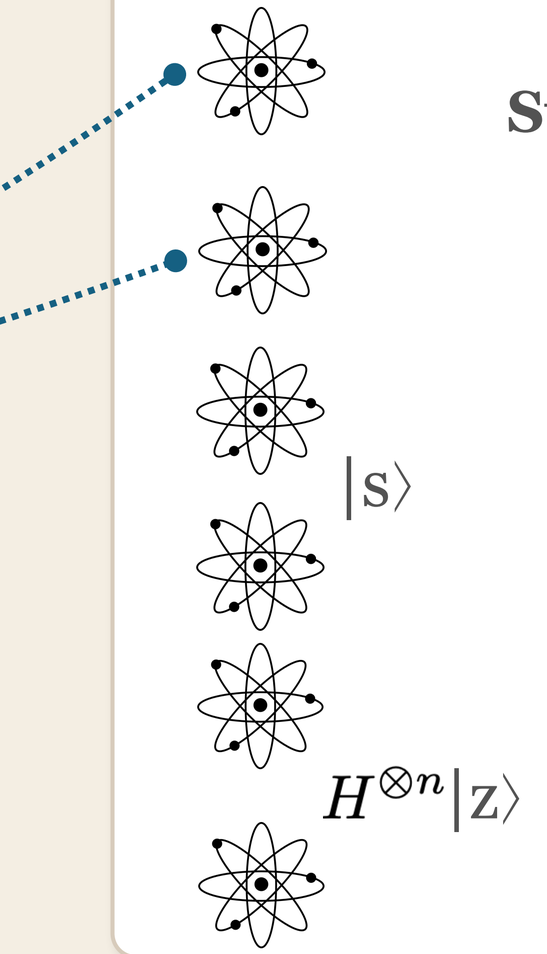

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Randomly sample a change-of-basis matrix

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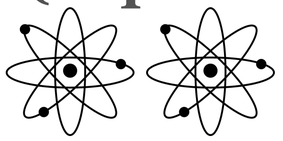

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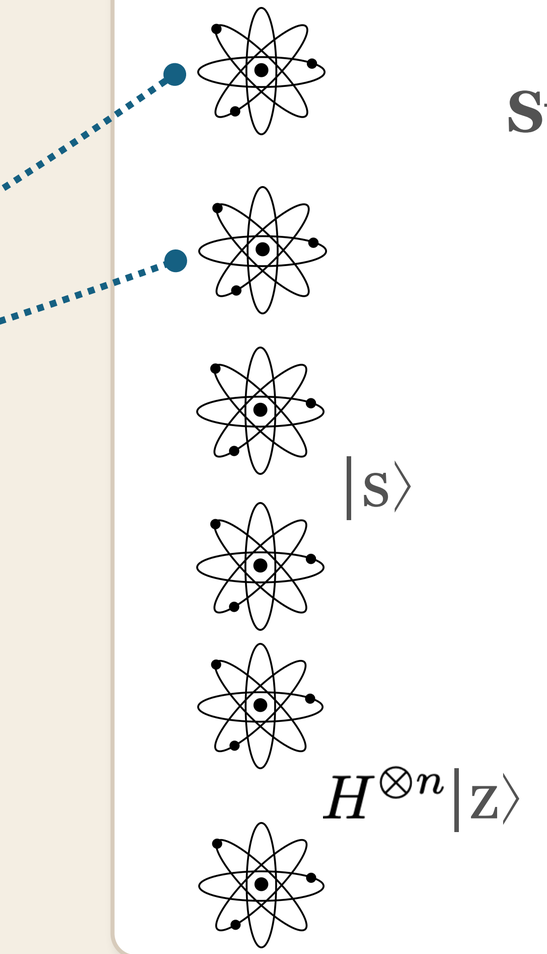

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$$T = \overbrace{S}^{S} \quad \overbrace{S^{\perp} \cap T}^{S^{\perp} \cap T} \quad \overbrace{T^{\perp}}^{T^{\perp}}$$

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Without knowing U_{shift}, u, v, entanglement is "hidden"

S=span(first n columns of U_{shift})

$$v = U_{\text{shift}}(s \times 0^{2n})$$

$$u = U_{\text{shift}}^{-T}(0^{2n} \times z)$$

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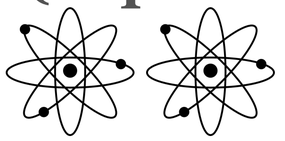

$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

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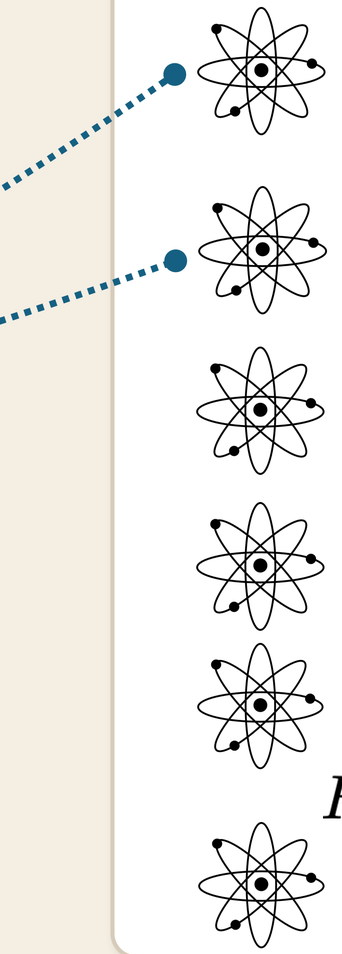

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New test: "is register B in the coset T+v" Non-destructive!

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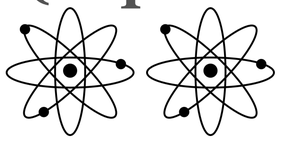
Register A (n qubits)

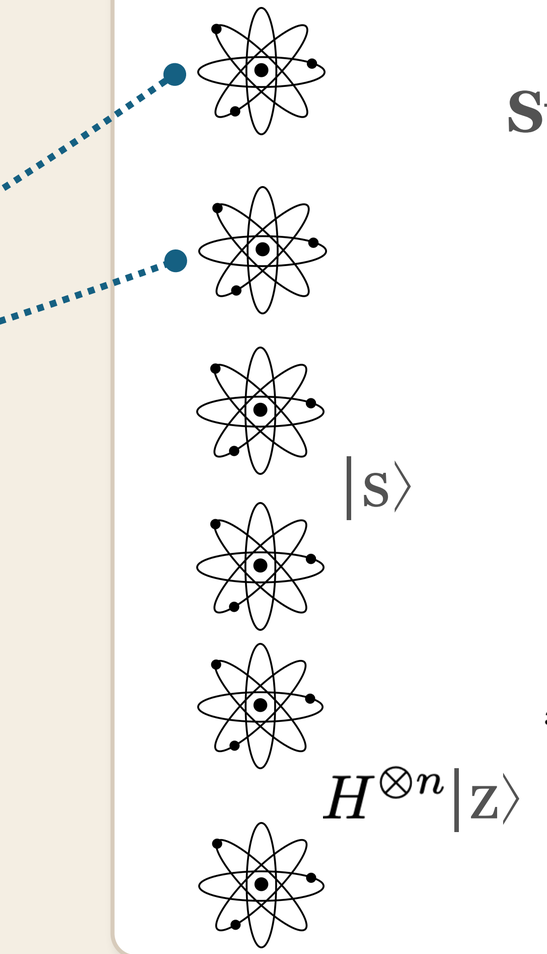


$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

Non-Destructive Entanglement Testing

Hiding Entanglement



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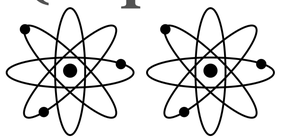

$$\sum_x |x\rangle|x\rangle$$

Register B (3n qubits)

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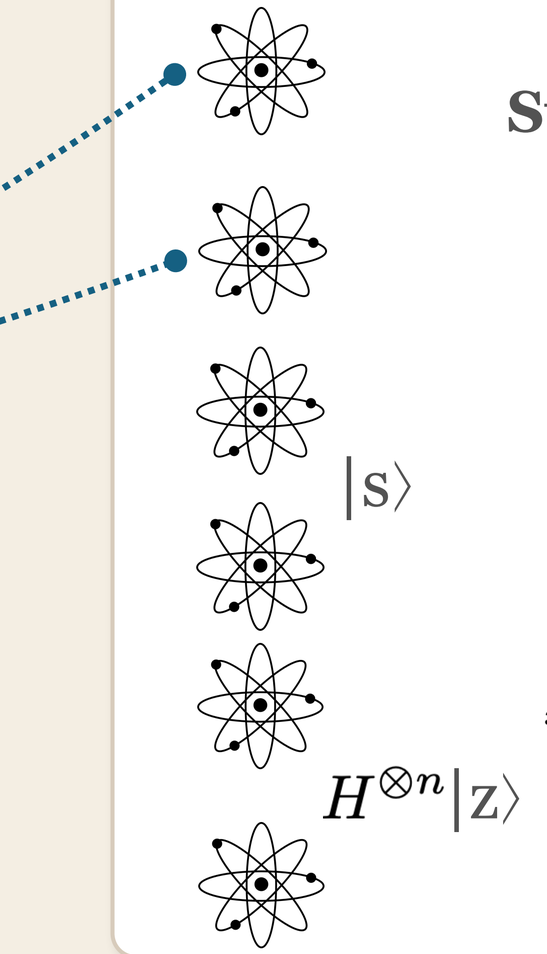

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Now: "is register B in the coset T^⊥ +u"

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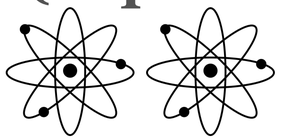
Register A (n qubits)



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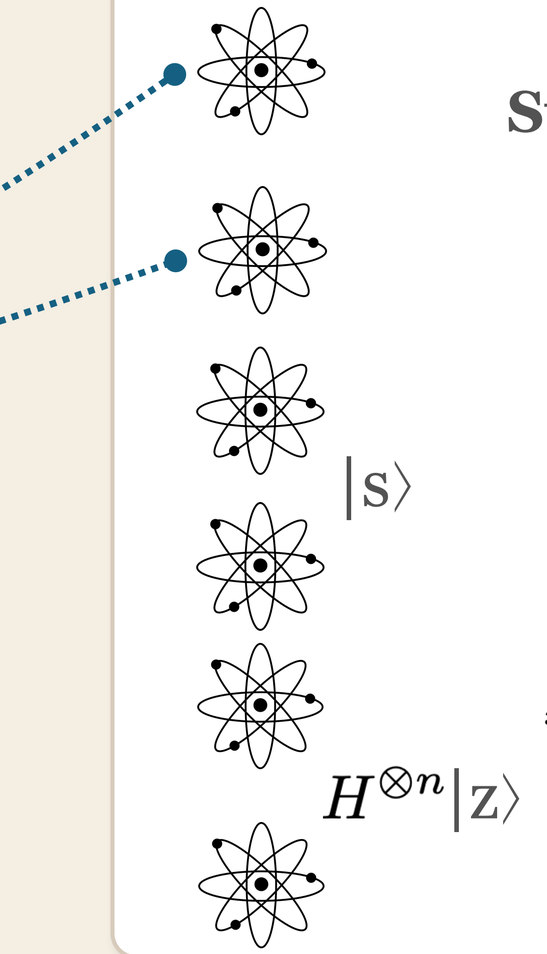

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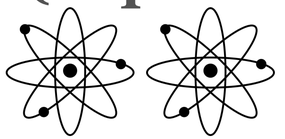

$$\sum_x |x\rangle|x\rangle$$

(Cf.) Coset state:

$$\sum_{a\in A} (-1)^{\langle a,z\rangle} |a + s\rangle$$

Register B (3n qubits)

Non-Destructive Entanglement Testing

Crash Course: Coset States

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Starting point: BB84 states

$$BB84^{\otimes n} := H^{\theta}|w\rangle$$

$$w|_{\theta_i=1} := z$$

$$w|_{\theta_i=0} := s$$

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$\mathcal{O}_{s,z,\theta} := H^\theta |w\rangle\langle w| H^\theta$

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Susceptible to qubit-by-qubit attack!

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$$|\psi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$$

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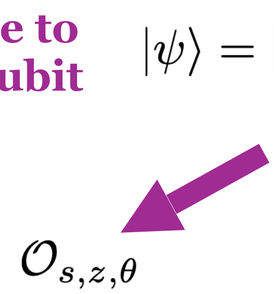

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Randomly sample a change-of-basis matrix

$$U_{\text{shift}} \in \mathbb{F}_2^{2n \times 2n}$$

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Non-Destructive Entanglement Testing

Crash Course: Coset States

Starting point: BB84 states

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$$w|_{\theta_i=1} := z$$
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Want "Public verification oracle"

$$\mathcal{O}_{s,z,\theta} := H^\theta |w\rangle\langle w| H^\theta$$

Susceptible to qubit-by-qubit attack!

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$$I \otimes X \otimes \cdots \otimes I$$
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Coset MoE property [CV'22]:

$|A_{s,z}\rangle$

$\mathcal{O}_{s,z,A}$

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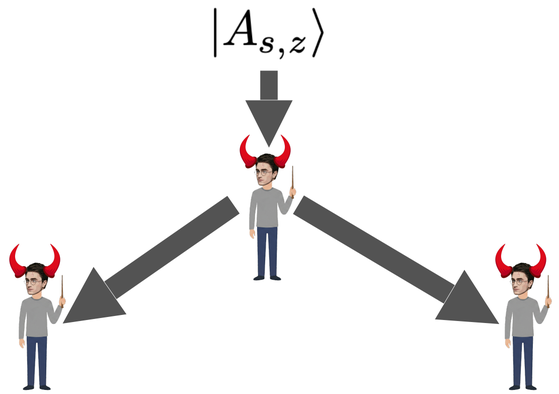

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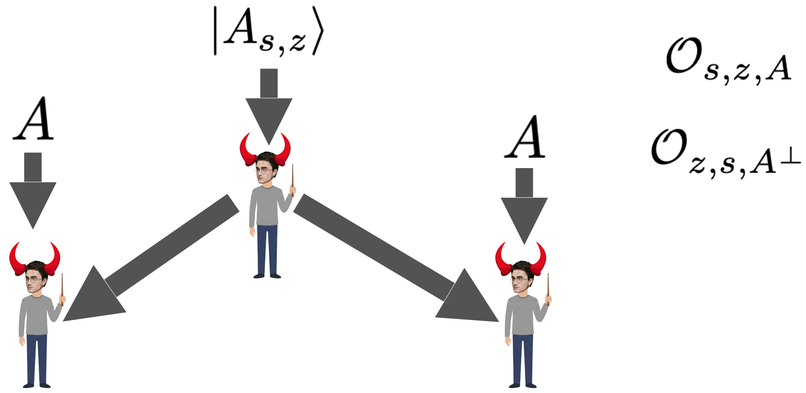

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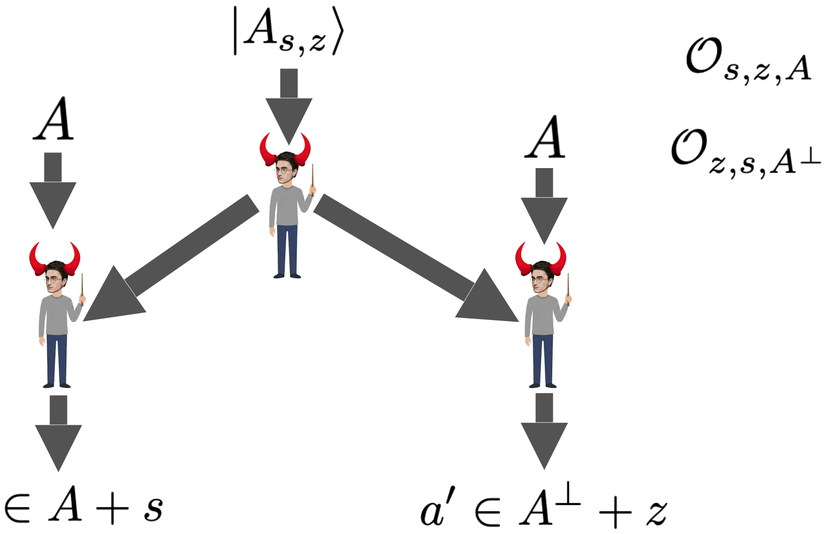

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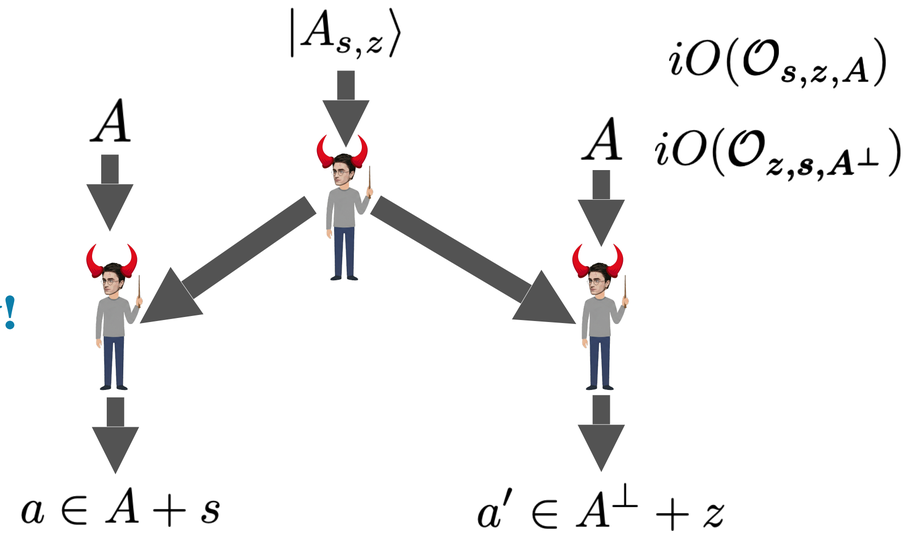
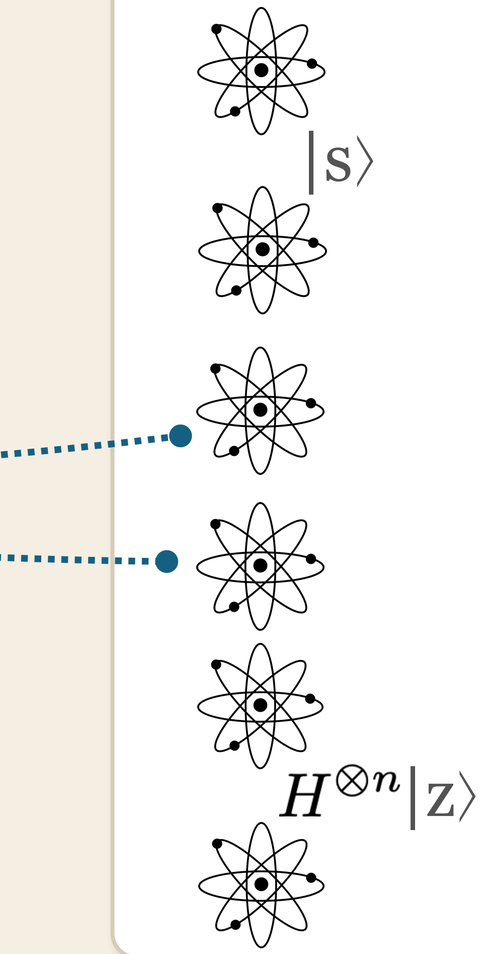


Non-Destructive Entanglement Testing

Hiding Entanglement

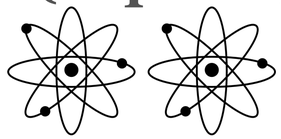


State: $\left(\sum_x |x\rangle\right) \otimes \left(\sum_a (-1)^{\langle a,z\rangle}|a\rangle\right) \otimes |s\rangle$

$\mathcal{U}_{\text{shift}}|x\rangle := |U_{\text{shift}}x\rangle$

$$\sum_{x\in\mathbb{F}_2^n} |x\rangle \text{⚓} \sum_{a\in S} (-1)^{\langle a,u\rangle} |a + U_{\text{shift}}(0^n \times x \times 0^n) + v\rangle \text{☠}$$

Register ⚓ (n qubits)



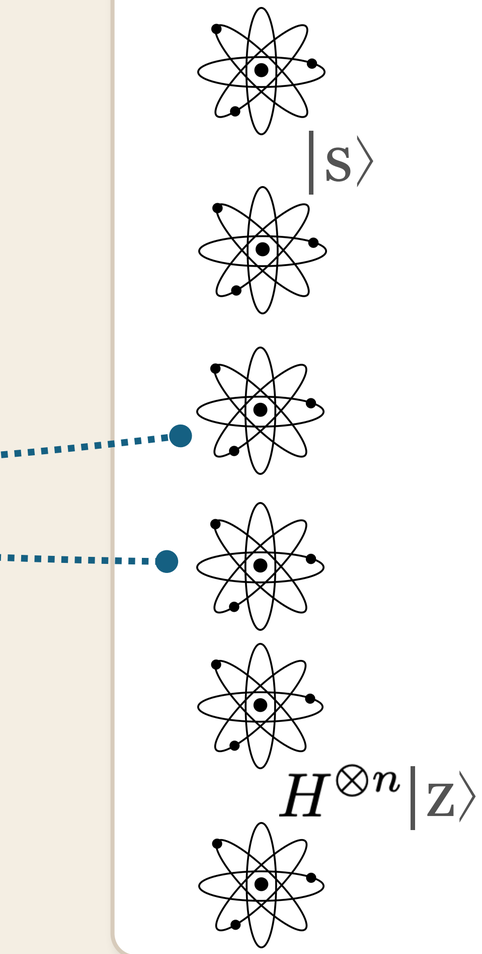
$\sum_x |x\rangle|x\rangle$

1. Verifier generates ⚓☠, sends ☠ to prover
2. Verifier gives out oracles $\mathcal{O}^{b_0}_{S,v}$ and $\mathcal{O}^{b_1}_{T^\perp,u}$, which hide secret random strings b0 and b1
3. Prover must find b0 and b1 by querying the oracles on the ☠ register
4. We've now non-destructively "taken standard and Hadamard measurements" of the entangled qubits

Register ☠ (3n qubits)

Non-Destructive Entanglement Testing

Hiding Entanglement



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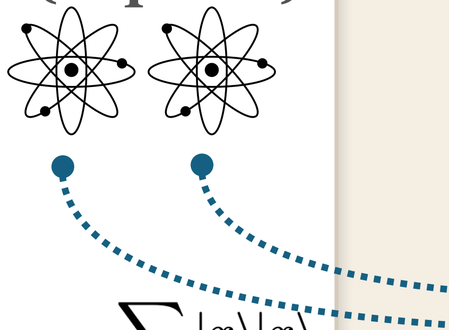

$$\sum_x |x\rangle|x\rangle$$

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$$\mathcal{O}^{b_0}_{S,v}(a) = \begin{cases} b_0, & \text{if } a \in S + v, \\ \perp, & \text{otherwise} \end{cases}$$

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Register 🏴‍☠️ (3n qubits)

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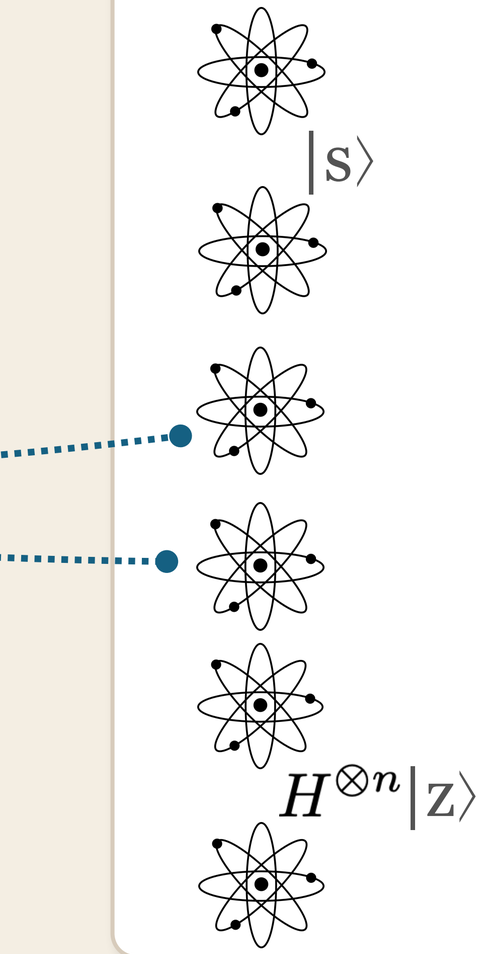

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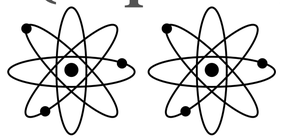
$H^{\otimes n}$

$$\sum_{x\in\mathbb{F}_2^n} |x\rangle_{\mathbf{A}} \sum_{a\in T^\perp} (-1)^{\langle a,v\rangle} |a + U_{\text{shift}}^{-T}(0^n \times x \times 0^n) + u\rangle_{\mathbf{B}}$$

T=span(last n columns of U_{shift})

Register ⚓ (n qubits)



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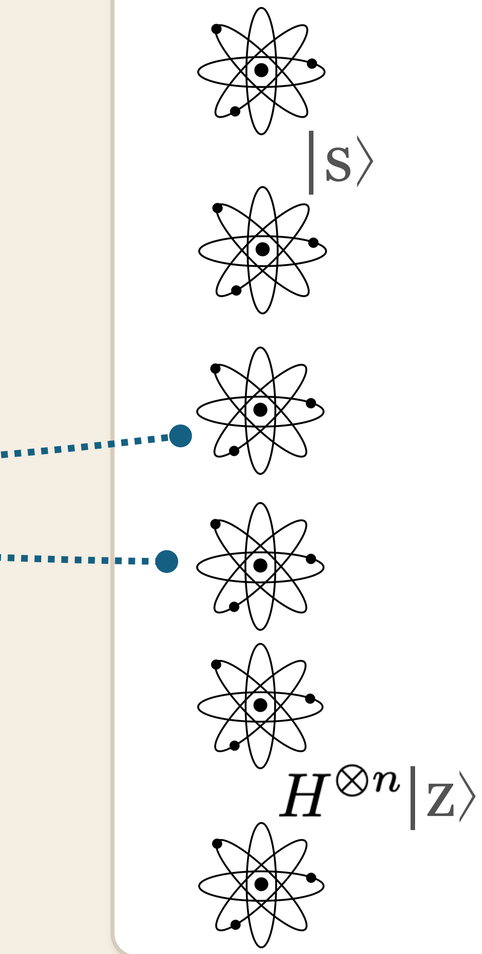

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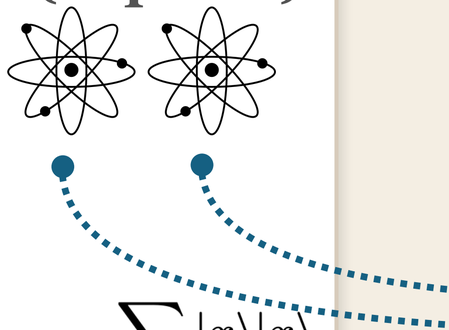

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Protocol: Trajectory Verification

Time

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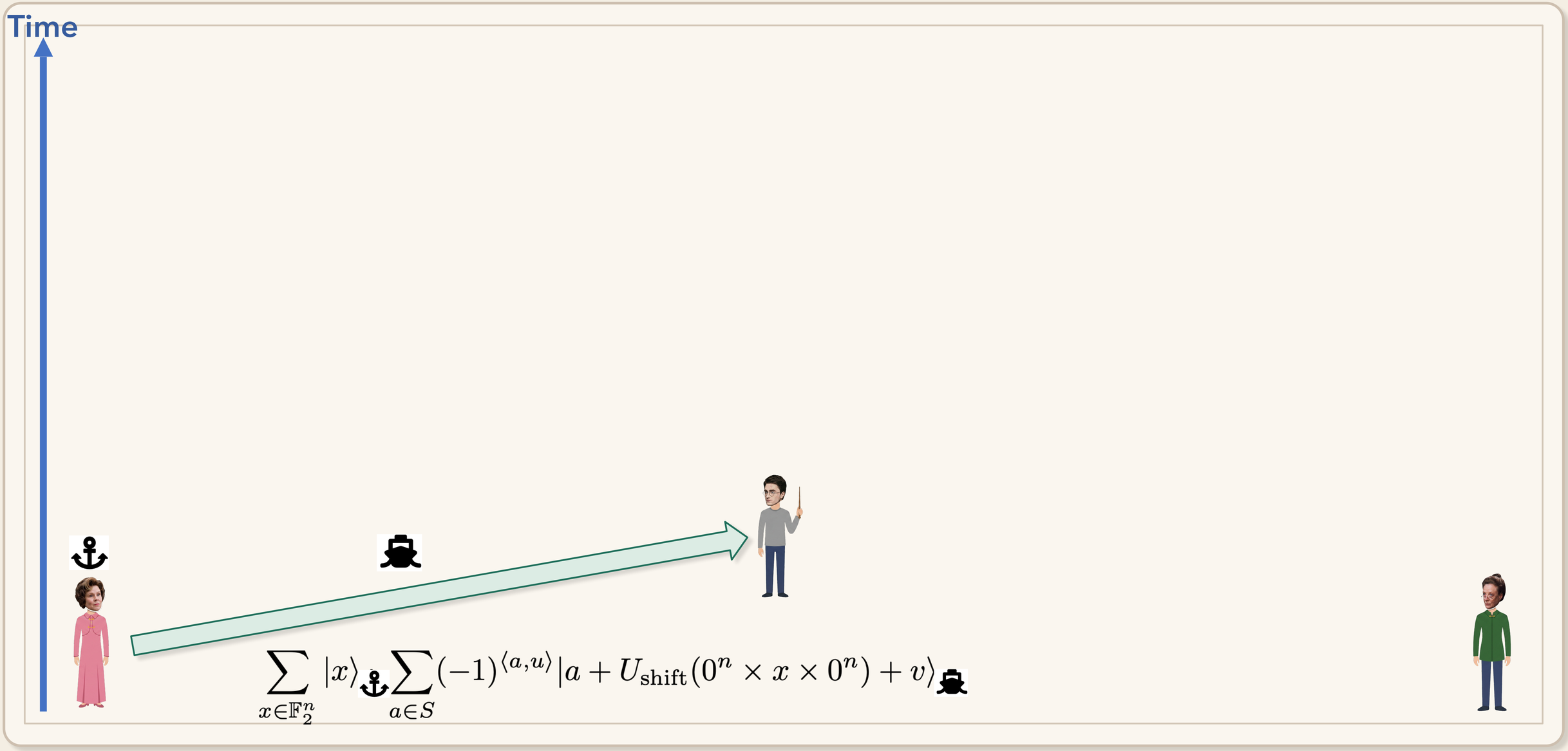

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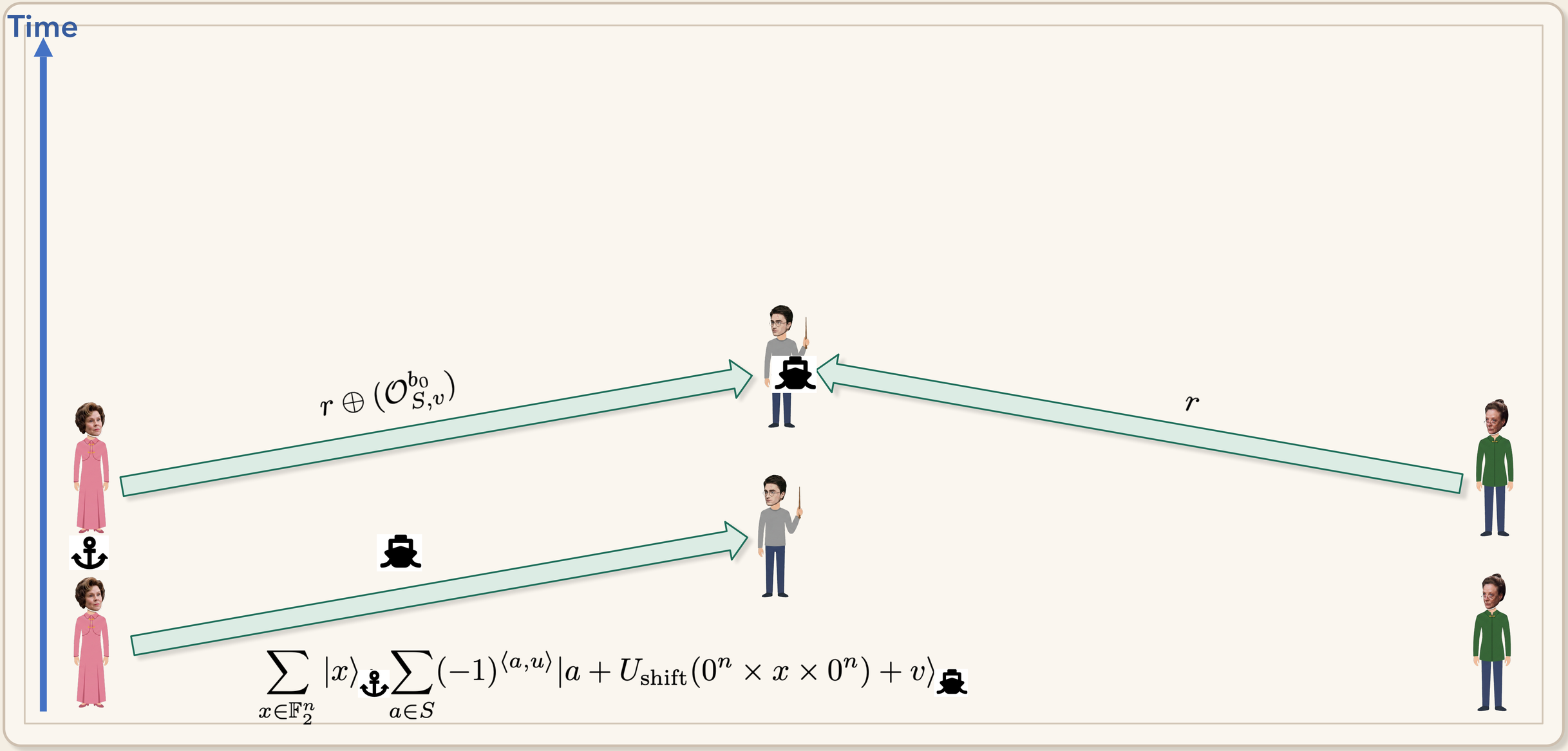

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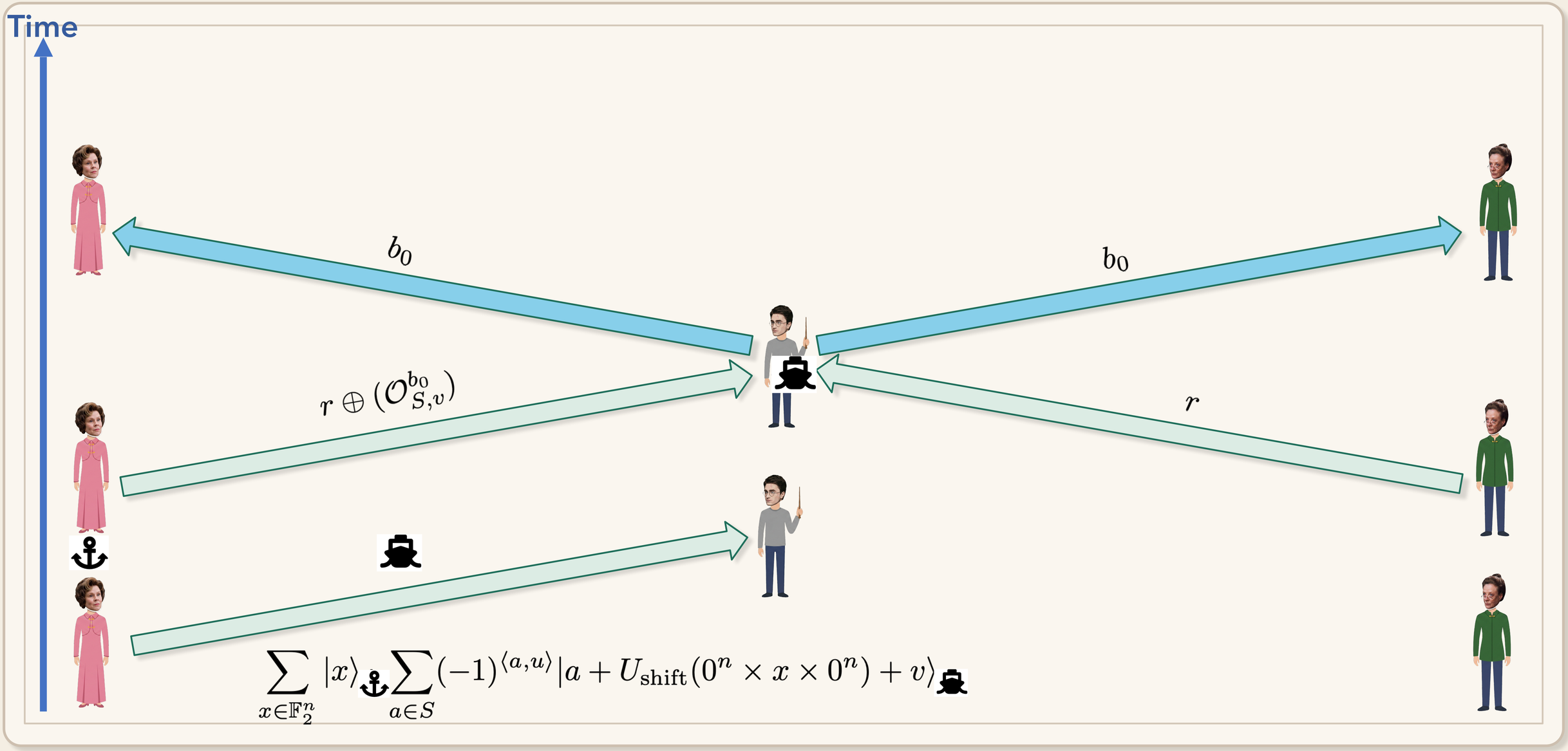

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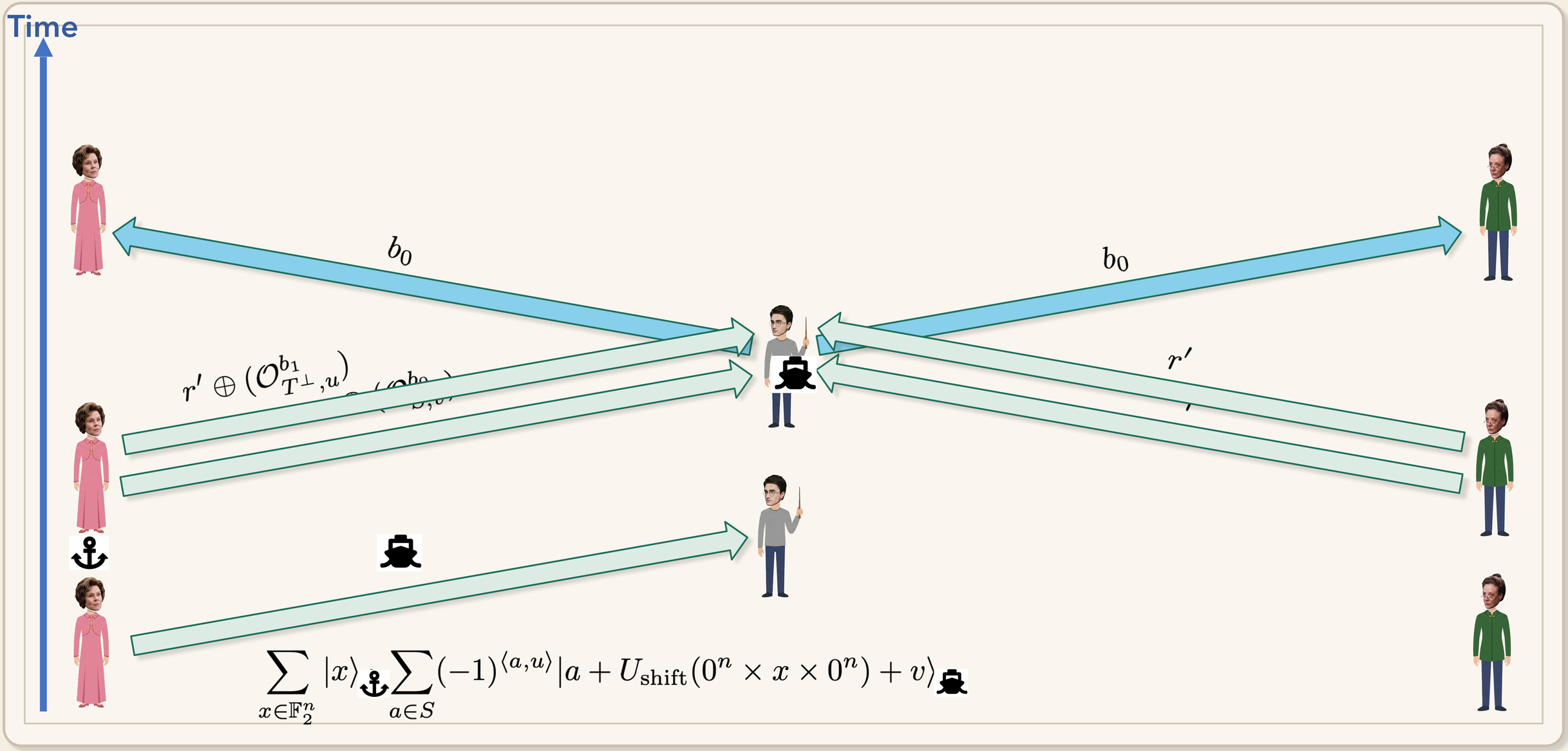

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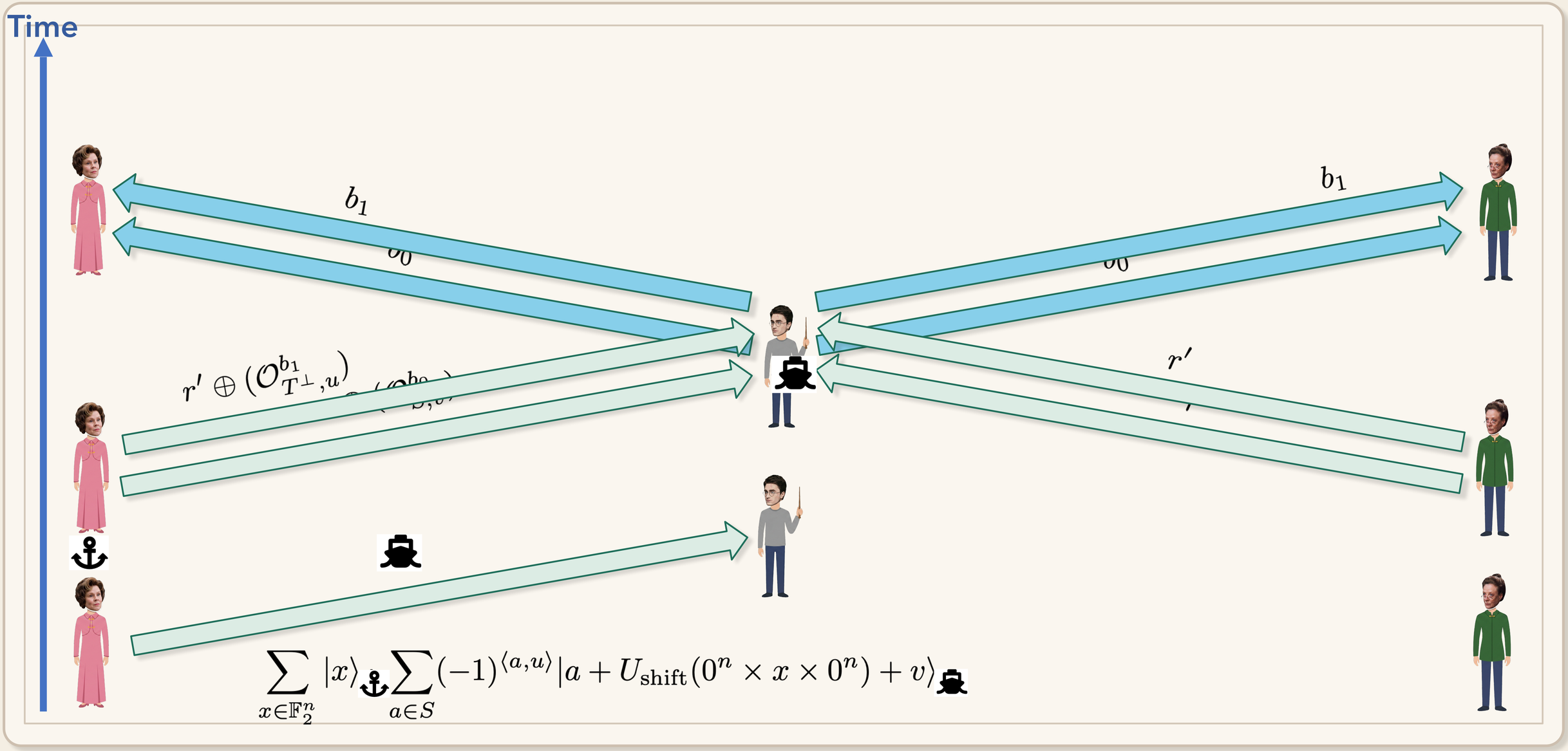

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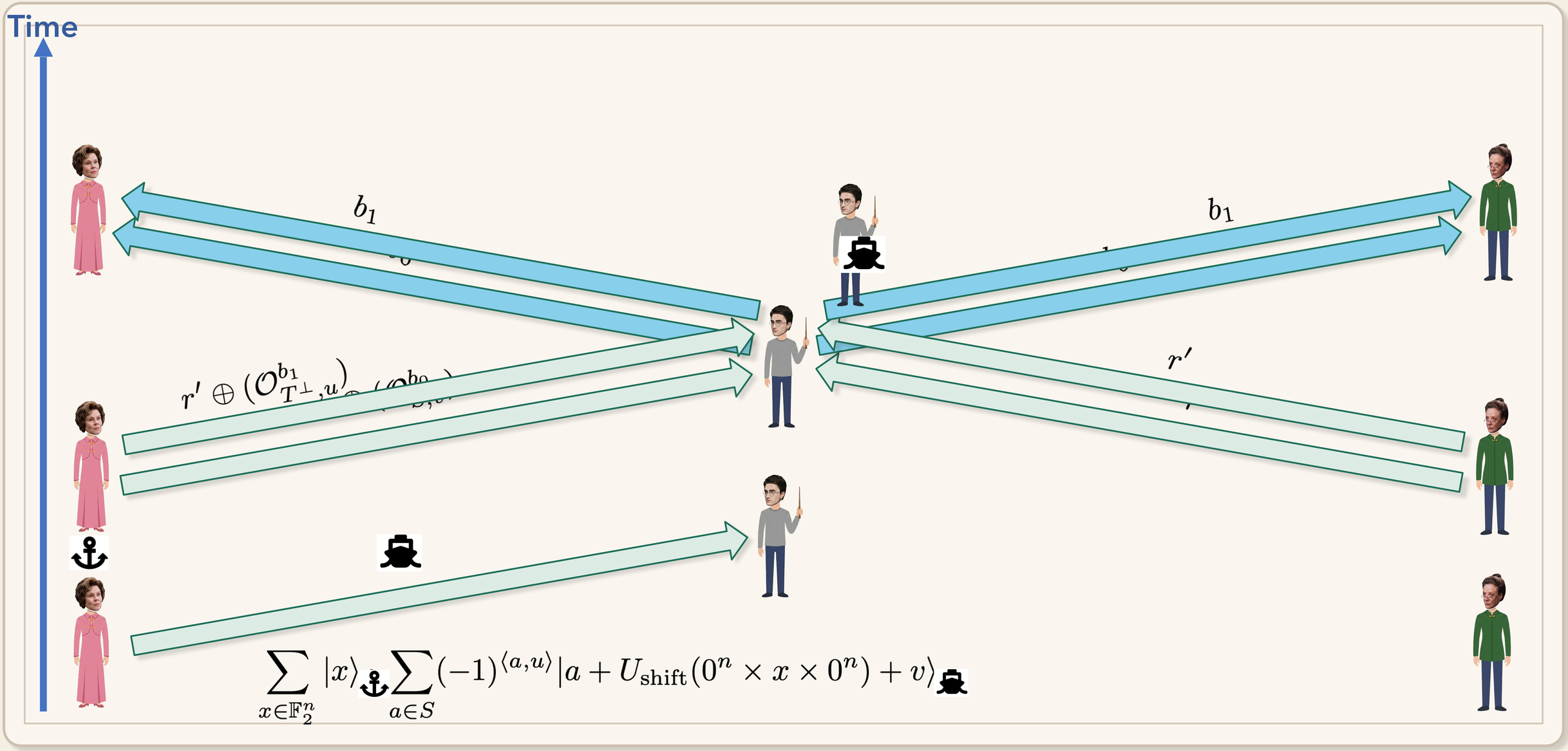

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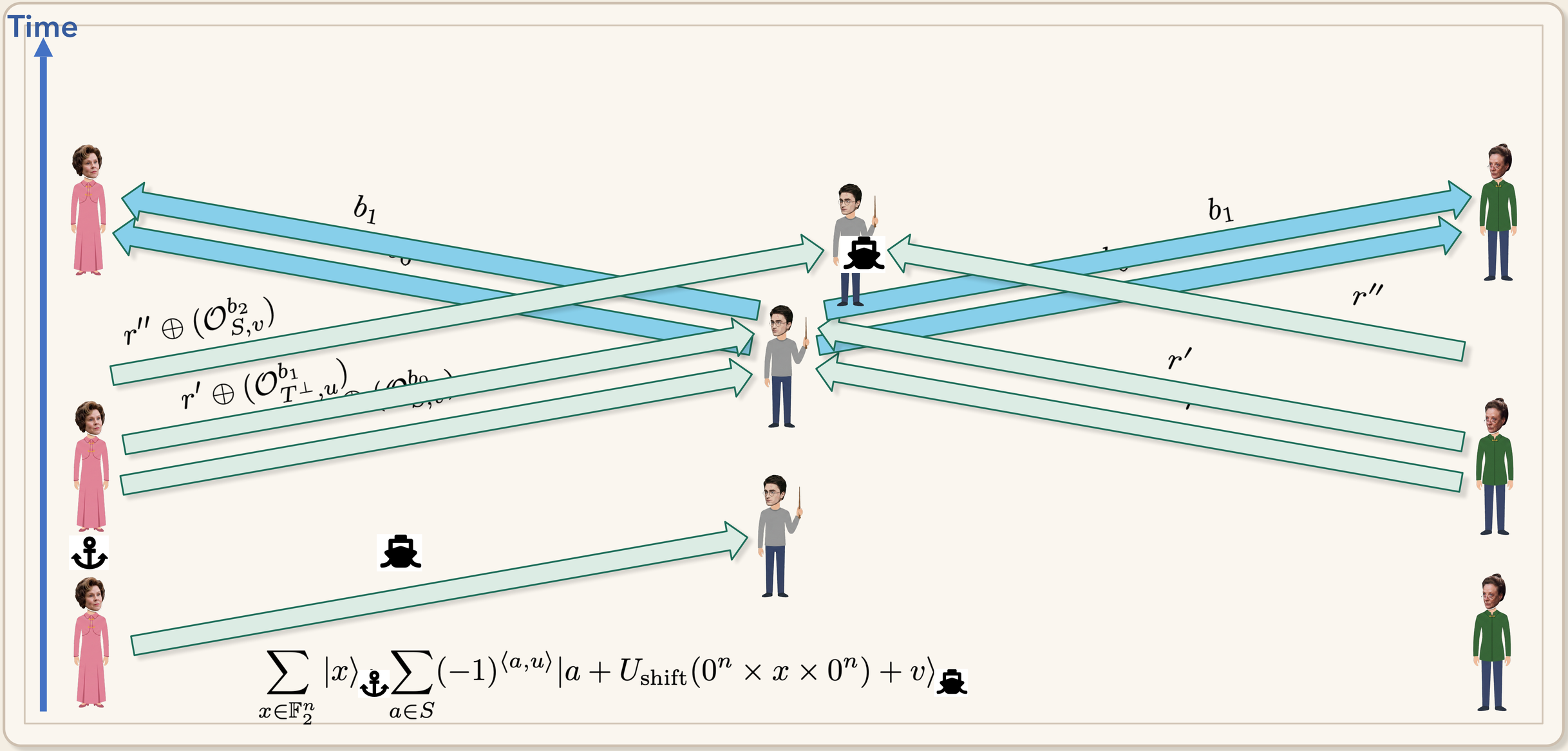

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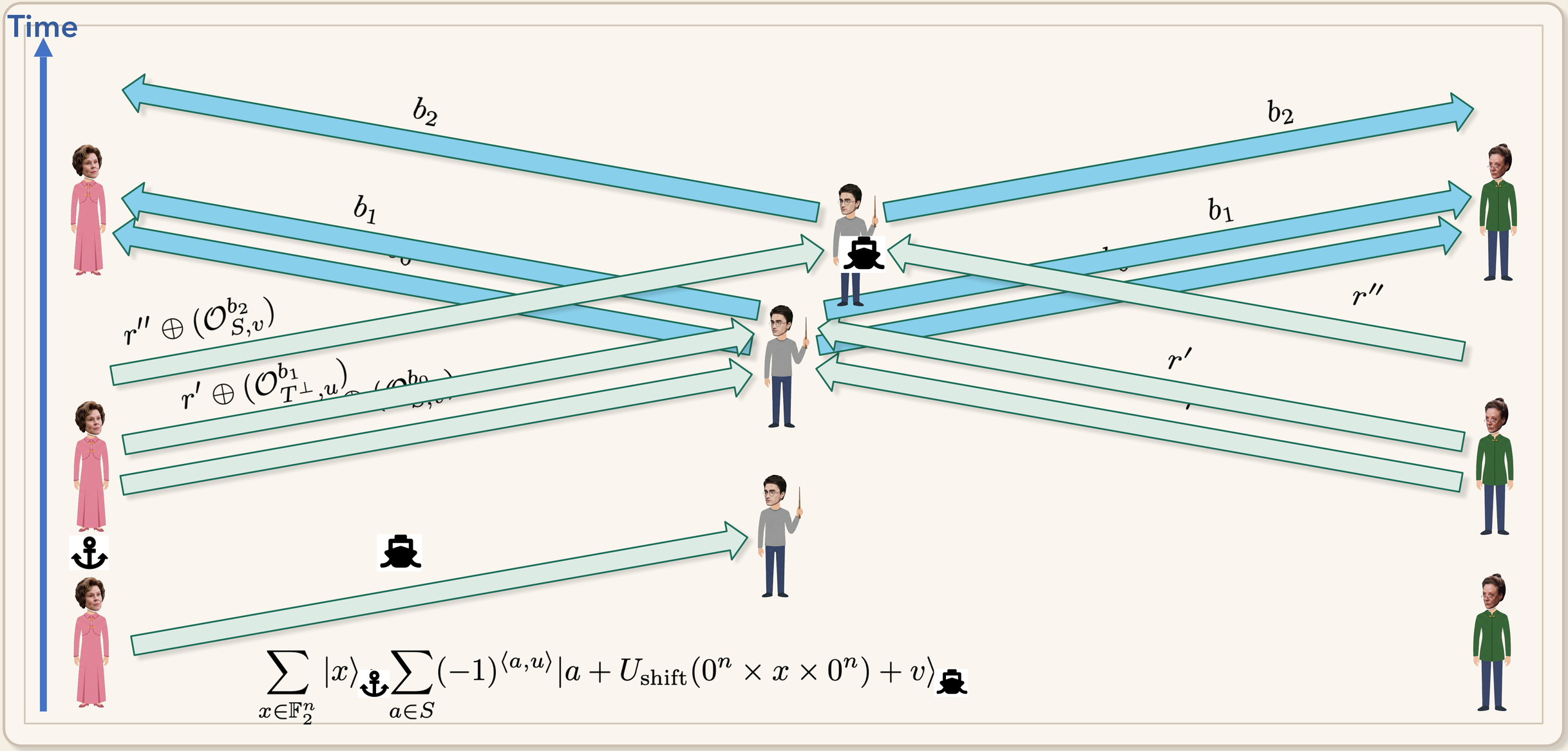

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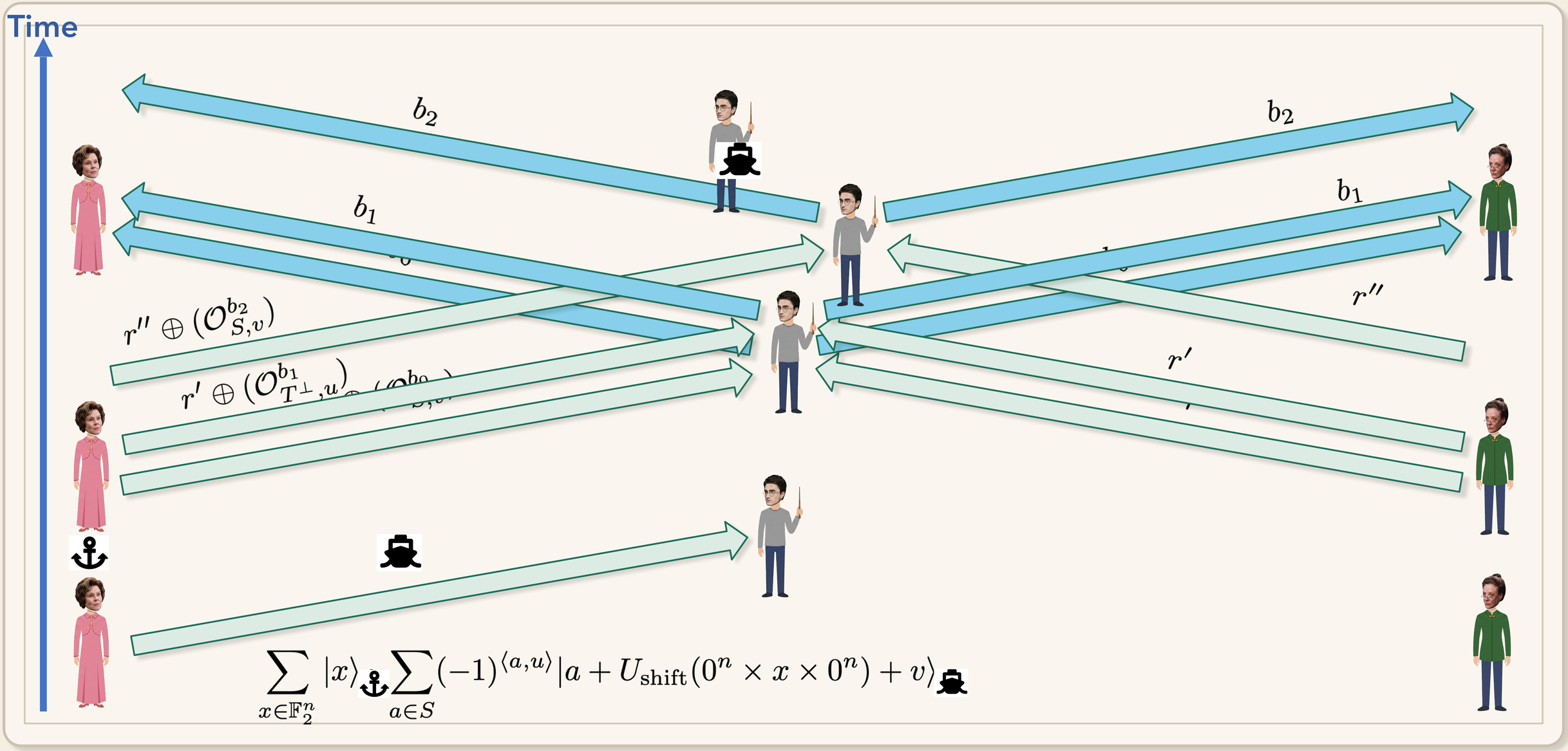

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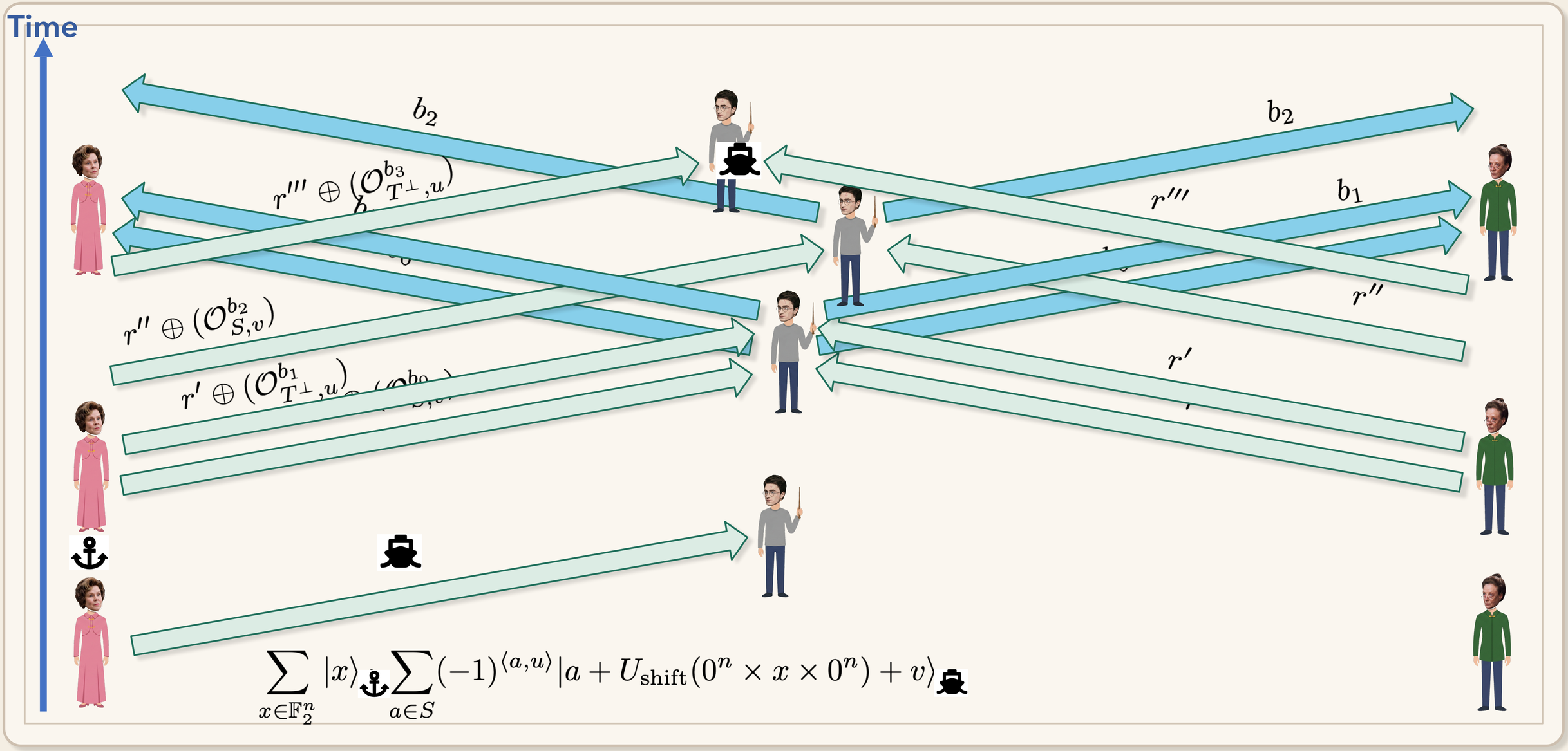

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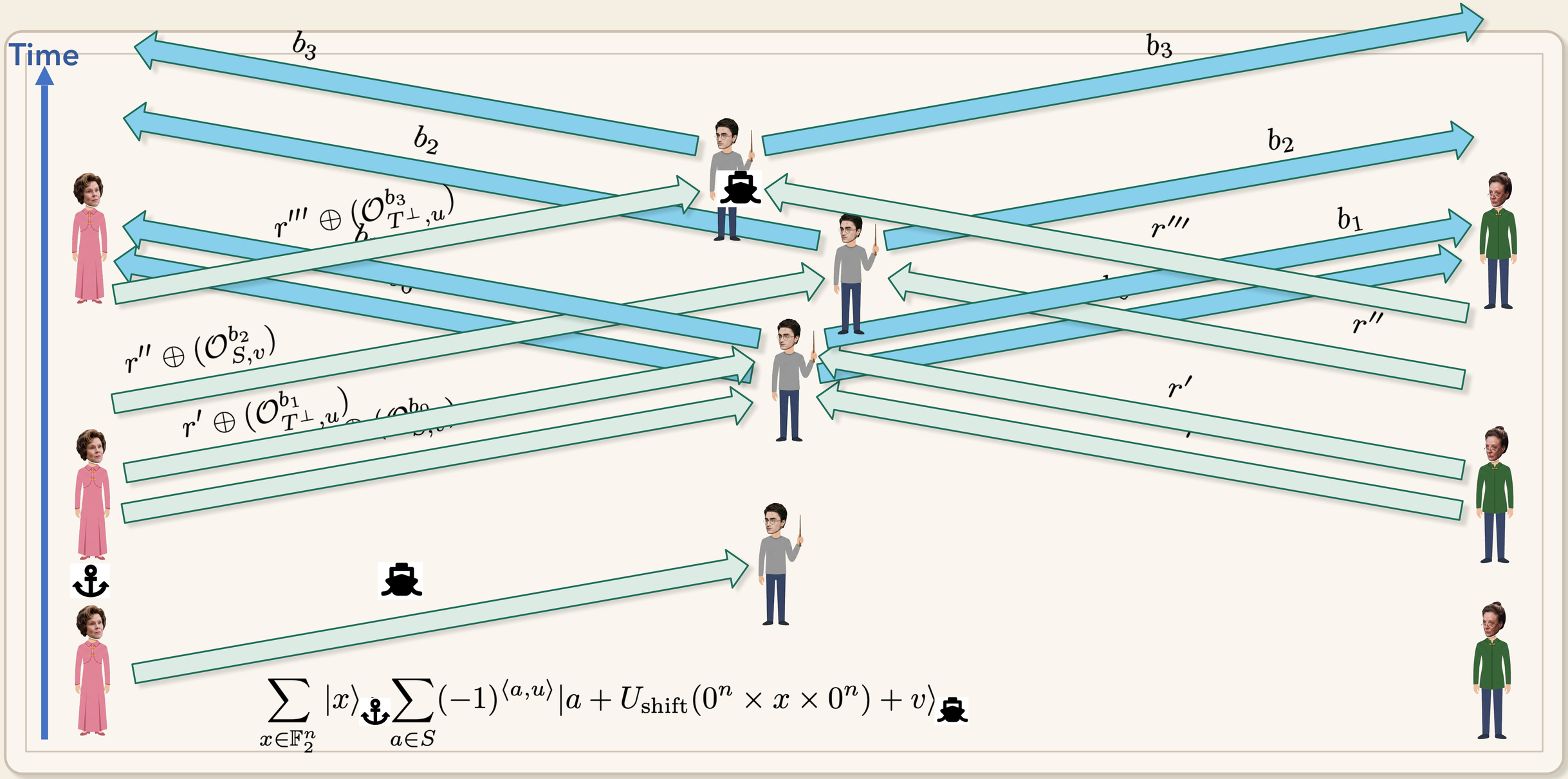

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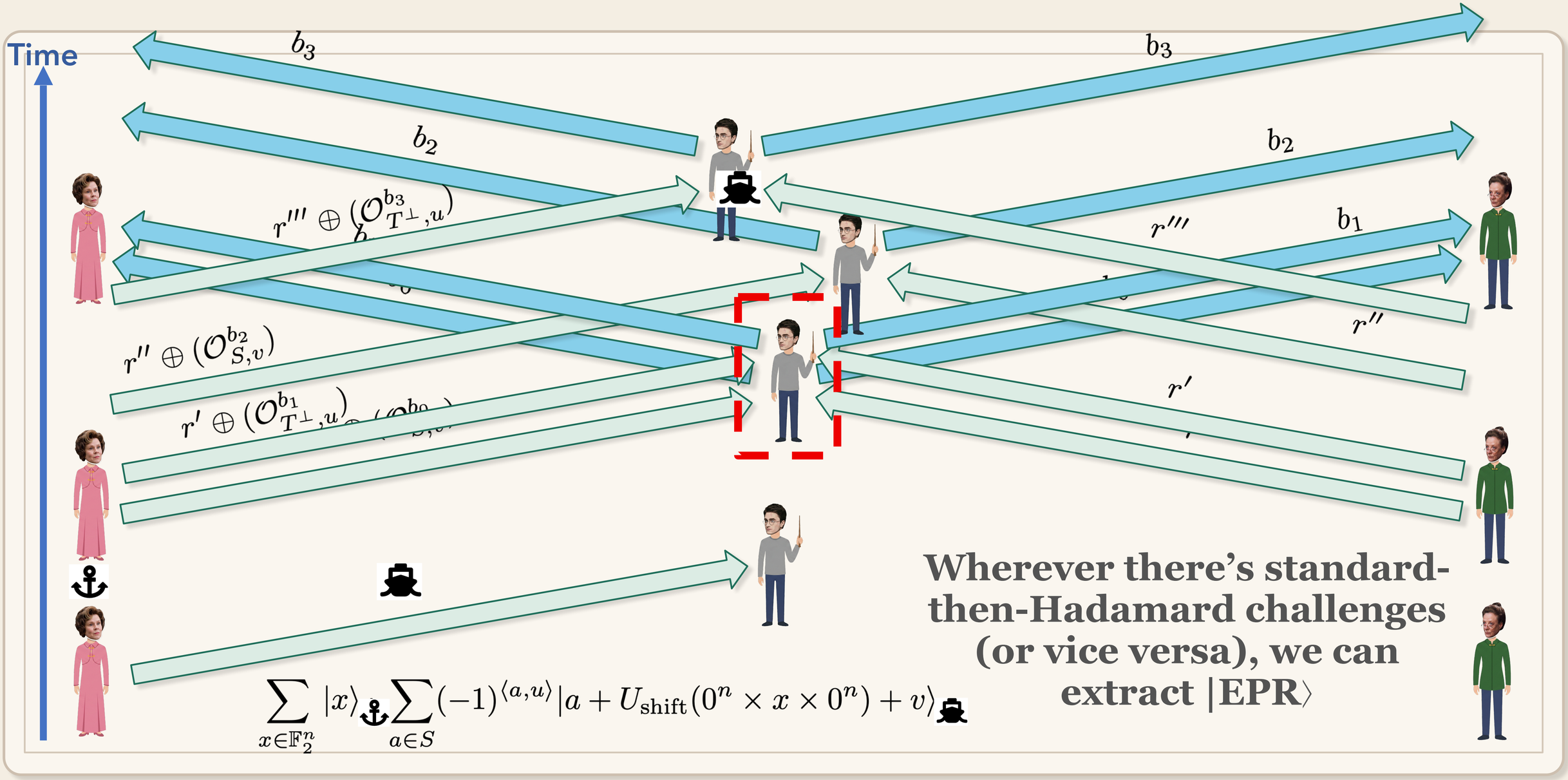

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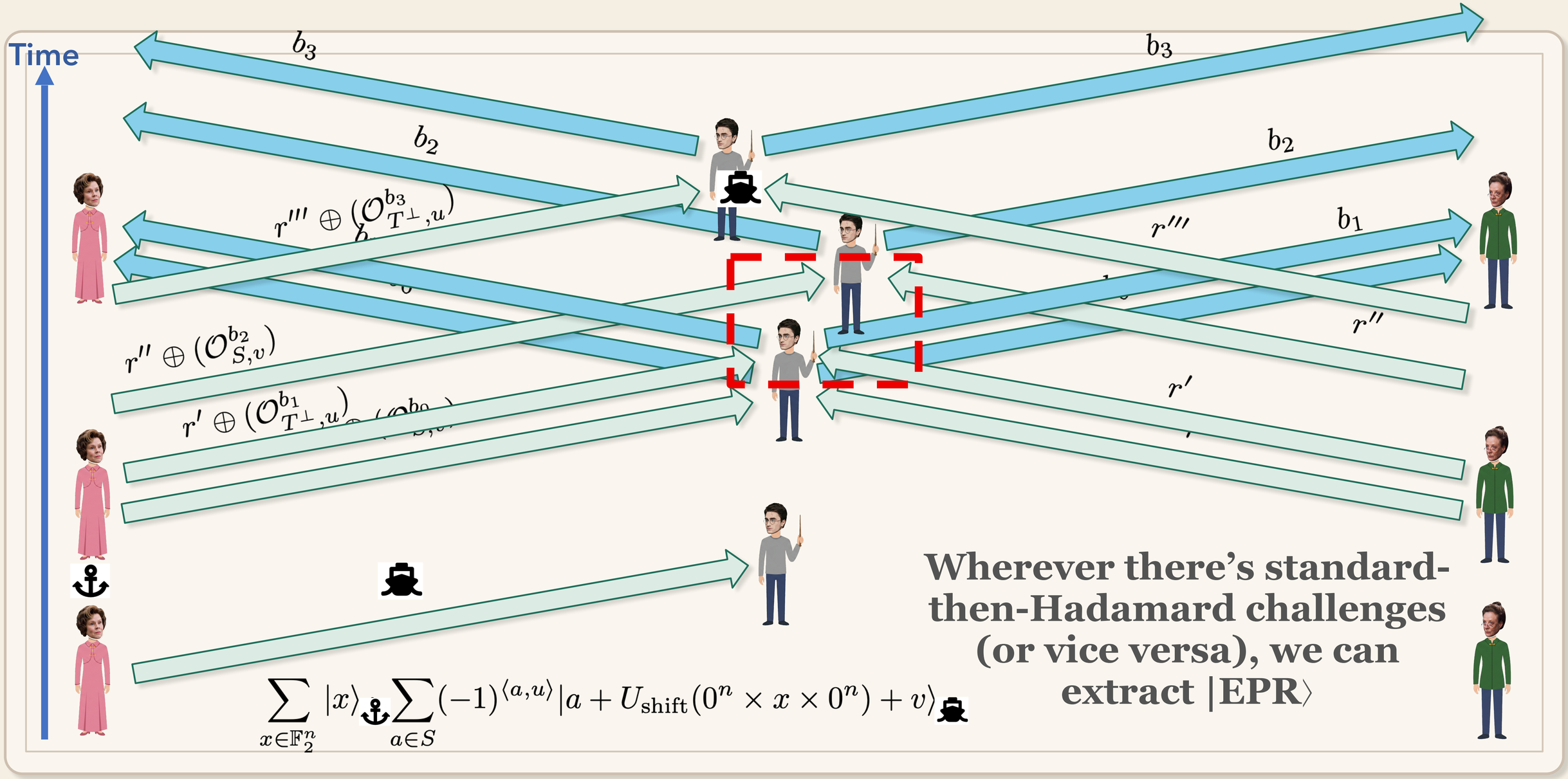

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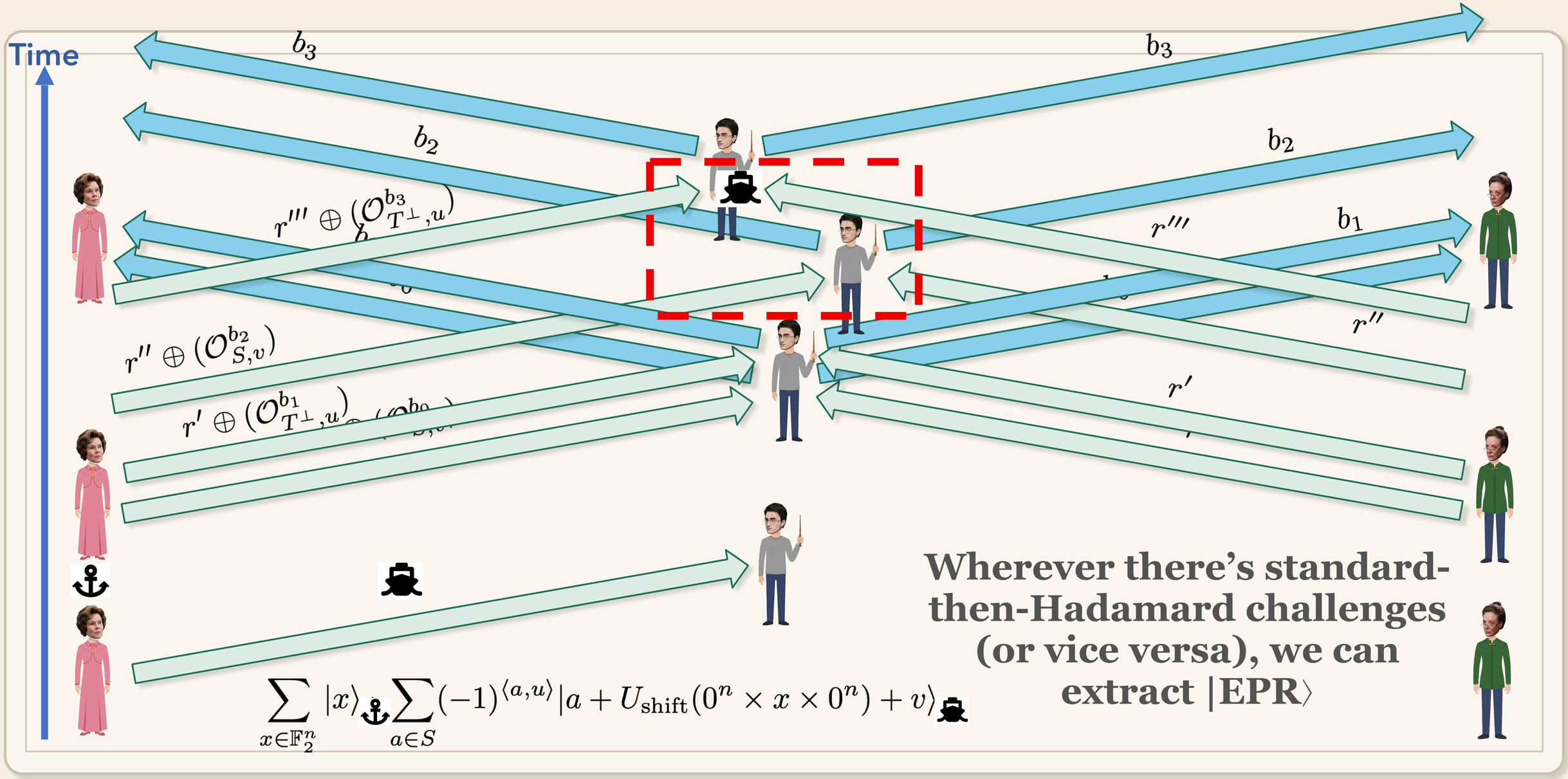

Towards a Stronger Position-Based Authentication

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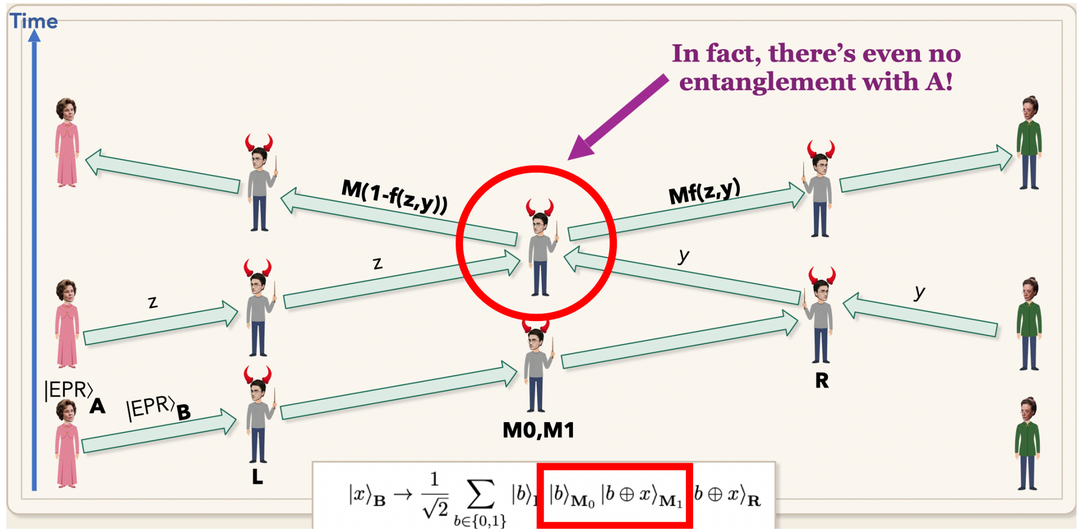
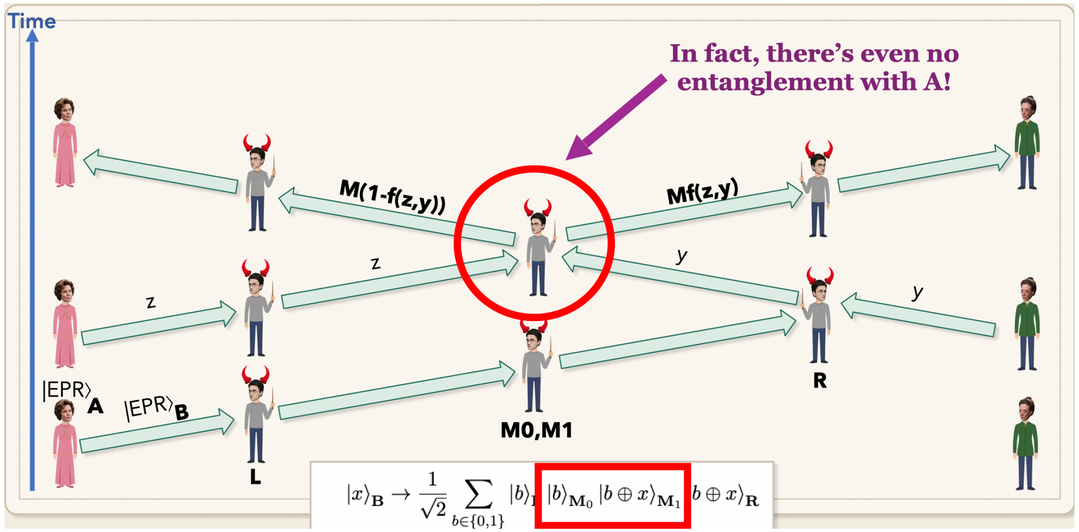


When position-based authentication is built from f-BB84, there is a non-local strategy which succeeds.

Towards a Stronger Position-Based Authentication



When position-based authentication is built from f-BB84, there is a non-local strategy which succeeds.

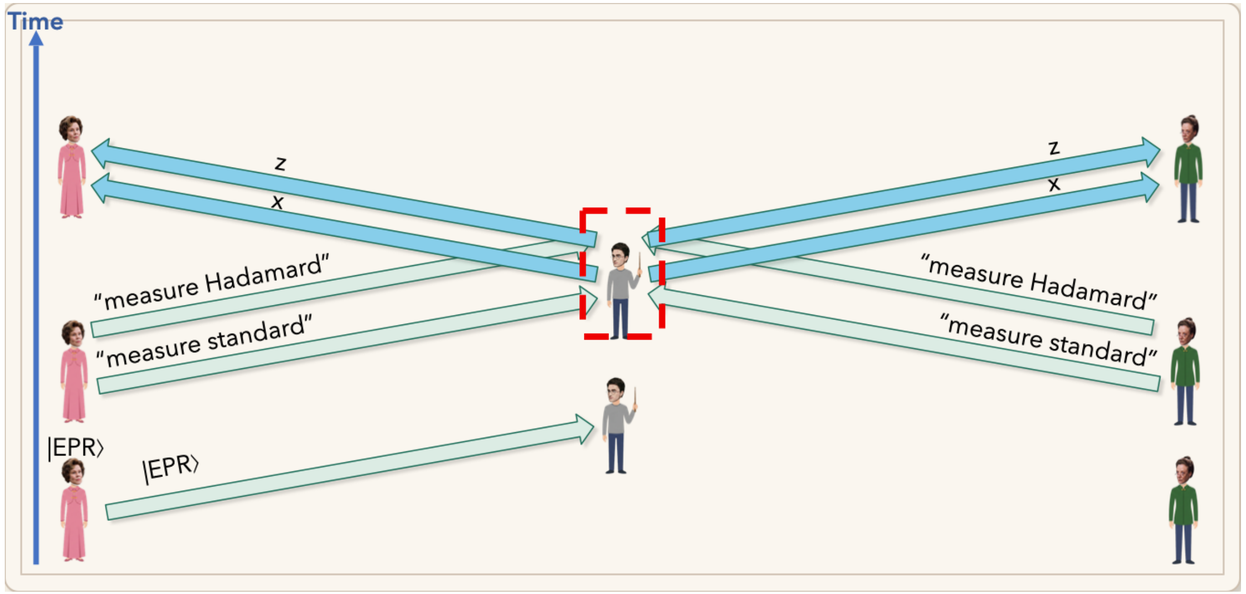
Can this be patched with localization?

Towards a Stronger Position-Based Authentication

Functionality Localization

Towards a Stronger Position-Based Authentication

Functionality Localization



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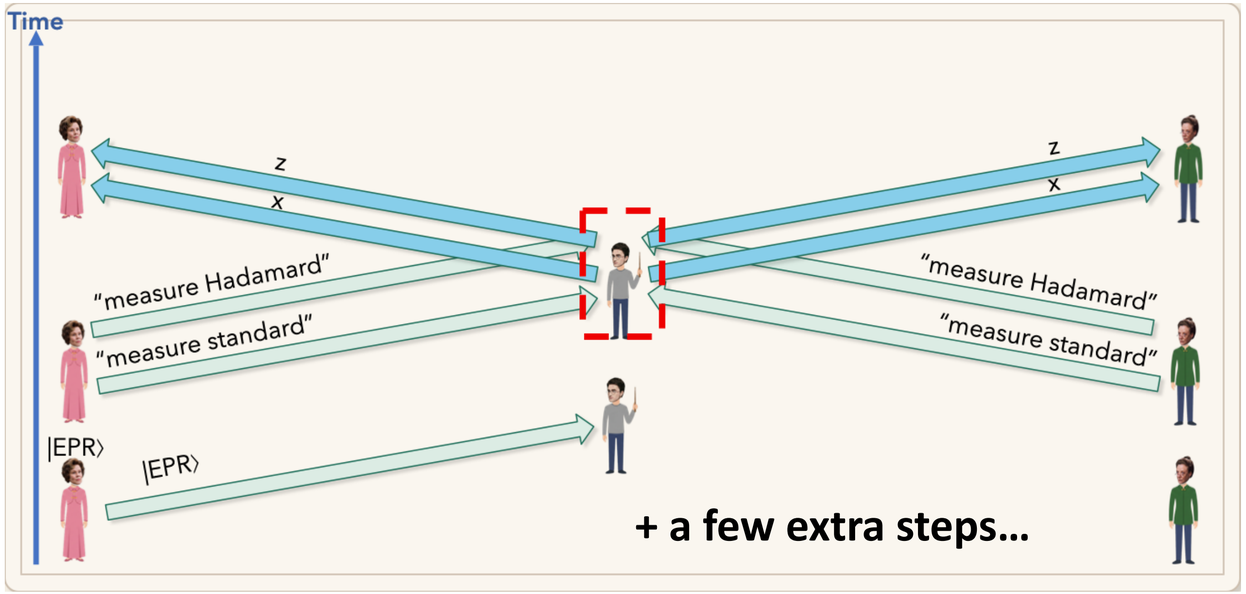

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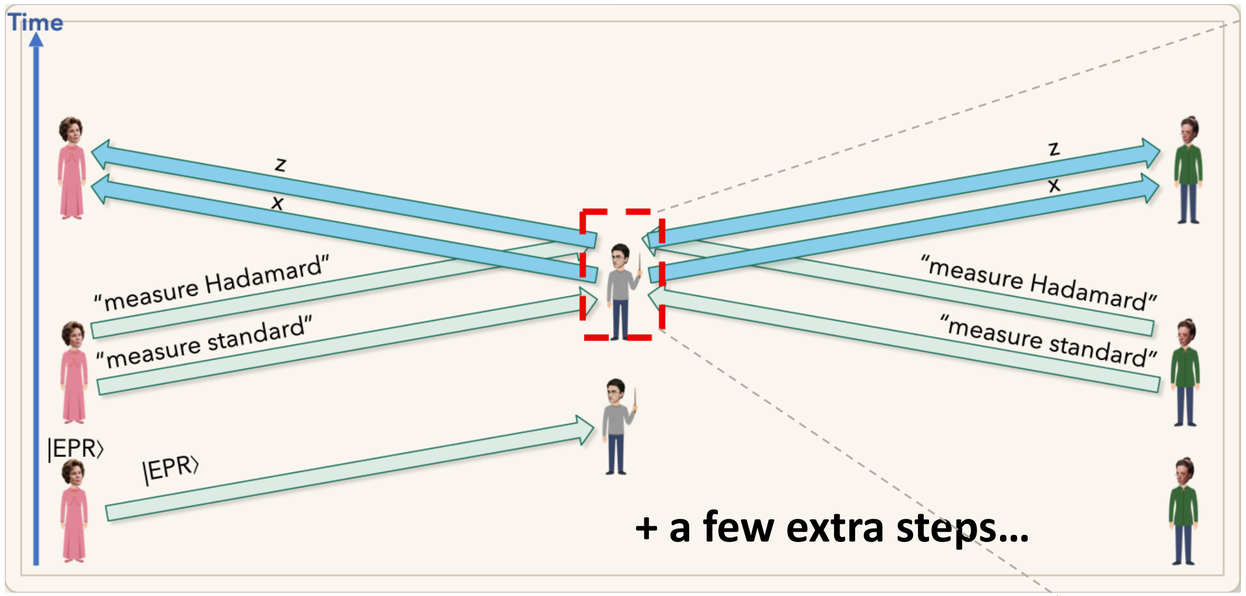

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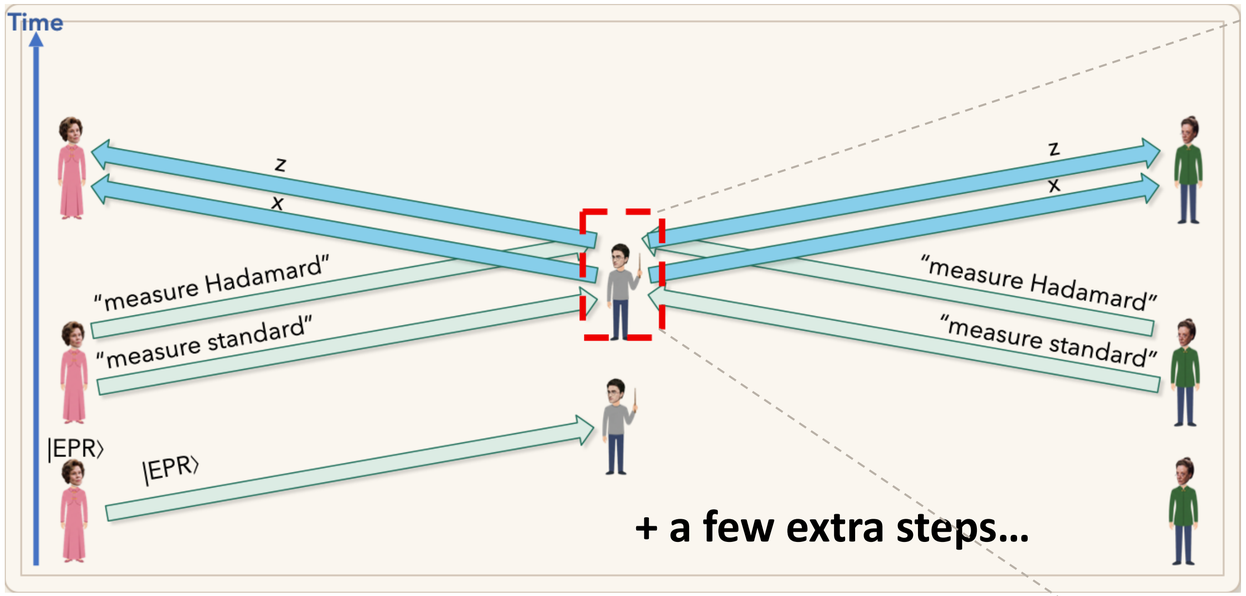

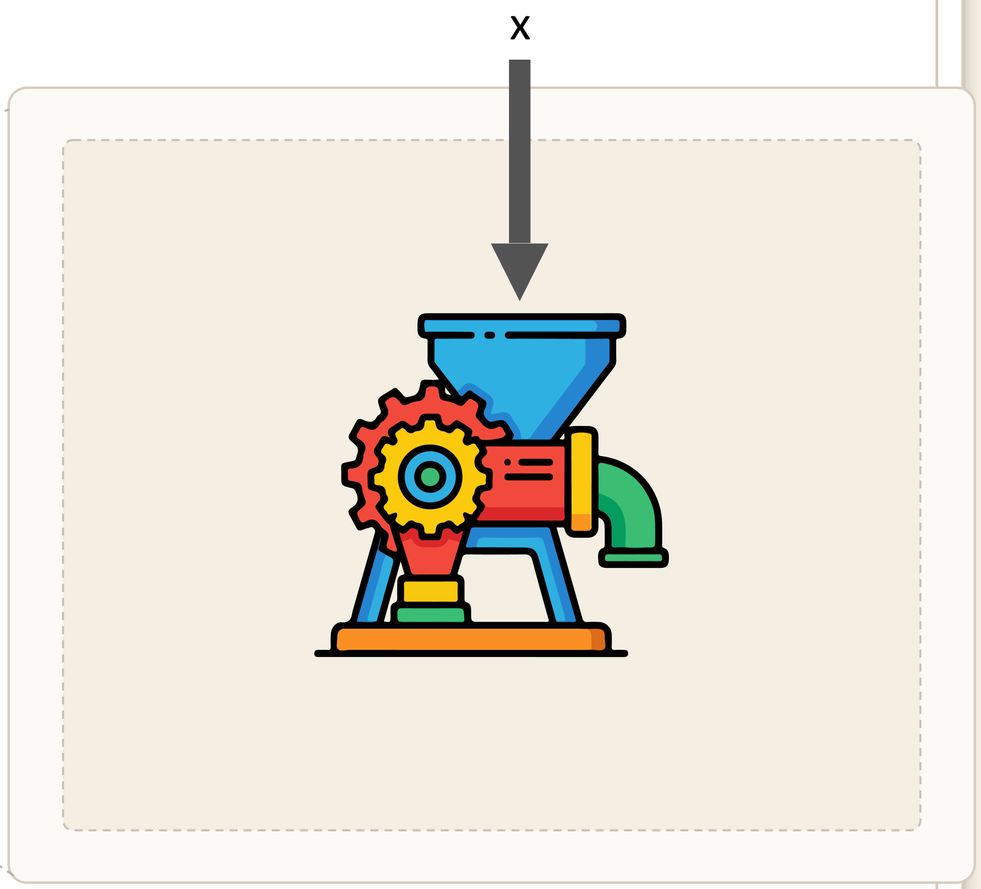

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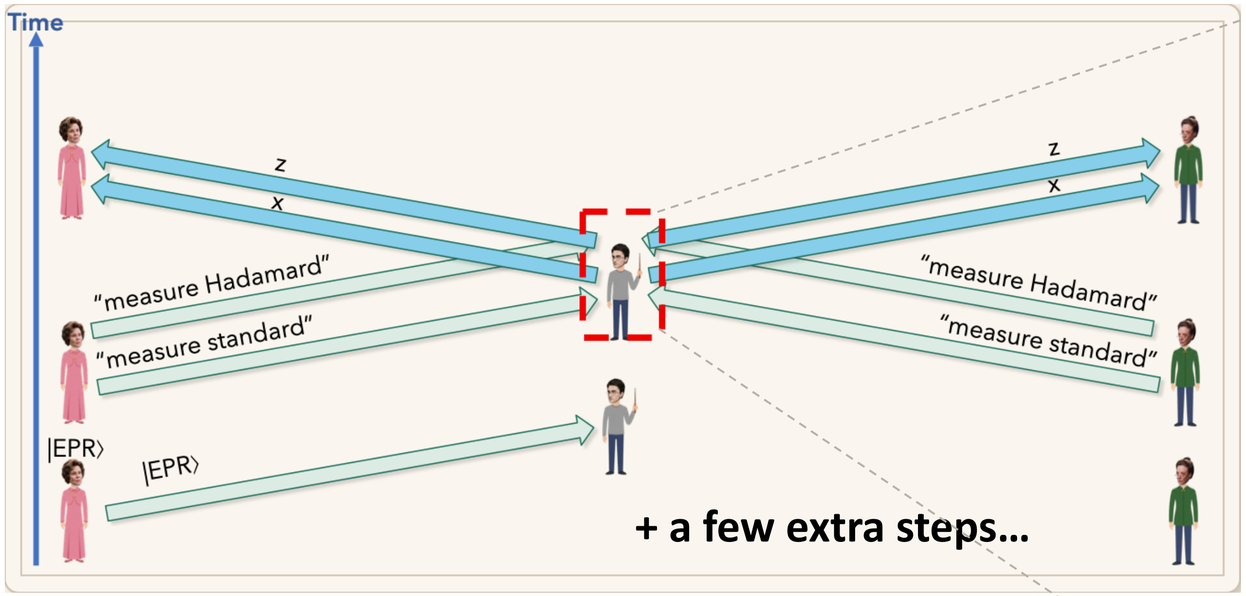

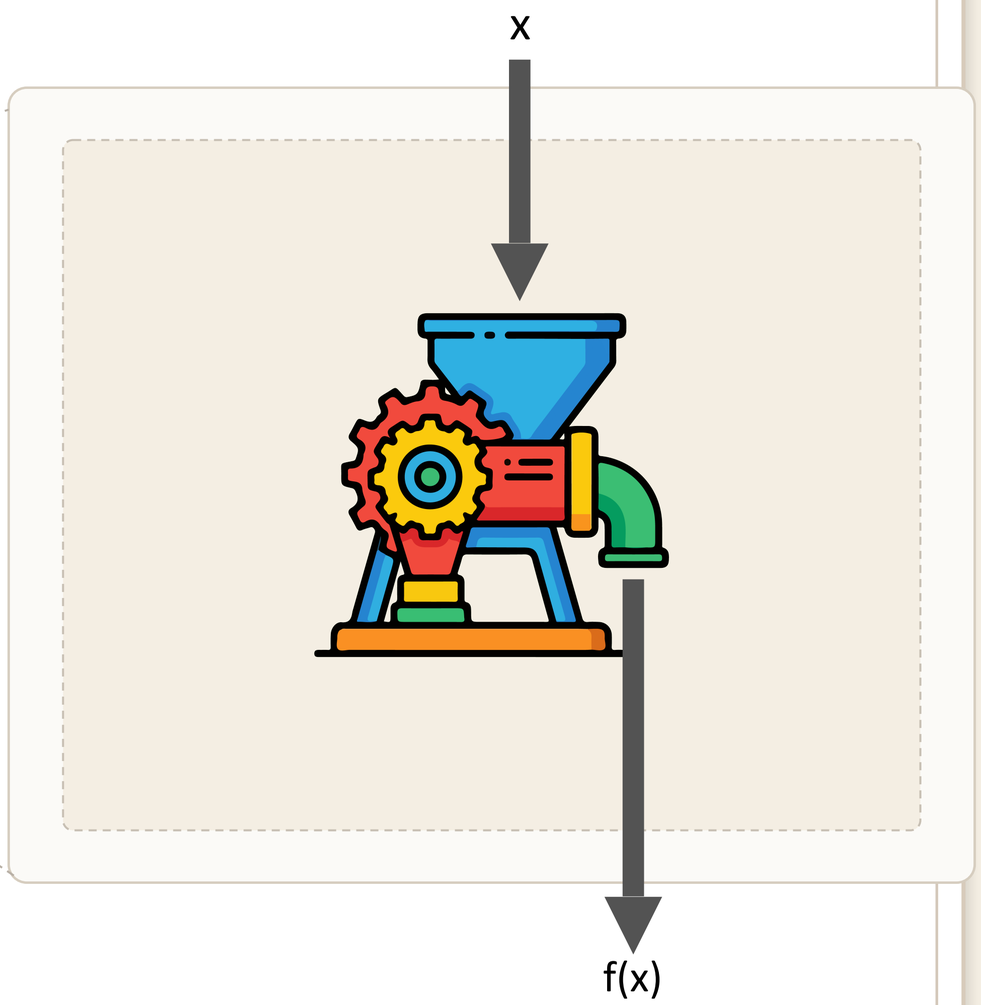

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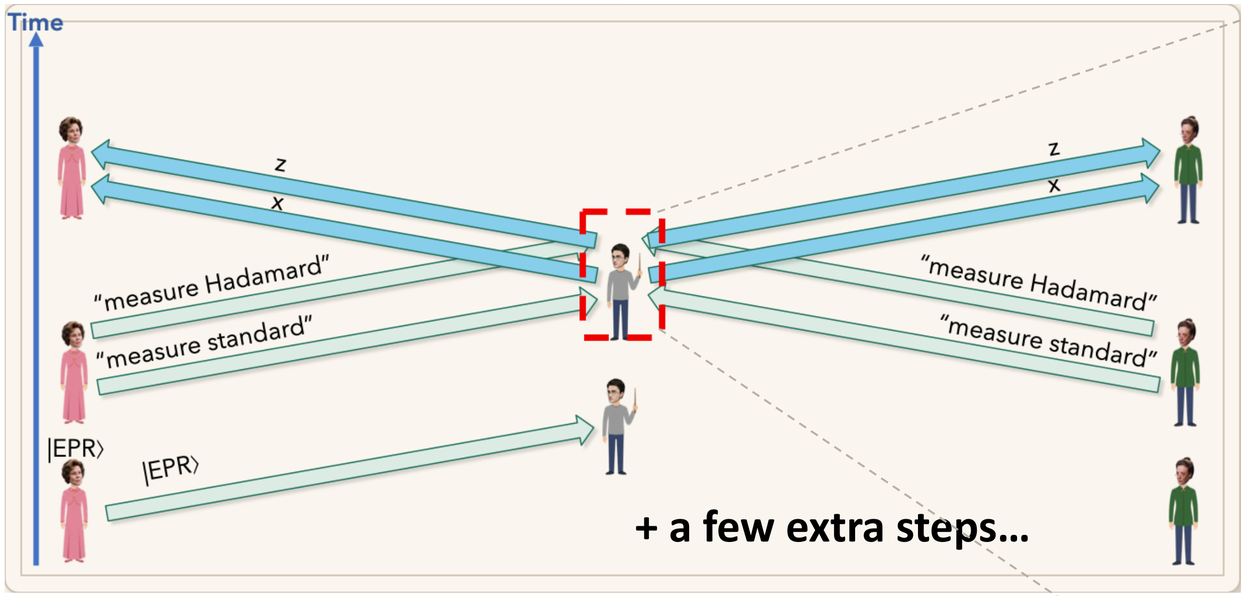

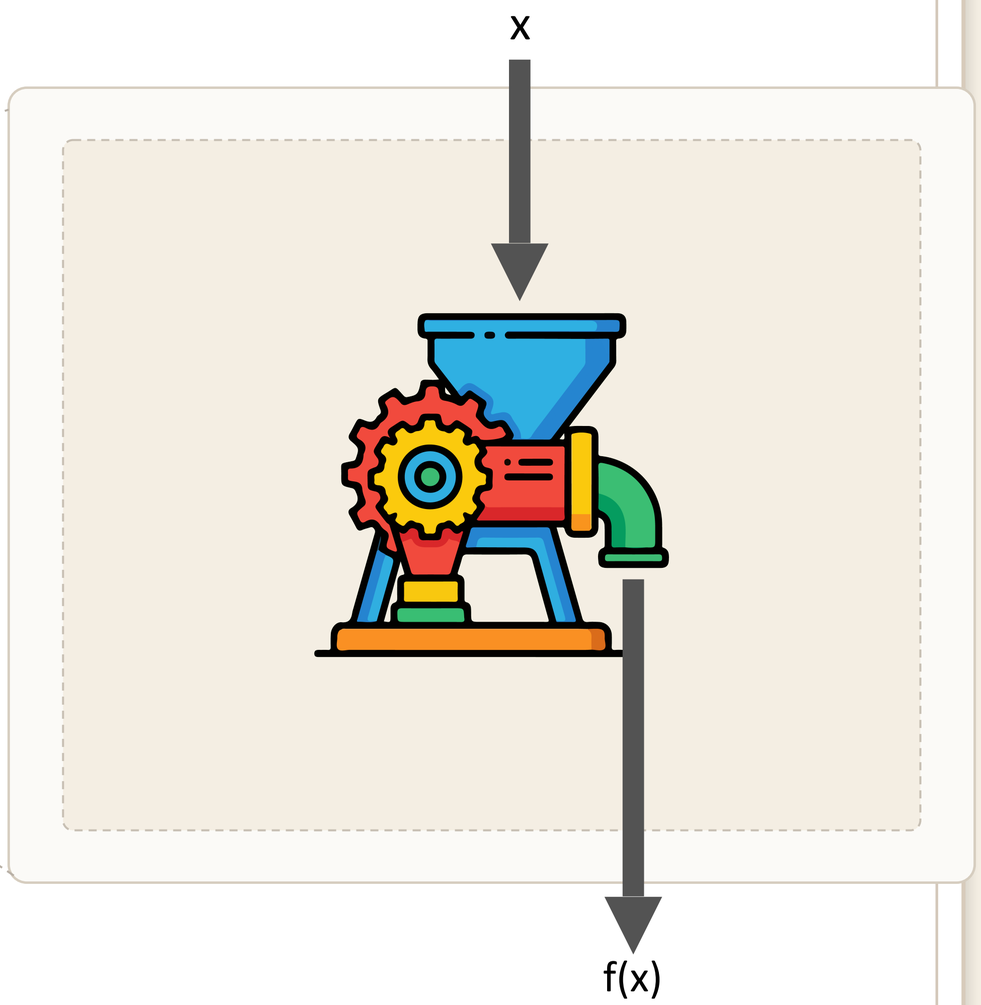

Can localize a function-like state.
If it's copy protected, we localize the *whole functionality*…

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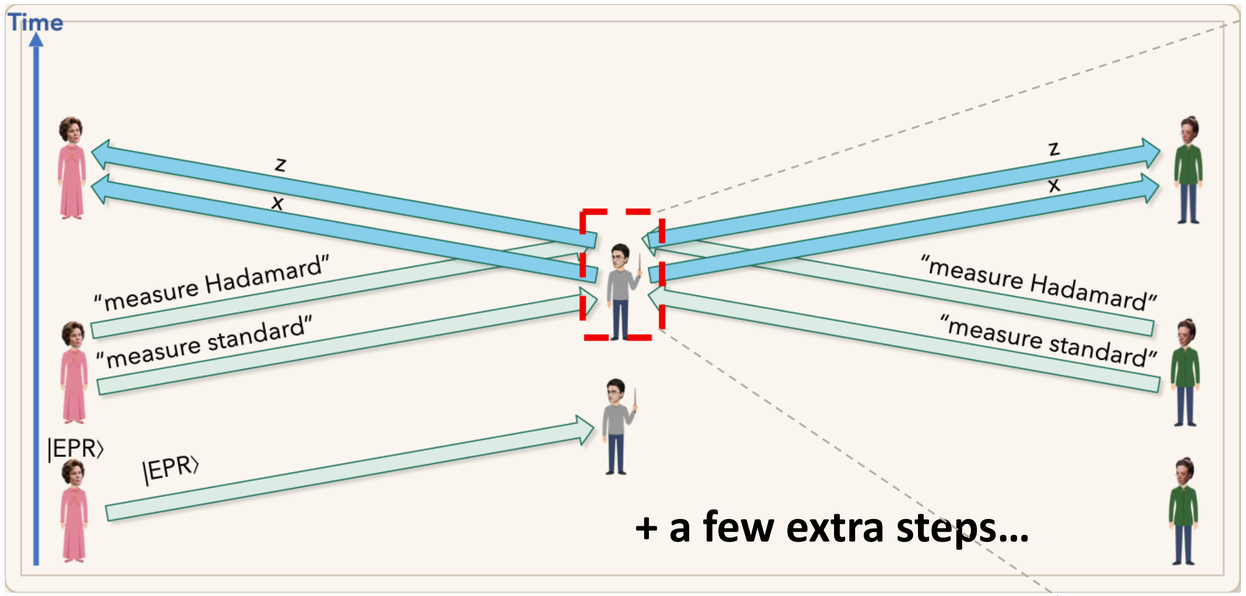
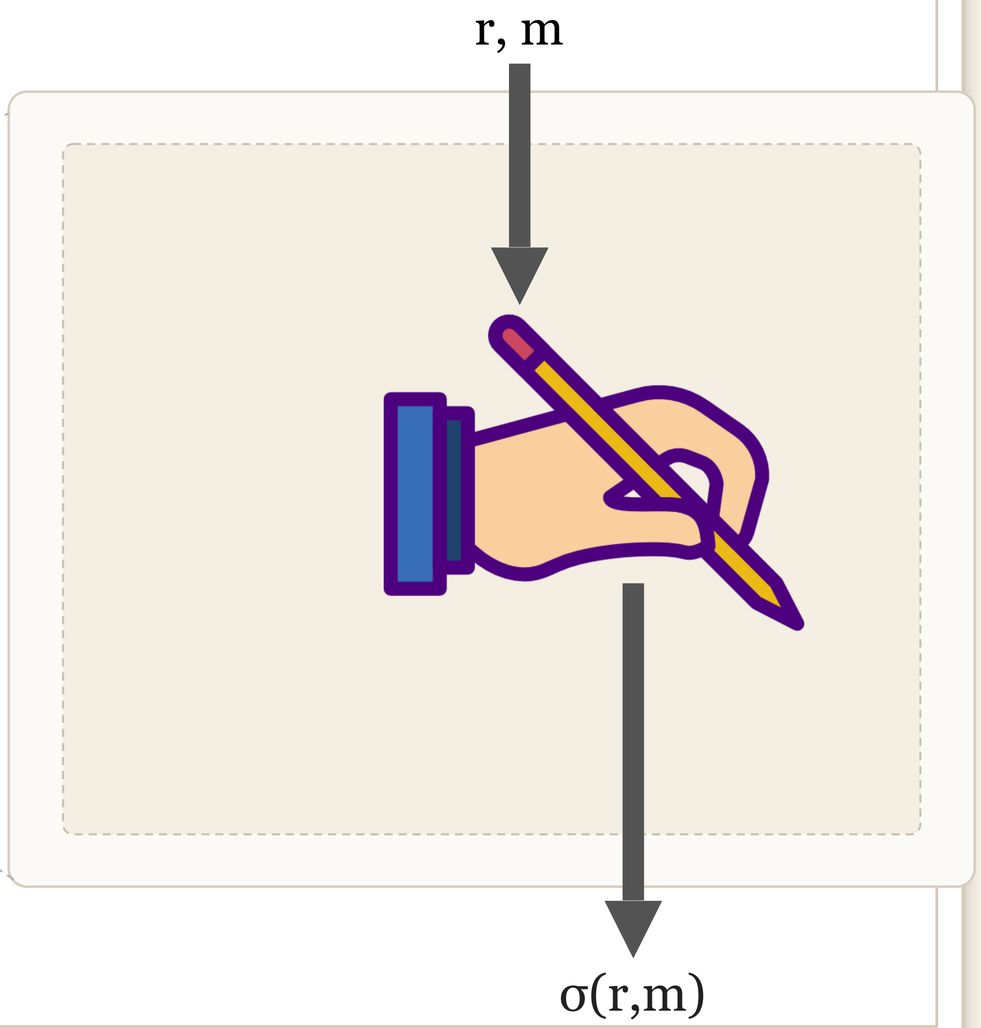
Functionality Localization





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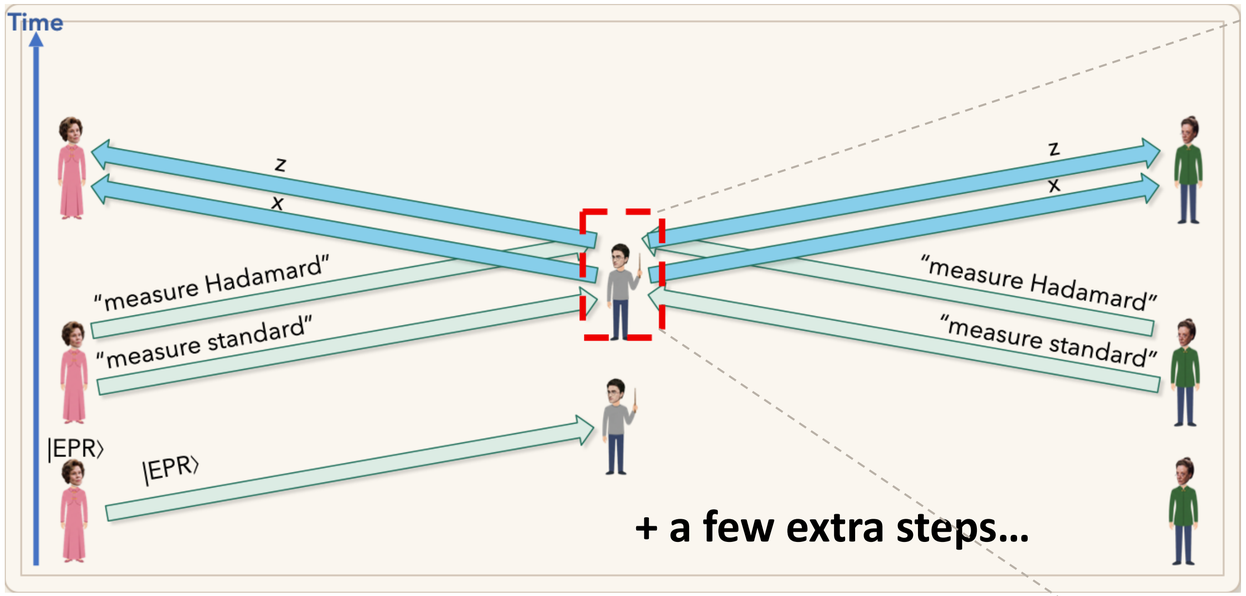
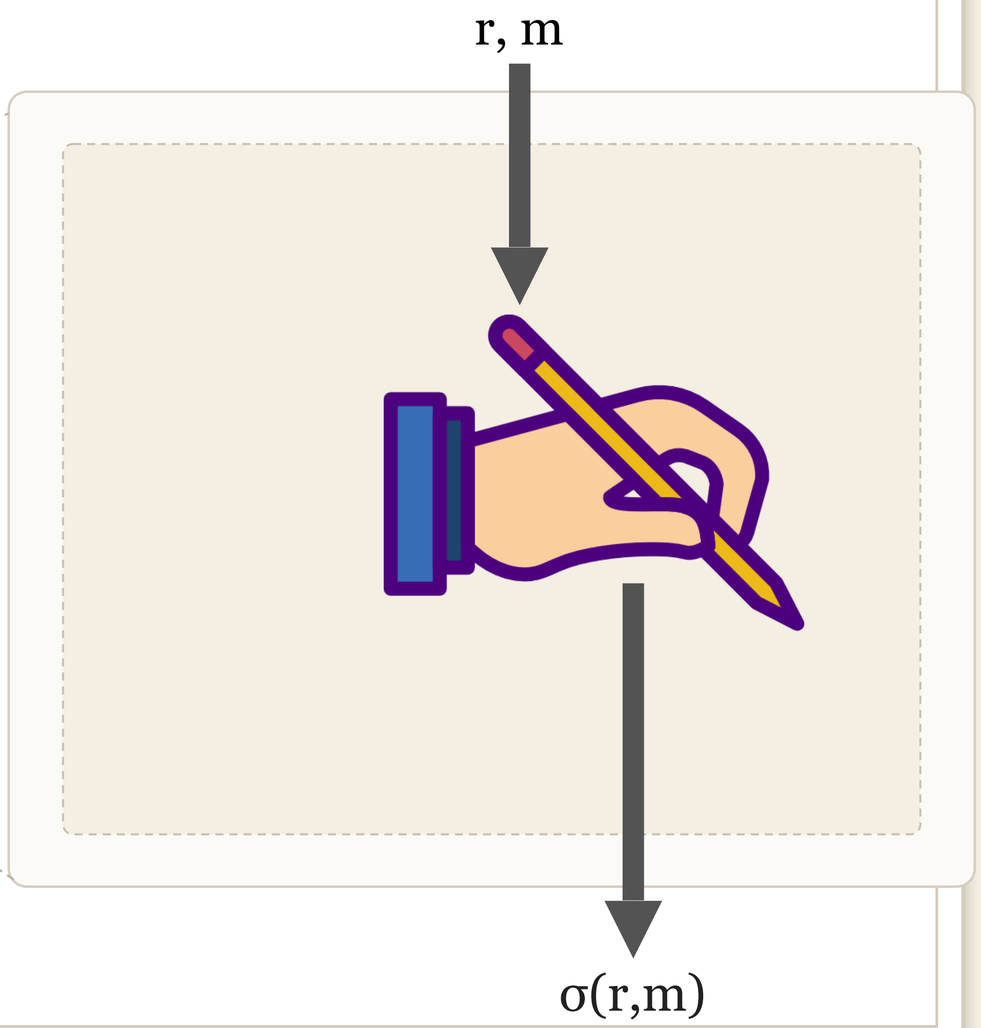


1. **Copy-protect a digital signature functionality into an unclonable state $|\Psi\rangle$**



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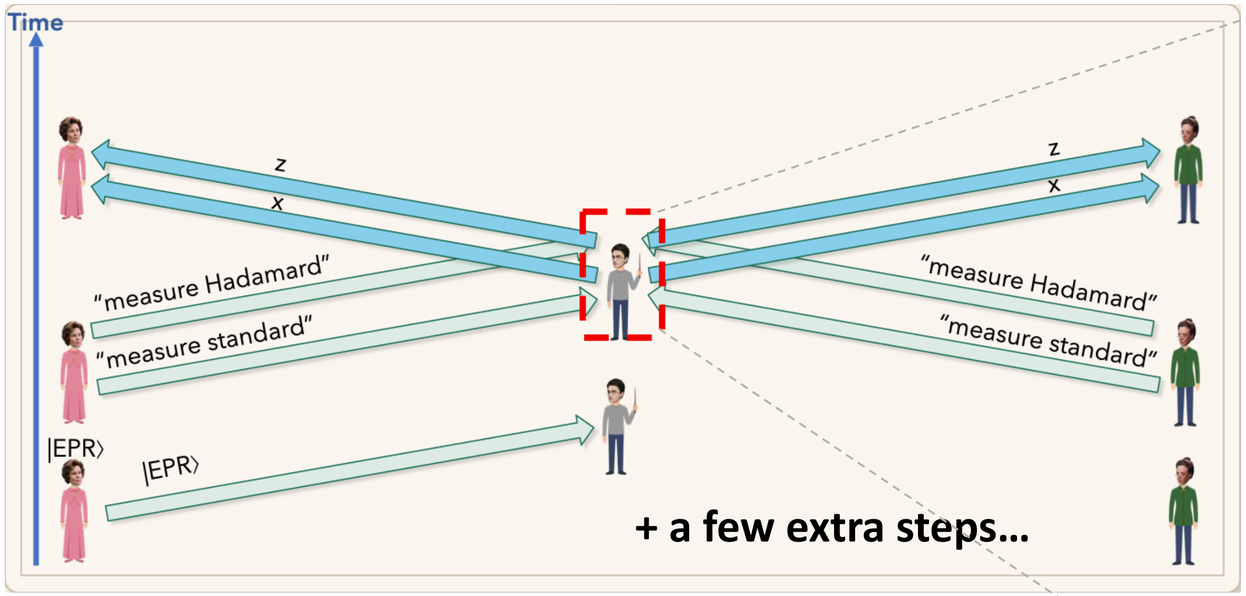

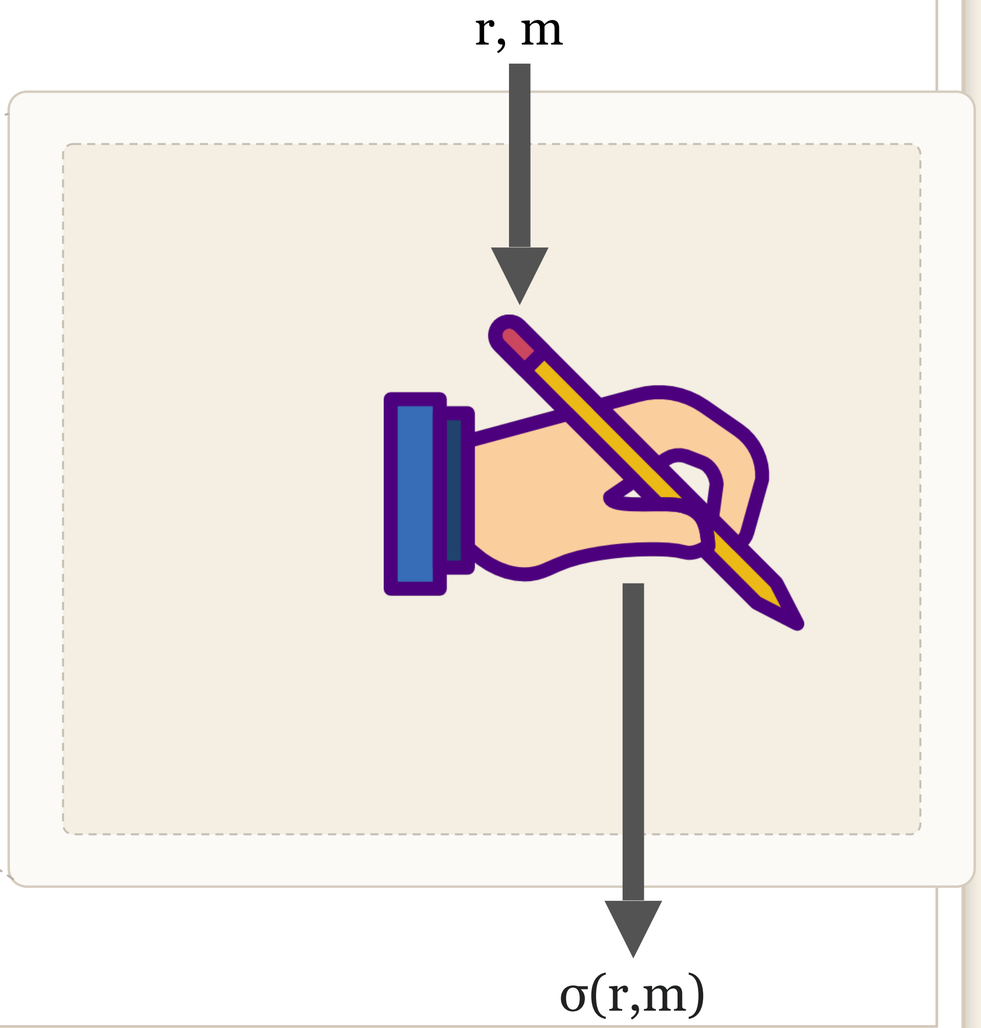

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Towards a Stronger Position-Based Authentication

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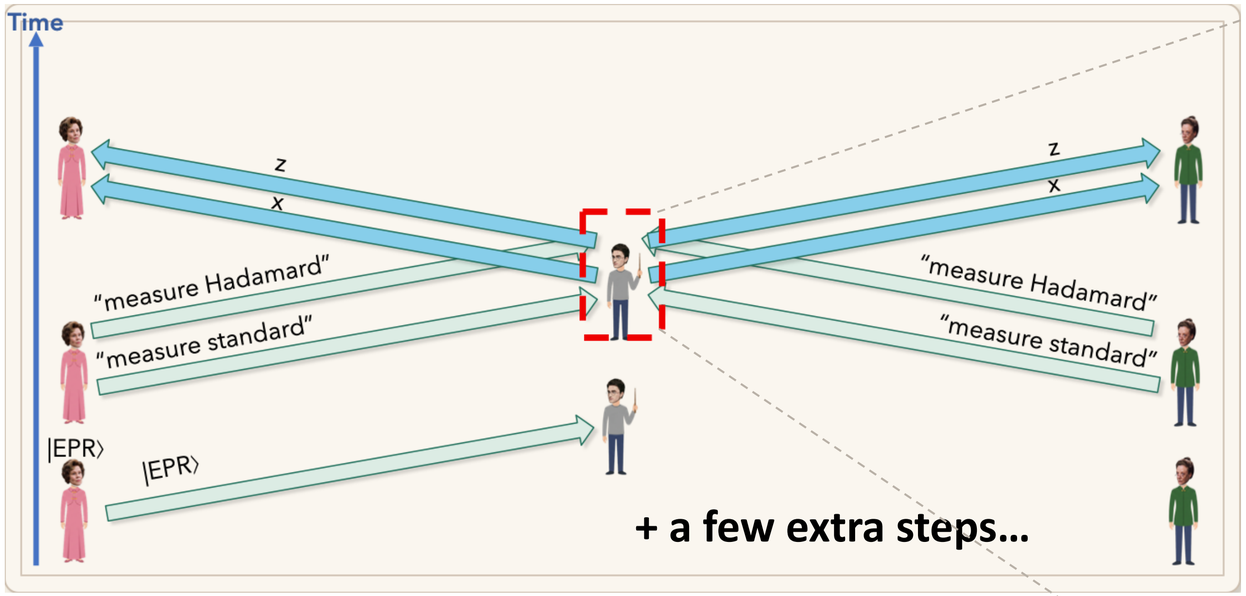

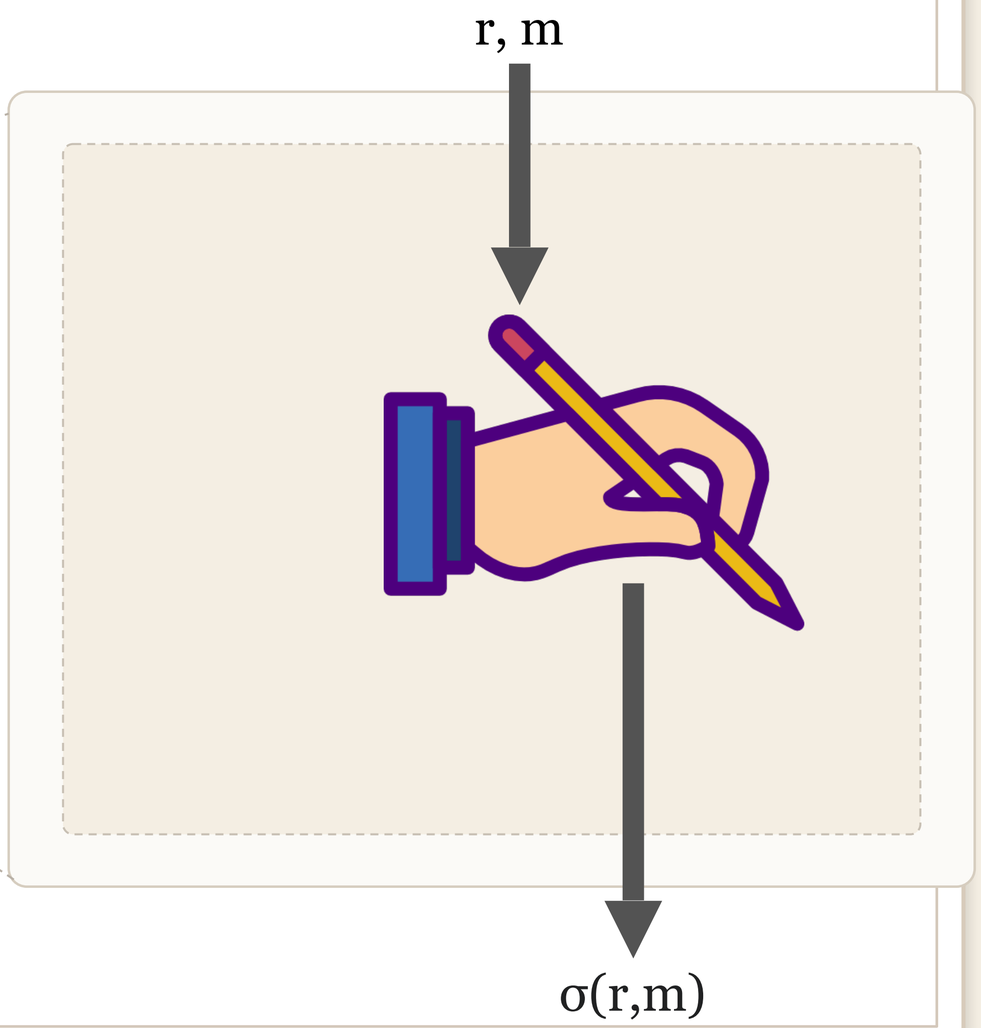

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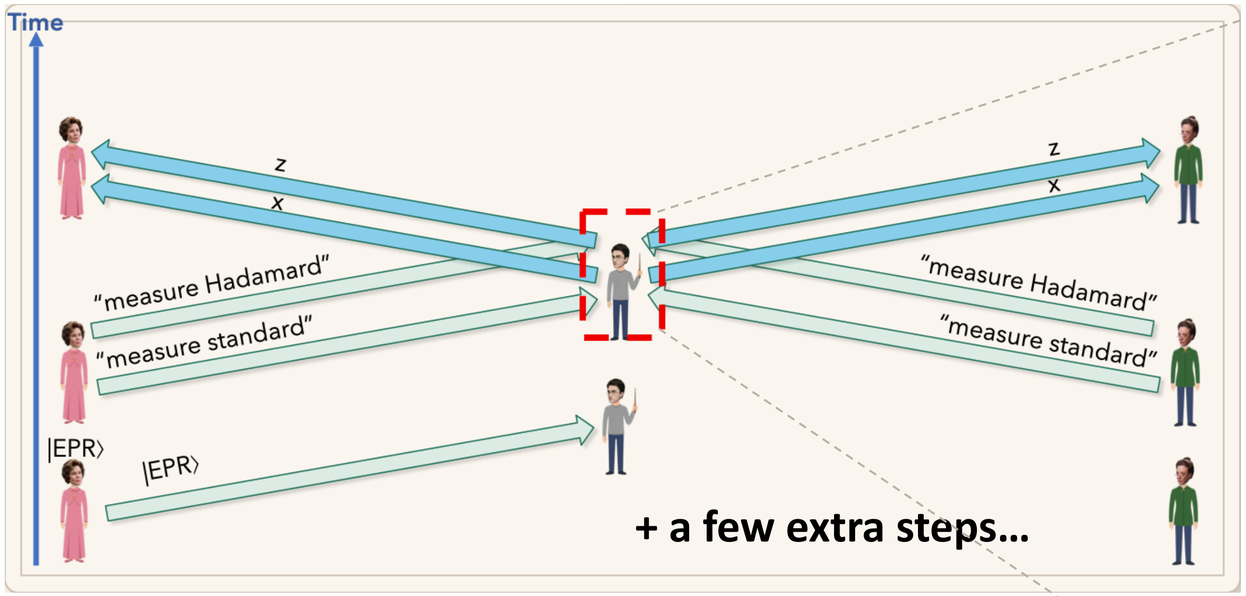

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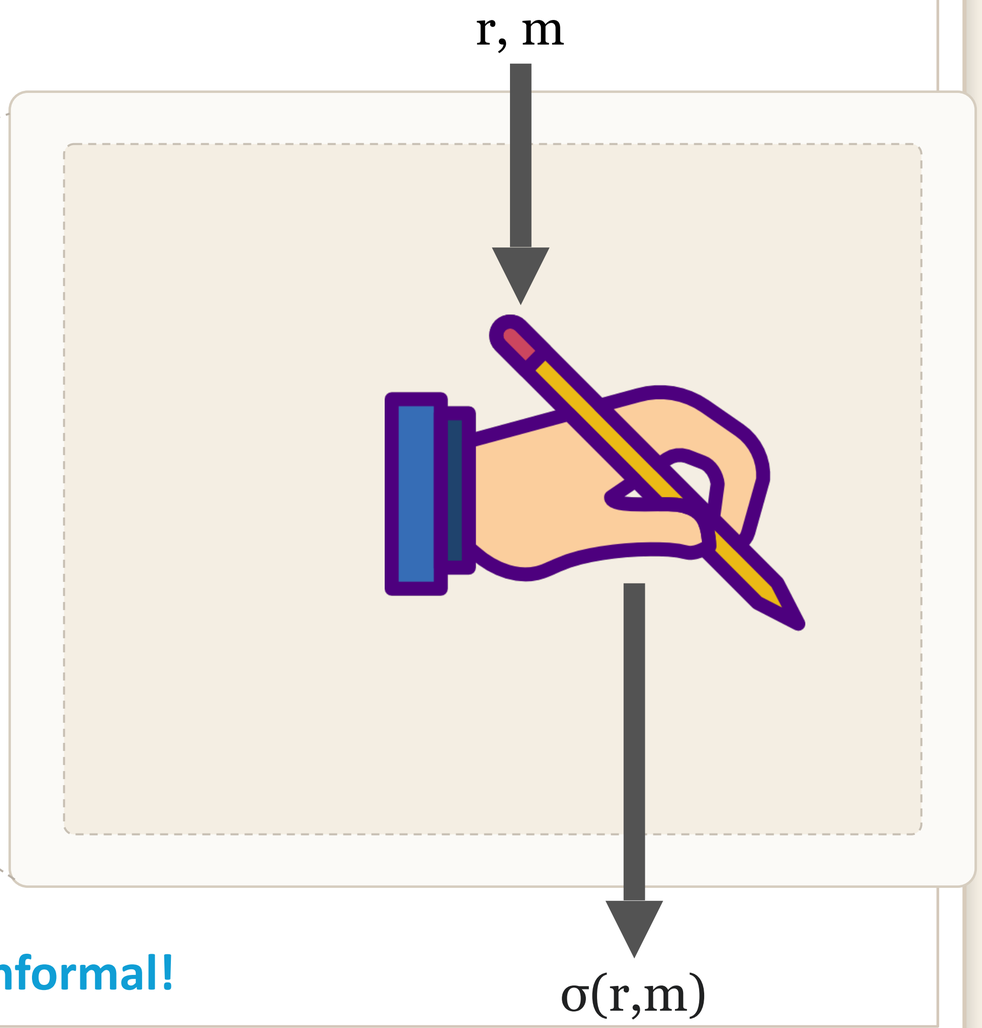

Informal!

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And more (read our paper! There are lots of pretty pictures 😊)

Thank you!